Muon anomalous magnetic moment

Ivan Logashenko (BINP)

Moscow
International
School of Physics

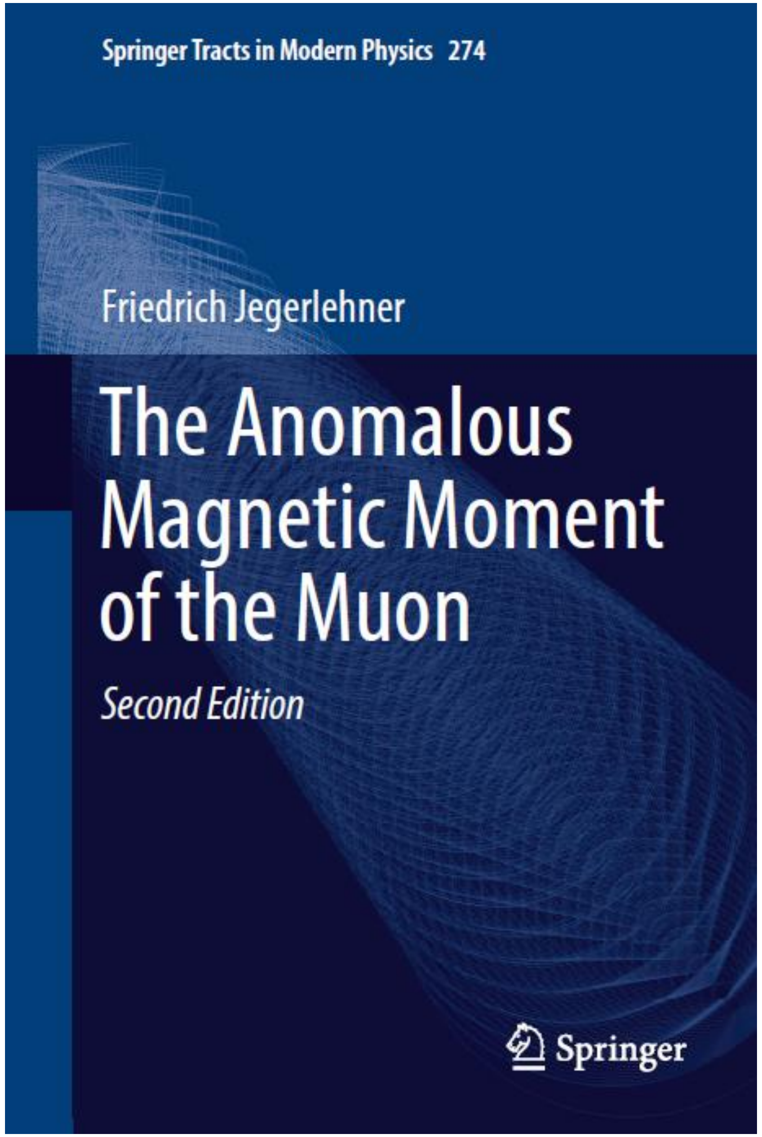
MISP-2024

From F.Jegerlehner book

The closer you look the more there is to see

“It seems to be a strange enterprise to attempt write a physics book about a single number. It was not my idea to do so, but why not. In mathematics, maybe, one would write a book about π. Certainly, **the muon’s anomalous magnetic moment is a very special number and today reflects almost the full spectrum of effects incorporated in today’s Standard Model (SM)** of fundamental interactions, including the electromagnetic, the weak and the strong forces….”

693 pages book on muon (g-2)!


Springer Tracts in Modern Physics 274

Friedrich Jegerlehner

The Anomalous Magnetic Moment of the Muon

Second Edition

Springer

Introduction

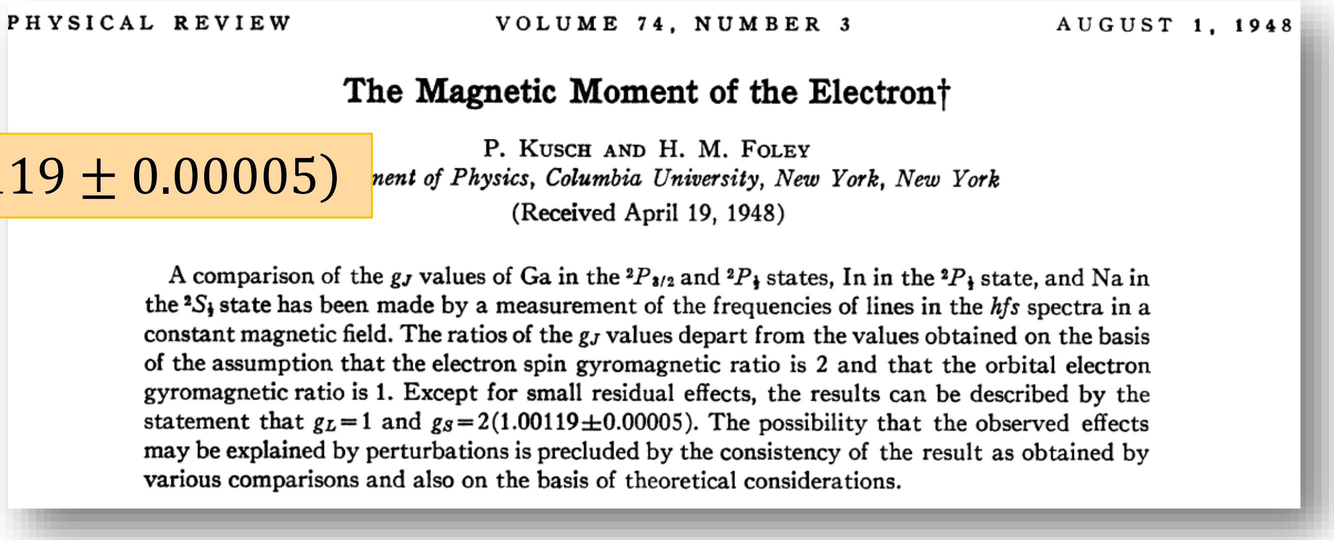
Gyromagnetic factor

- The magnetic moment of the particle relates to its spin angular momentum via the gyromagnetic factor, g:

$$\vec{\mu}_S = g\frac{e}{2m}\vec{S}$$

- In Dirac theory, point-like, spin ½ particle has $g = 2$ exactly
- Experimental values:

$$g_s \approx 2(1.00119 \pm 0.00005)$$



PHYSICAL REVIEW VOLUME 74, NUMBER 3 AUGUST 1, 1948

The Magnetic Moment of the Electron†

P. KUSCH AND H. M. FOLEY
ment of Physics, Columbia University, New York, New York
(Received April 19, 1948)

A comparison of the g_J values of Ga in the $^2P_{3/2}$ and $^2P_{\frac{1}{2}}$ states, In in the $^2P_{\frac{1}{2}}$ state, and Na in the $^2S_{\frac{1}{2}}$ state has been made by a measurement of the frequencies of lines in the *hfs* spectra in a constant magnetic field. The ratios of the g_J values depart from the values obtained on the basis of the assumption that the electron spin gyromagnetic ratio is 2 and that the orbital electron gyromagnetic ratio is 1. Except for small residual effects, the results can be described by the statement that $g_L=1$ and $g_S=2(1.00119\pm0.00005)$. The possibility that the observed effects may be explained by perturbations is precluded by the consistency of the result as obtained by various comparisons and also on the basis of theoretical considerations.

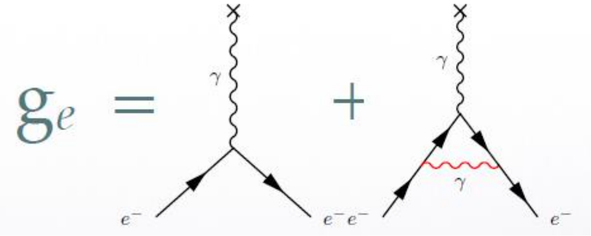
Anomalous magnetic moment

Anomalous magnetic moment: $a = (g-2)/2$

- Schwinger correction

Schwinger(1948), Feynman(1949)

$$\frac{g_e-2}{2} \approx \frac{\alpha}{2\pi}, \quad \alpha = \frac{e^2}{\hbar c} \approx 1/137$$



- Experimental values:

$$\left.\begin{array}{l} g_e \approx 2.002 \\ g_\mu \approx 2.002 \end{array}\right\} \text{point-like particles}$$

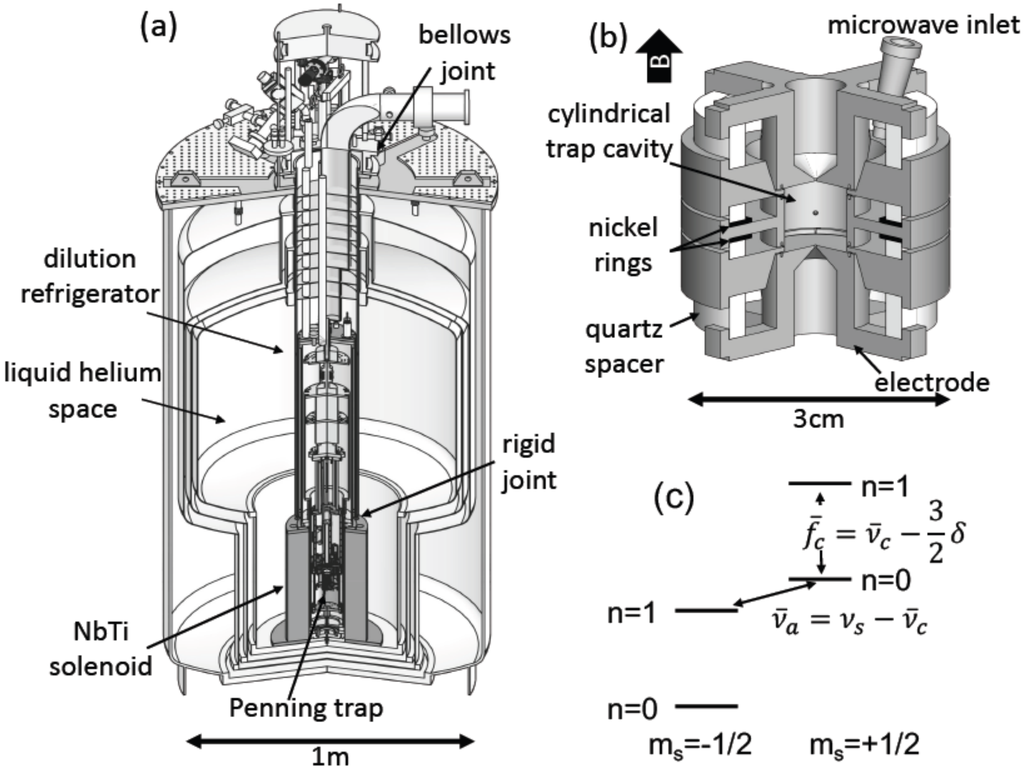
$$\left.\begin{array}{l} g_p \approx 5.586 \\ g_n \approx -3.826 \end{array}\right\} \text{compound particles}$$

$$a \approx 10^{-3}$$

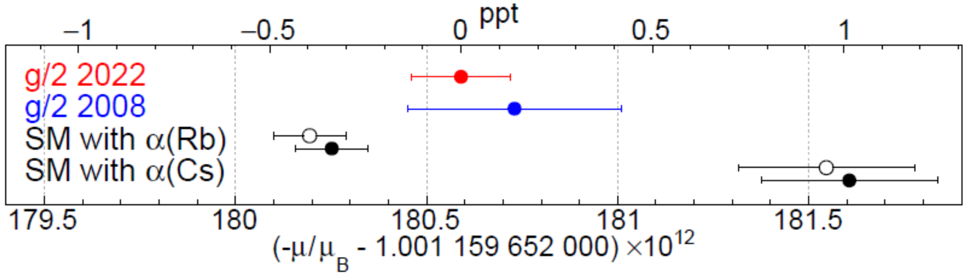
Anomalous magnetic moment of electron

The best precision is achieved for electrons (g-2). The value of a_e is used to get the determination of fine-structure constant α.

X. Fan, T. G. Myers,
B. A. D. Sukra, G. Gabrielse,
Phys.Rev.Lett.
130 (2023) 7, 071801



$$a_e = 1\,159\,652\,180\,59\,(13) \times 10^{-14}\ \ (0.11\ \text{ppb})$$



From electron to muon

The muon was discovered in a cosmic-rays in **1936** by Carl D. Anderson and Seth Neddermeyer.

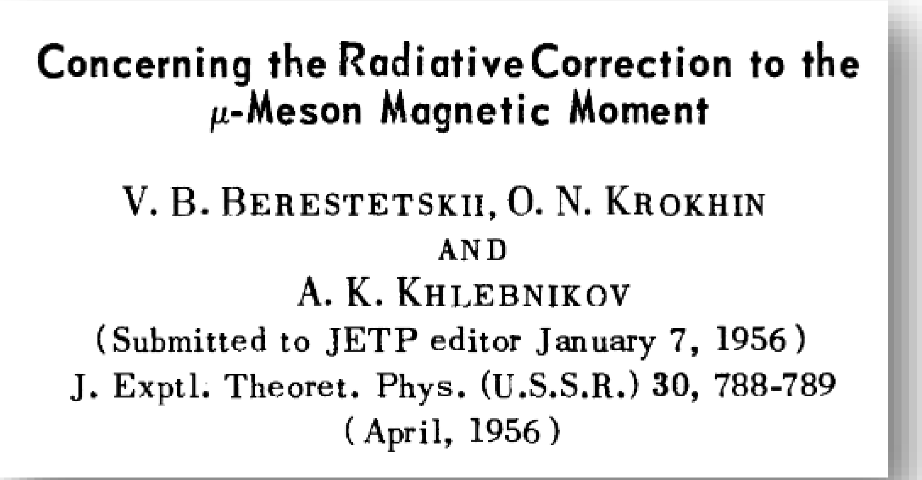
Eventually understood as heavy “electron”

I.Rabi: “Who ordered that?”

Why not to measure (g-2) for muon?

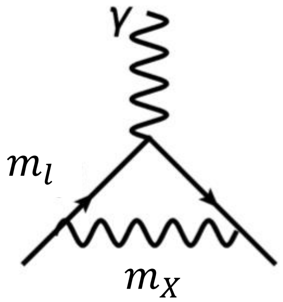
Berestetskii et al. (1956): since the muon is heavy, it is more sensitive to massive fields than the electron.

$$\left(\frac{m_\mu}{m_e}\right)^2 \approx 43000$$



Concerning the Radiative Correction to the μ-Meson Magnetic Moment

V. B. BERESTETSKII, O. N. KROKHIN
AND
A. K. KHLEBNIKOV
(Submitted to JETP editor January 7, 1956)
J. Exptl. Theoret. Phys. (U.S.S.R.) 30, 788-789
(April, 1956)



$$\Delta a \sim \left(\frac{m_l}{m_X}\right)^2$$

(g-2) of muon as a test of Standard Model

Idea of experiment: by comparing measured value of $\boldsymbol{a}$ with the theory prediction we probe extra contributions beyond theory expectations

$$a_\mu = a_\mu^{QED} + a_\mu^{Had} + a_\mu^{Weak} + a_\mu^{New\,Physics}$$

$$1{,}000{,}000 \quad : \quad 60 \quad : \quad 1.3 \quad : \quad \propto \left(m_\mu / m_X\right)^2$$

High precision is absolutely required:

$$a_\mu(strong)/a_\mu(QED) \approx 6 \times 10^{-5} \qquad a_\mu(weak)/a_\mu(QED) \approx 10^{-6}$$

Unique combination for $\boldsymbol{\mu}$: able to measure to high precision, able to calculate to high accuracy, sensitive to potential BSM contribution

$\boldsymbol{\tau}$ lepton: able to calculate to high accuracy, even more sensitive to potential BSM contribution, but cannot measure to high precision

Electron: able to measure to high precision, able to calculate to high accuracy, but not sensitive to potential BSM contribution

Muon – unique physics laboratory

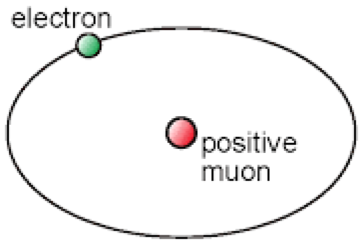
I.Rabi: “Who ordered that?”

Muon was invented so that physicists can understand the world

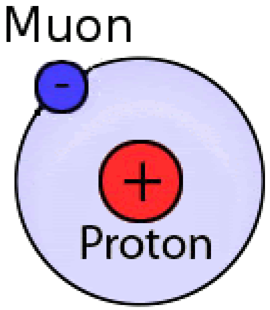
- Heavy: ~200 electron mass
  - Sensitivity to heavy fields
  - Compact wave function
- Does not have strong charge
  - No QCD complications
- Perfect lifetime: ~2.2 μs
  - Long enough to store and manipulate
  - Short enough to use decays
- Born polarized and decays keeps polarization information
- Forms muonium (pure QED atom) and muonic atoms

Muon

τ_μ (G_F), V-A structure, $(g-2)_\mu$, EDM, ultra rare decays $\mu \to e\gamma$, $\mu N \to eN$



Muonium



Muonic atom

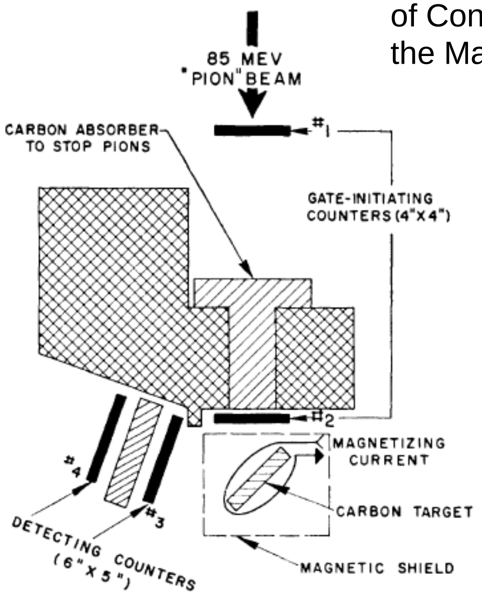
m_μ/m_e, μ_μ/μ_p, lamb shift, proton radius, bound QED test, muon capture

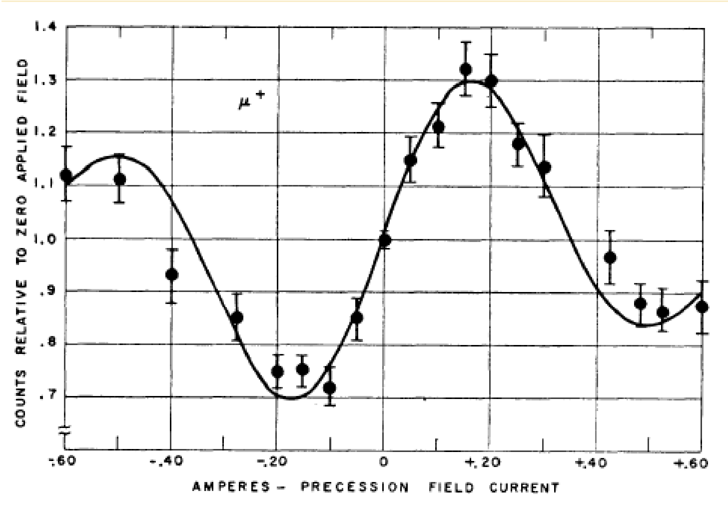
History of muon (g-2) measurements

Generations of a_μ measurements

R.L. Garwin, L.M. Lederman, M. Weinrich, Observations of the Failure of Conservation of Parity and Charge Conjugation in Meson Decays: the Magnetic Moment of a Free Muon, Phys. Rev. 105, 1415 (1957).

NEVIS (USA)





Muon is heavy electron! Parity is not conserved!

$$g_\mu(\text{эксп}) \quad = 2.00\ (10)$$

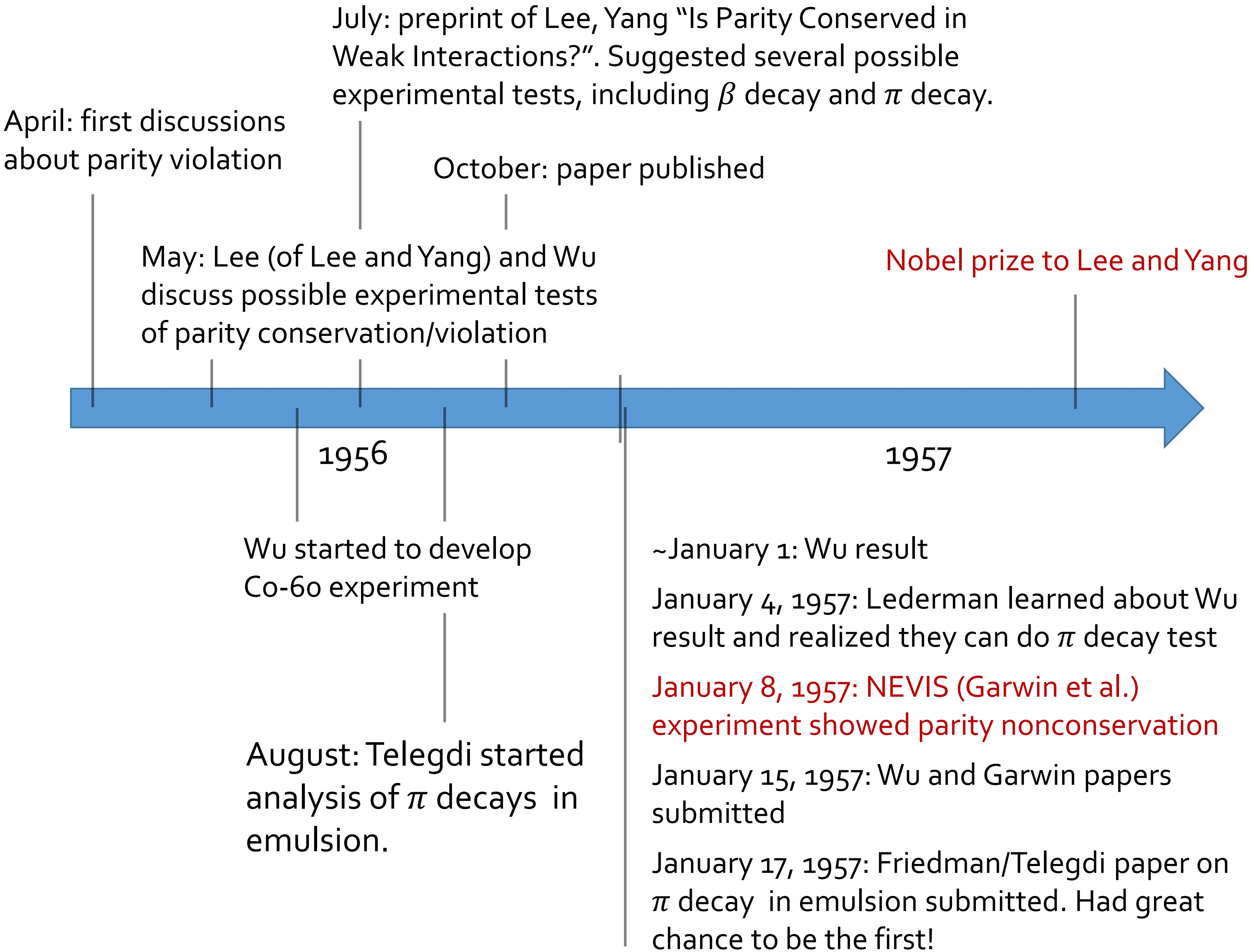
QED

Strong

Weak

Contributions of known interactions

Case study: how to get Nobel prize



April: first discussions about parity violation

May: Lee (of Lee and Yang) and Wu discuss possible experimental tests of parity conservation/violation

July: preprint of Lee, Yang "Is Parity Conserved in Weak Interactions?". Suggested several possible experimental tests, including β decay and π decay.

October: paper published

Wu started to develop Co-60 experiment

August: Telegdi started analysis of π decays in emulsion.

1956

1957

Nobel prize to Lee and Yang

~January 1: Wu result

January 4, 1957: Lederman learned about Wu result and realized they can do π decay test

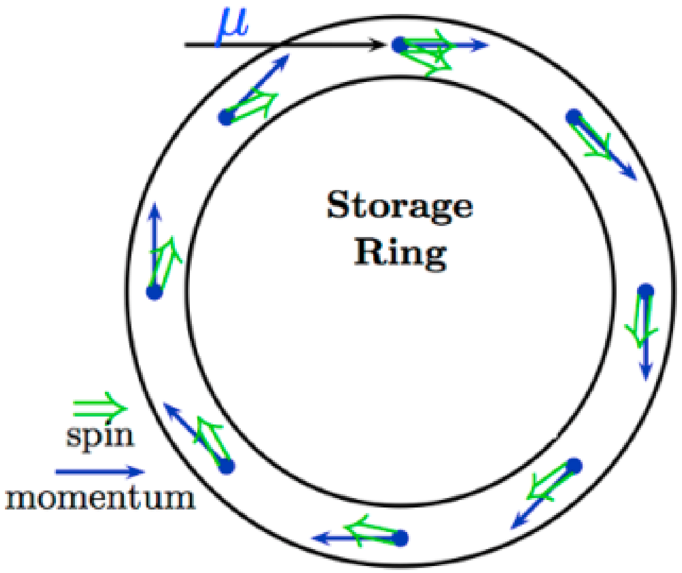
January 8, 1957: NEVIS (Garwin et al.) experiment showed parity nonconservation

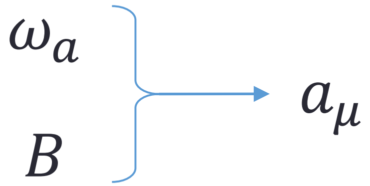
January 15, 1957: Wu and Garwin papers submitted

January 17, 1957: Friedman/Telegdi paper on π decay in emulsion submitted. Had great chance to be the first!

From muons at rest to muons at flight

- Store polarized muons in the uniform magnetic field B
- Momentum rotates with cyclotron frequency:
  $$\omega_c = eB/\gamma mc$$
- Spin rotates with Larmor+Thomas frequency:
  $$\omega_s = geB/2mc + (1-\gamma)eB/\gamma mc$$
- Spin precesses relative to momentum with frequency ω_a:
  $$\omega_a = \omega_s - \omega_c = a_\mu eB/mc$$




ω_a

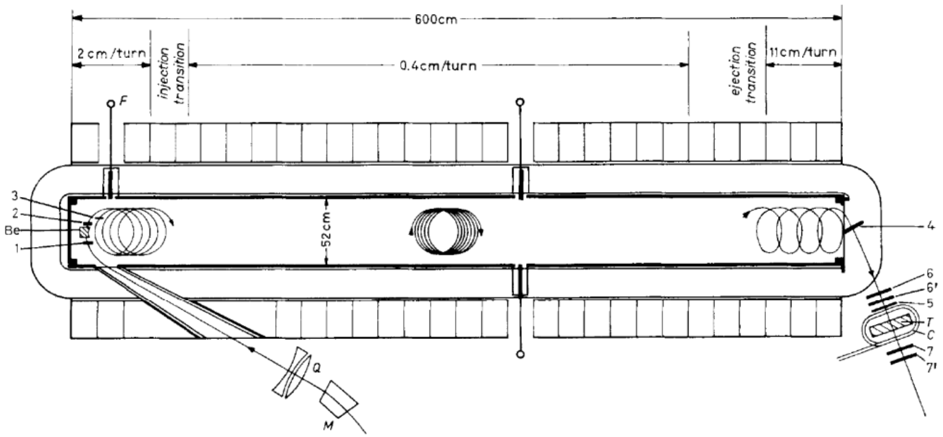
B

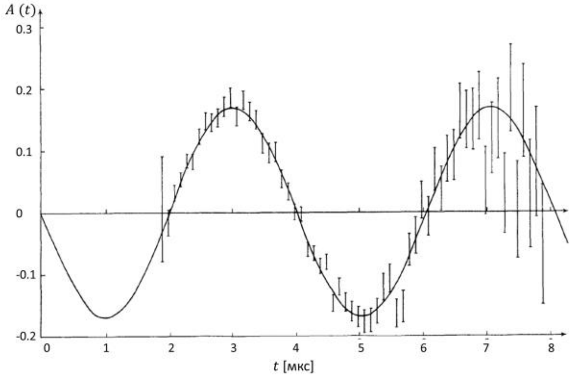
a_μ

Muons at flight allow to measure a_μ directly!
Factor 1000 improvement in precision "for free".

Generations of a_μ measurements

CERN I





The first CERN g-2 team: Sens, Charpak, Muller, Farley, Zichichi (CERN/1959)

1957 1965

$$a_\mu(\text{эксп}) \quad = 0.001\,162\,(5)$$

4300 ppm

QED

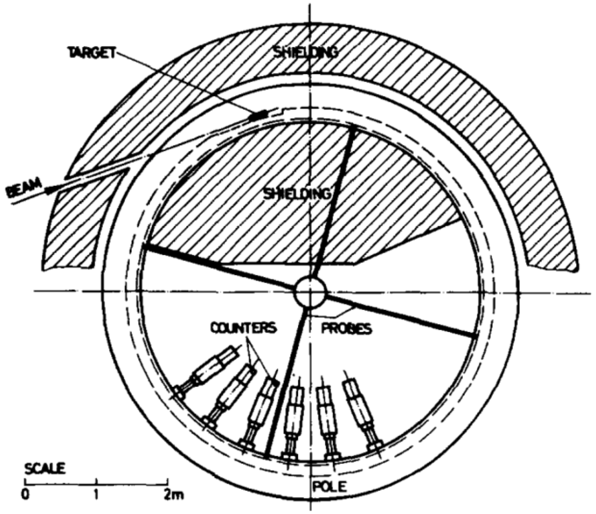
Strong

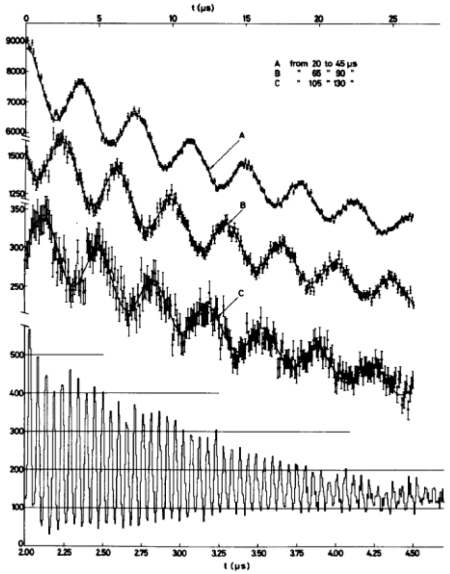
Weak

Contributions of known interactions

Generations of a_μ measurements

CERN II






1957
1965
1968

$$a_\mu(\text{эксп}) \quad = 0.001\,166\,16\,(31)$$

270 ppm


QED
Strong
Weak

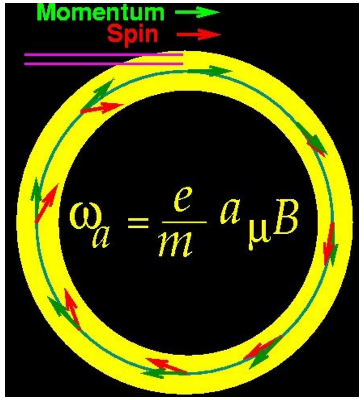
Contributions of known interactions

Magic γ (CERN-III)

Anomalous magnetic moment is independent of γ. The larger γ, the longer muon lifetime, the more g-2 circles observed – good! But there is a problem: particles are not stored in the uniform magnetic field.

Solution: introduce gradient with electric field to build a trap.

$$\bar{\omega} = -\frac{e}{m}\left[a_\mu \bar{B} - \underbrace{\cancel{\left(a_\mu - \frac{1}{\gamma^2-1}\right)}}_{=0}\frac{\bar{\beta}\times\bar{E}}{c} + \underbrace{\cancel{\frac{\eta}{2}\left(\bar{\beta}\times\bar{B} + \frac{\bar{E}}{c}\right)}}_{=0}\right]$$



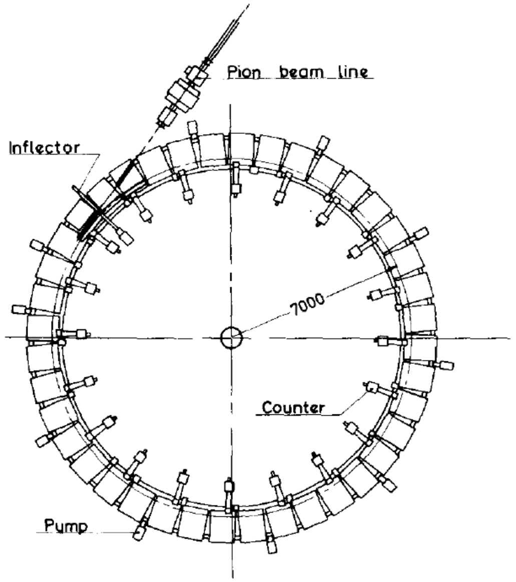
$$\gamma_{\text{magic}} = 29.3$$
$$p_{\text{magic}} = 3.09 \text{ GeV/c}$$

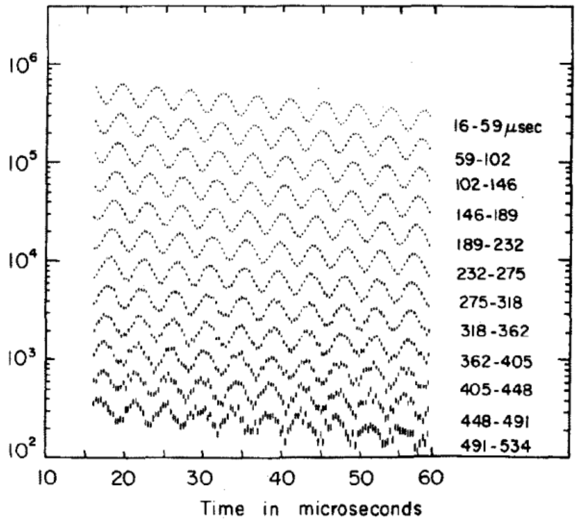
Contribution from potential EDM

Magic γ completely determines the size of the CERN-type experiment.

Generations of a_μ measurements

CERN III





1957

1965

1968

1976

“CERN-III-type experiments”

$$a_\mu(\text{эксп}) \quad = 0.001\,165\,924\,(8.5)$$

7 ppm

QED

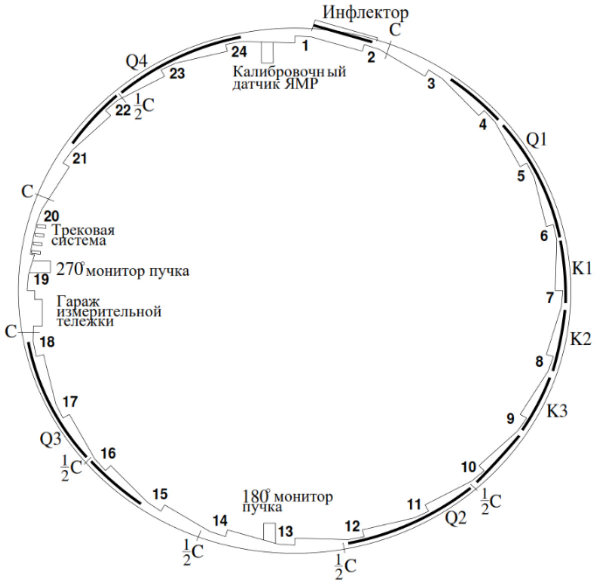
Strong

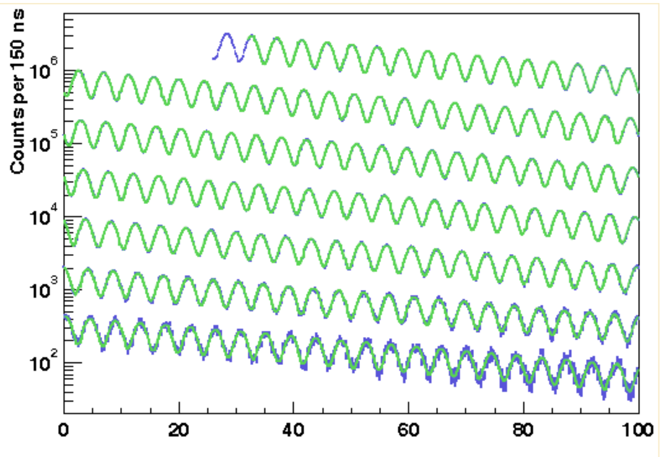
Weak

Contributions of known interactions

Generations of a_μ measurements

BNL (USA)





1957 1965 1968 1976 1997 2006

$$a_\mu(\text{эксп}) = 0.001\,165\,920\,89\,(63)$$

0.54 ppm

QED

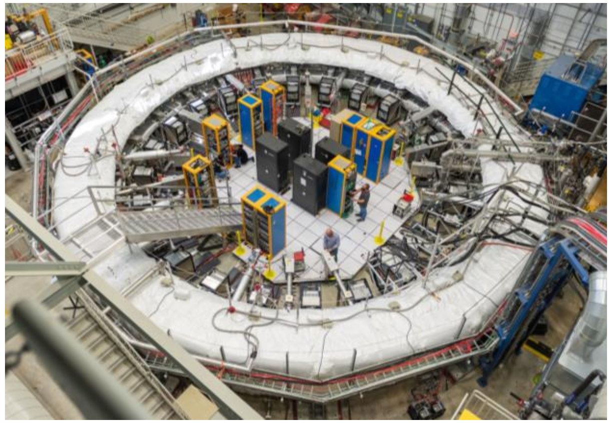
Strong

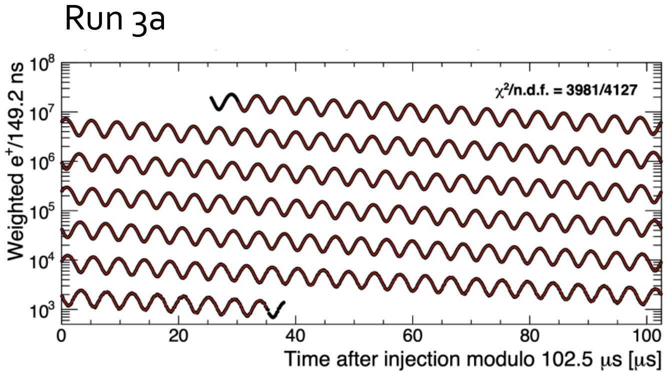
Weak

Contributions of known interactions

Generations of a_μ measurements

FNAL Run 2-3 (USA)




Run 3a


NEVIS
CERN 1-3
BNL
FNAL
1957
1965
1968
1976
2006
2021
2023

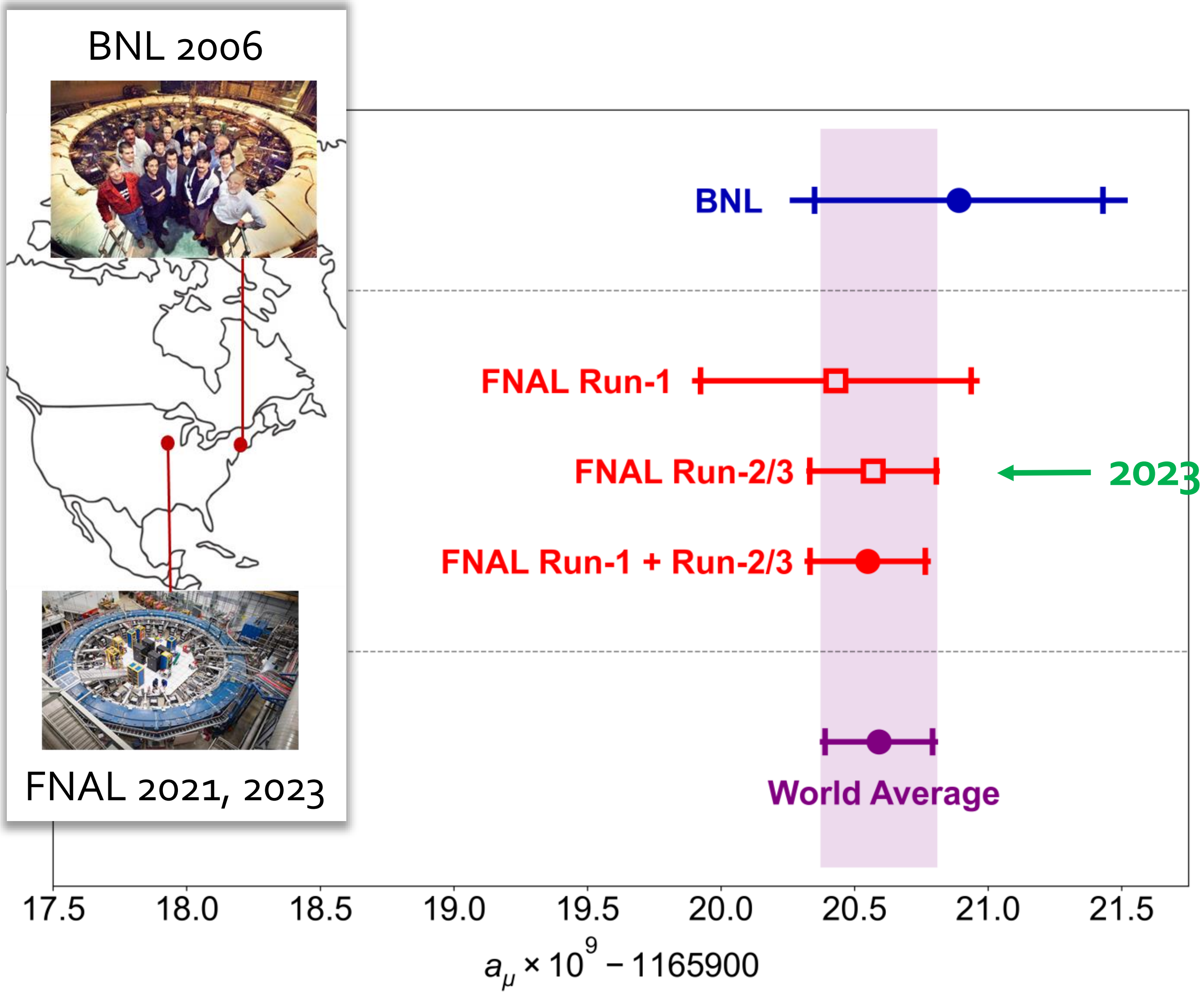
$$a_\mu(\text{эксп}) = 0.001\,165\,920\,55\,(24)$$ FNAL2023

0.21 ppm


QED
Strong
Weak

Contributions of known interactions

Muon G-2 2023 result



2023 result

$$a_\mu(\mathrm{Exp}) = 0.00\,116\,592\,059(22) \quad [190\ \mathrm{ppb}]$$

Measurement of muon (g-2) at Fermilab

Fermilab Muon G-2 collaboration

USA

- Boston
- Cornell
- Illinois
- James Madison
- Kentucky
- Massachusetts
- Michigan
- Michigan State
- Mississippi
- North Central
- Northern Illinois
- Regis
- Virginia
- Washington

USA National Labs

- Argonne
- Brookhaven
- Fermilab

181 collaborators
33 Institutions
7 countries

China

- Shanghai Jiao Tong

Germany

- Dresden
- Mainz

Italy

- Frascati
- Molise
- Naples
- Pisa
- Roma Tor Vergata
- Trieste
- Udine

Korea

- CAPP/IBS
- KAIST

Russia

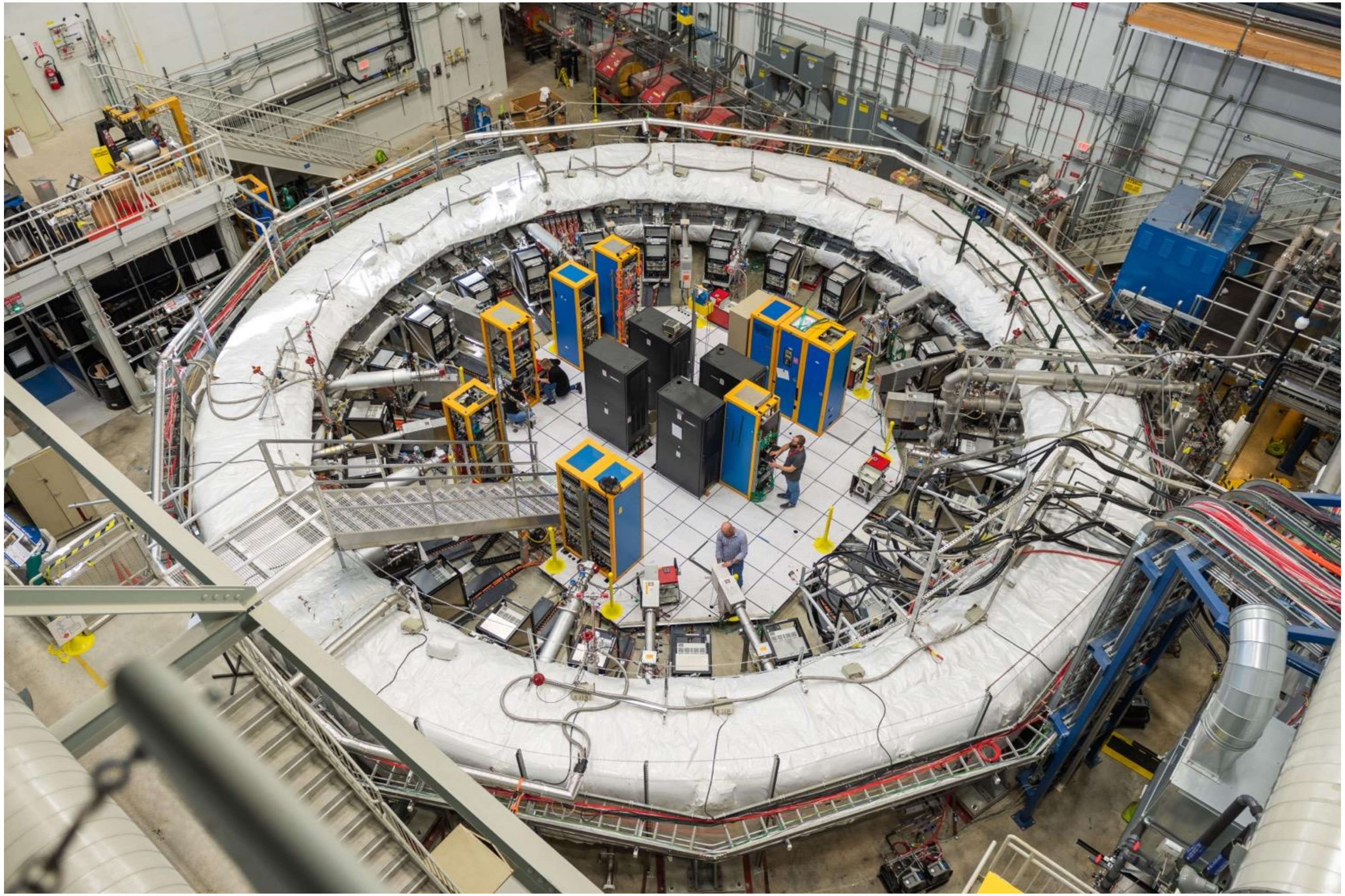
- Budker/Novosibirsk
- JINR Dubna

United Kingdom

- Lancaster/Cockcroft
- Liverpool
- Manchester
- University College London

Muon g-2 Collaboration Meeting @ Elba, May 2019

Muon G-2 Ring @FNAL



Principles of measurement

$$a_\mu = \frac{g-2}{2} \propto \frac{\omega_a}{B}$$

Beam of polarized muons

Polarized muons are obtained from pion decay

$\pi \to \mu + \nu_\mu$

Precession in uniform magnetic field

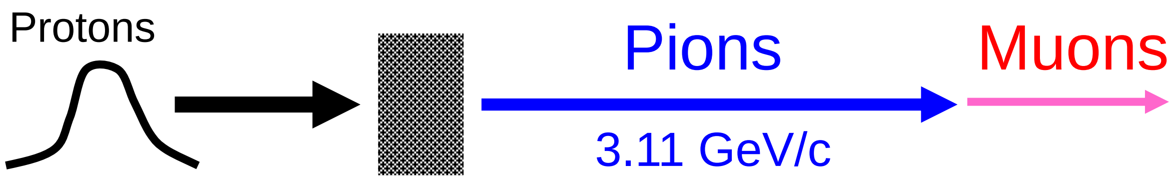
Muons are stored in the ring with ultra-uniform magnetic field, measured using NMR

Measure direction of spin at the moment of decay

Spin direction is measured using anisotropy of electrons produced in muon decay

$\mu \to e + \nu_e + \nu_\mu$

Generation of muons

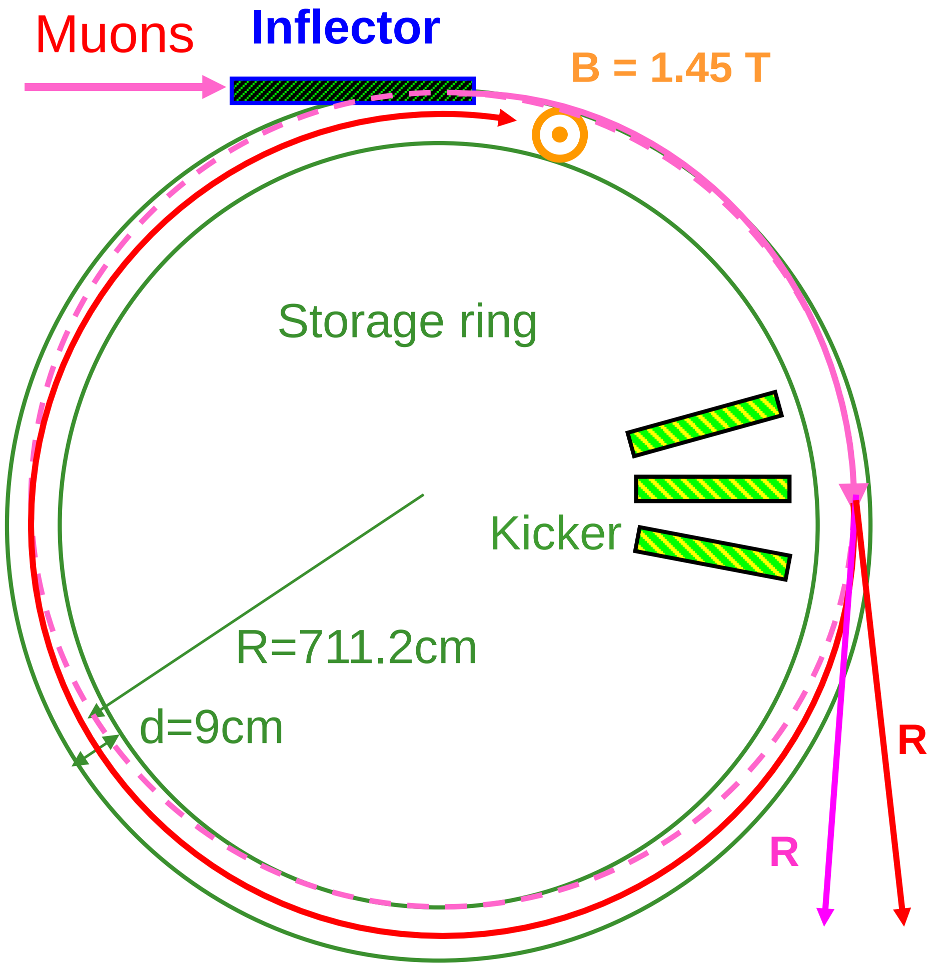


- Protons hit the target
- Pions with energy of 3.11 GeV are selected and transferred to long decay channel
- Muons from forward decay are selected – almost complete polarization (>95%)
- Muons with energy of 3.09 GeV are selected (“magic γ”)

Injecting muons

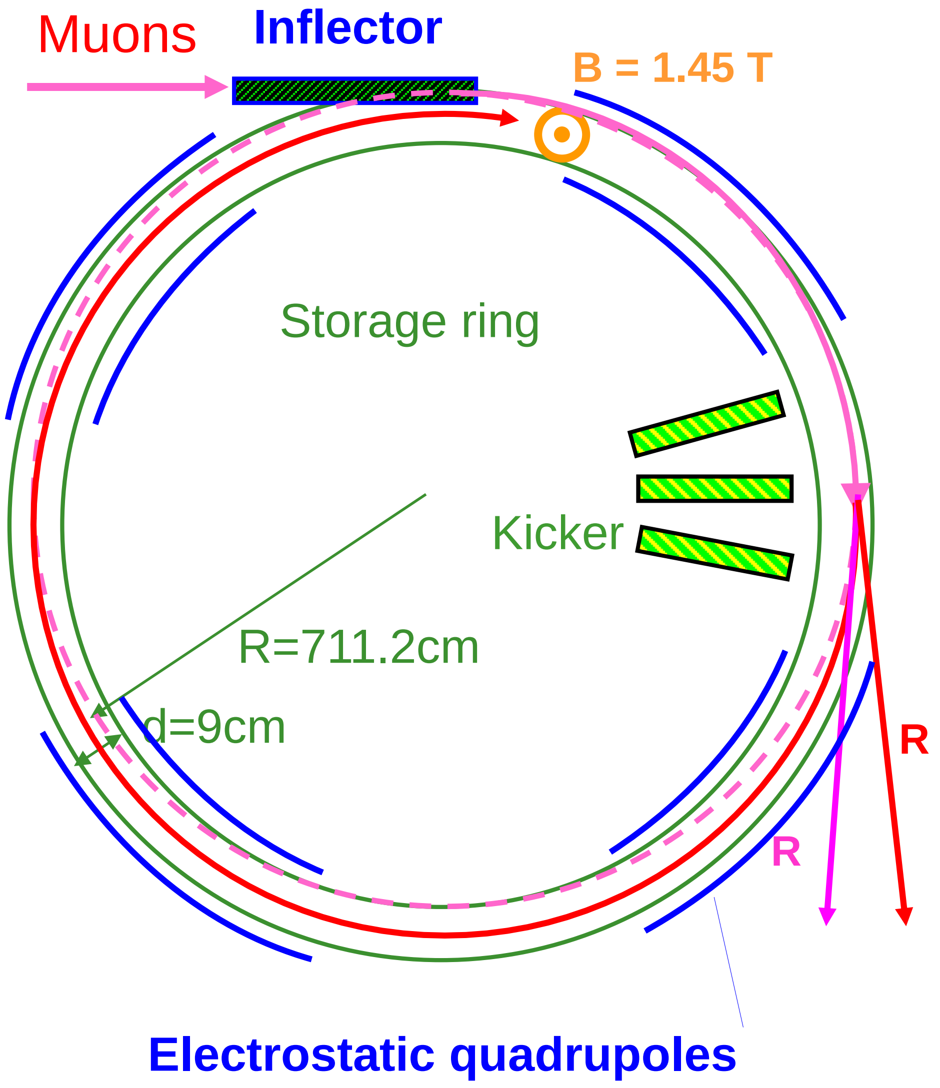
- Muons have to move from region of no field to region of 1.45 T -> inflector magnet

- Muons have to be kicked to get to storage orbid -> fast kicker magnets



Storing muons

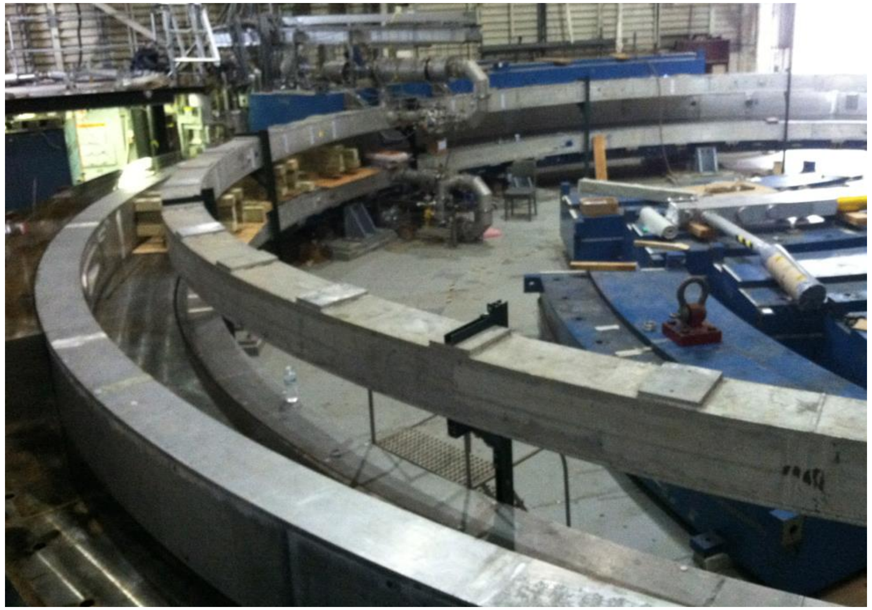
- Muons are stored in orbit with the help of electrostatic quadrupoles. They provide weak vertical focusing



Muon Campus at Fermilab

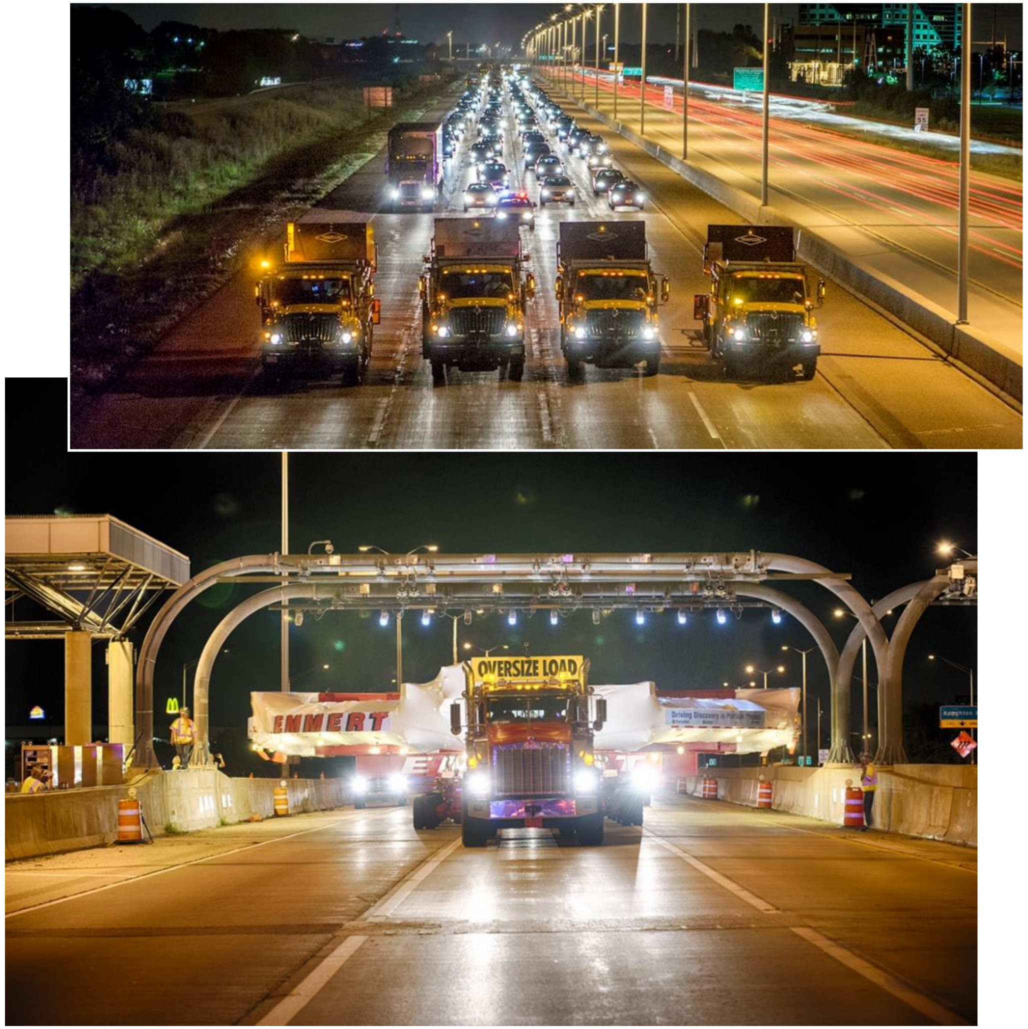
Moving the ring from BNL to FNAL

In order to save $, the most expensive piece from the BNL experiment – the storage ring itself, is reused. The steel, pole pieces etc. are disassembled and moved by trucks. But there are three coils inside the cryostats… - 15 m diameter, they cannot be broken in pieces, flexed > 3 mm



Moved in 2013 by truck and the sea

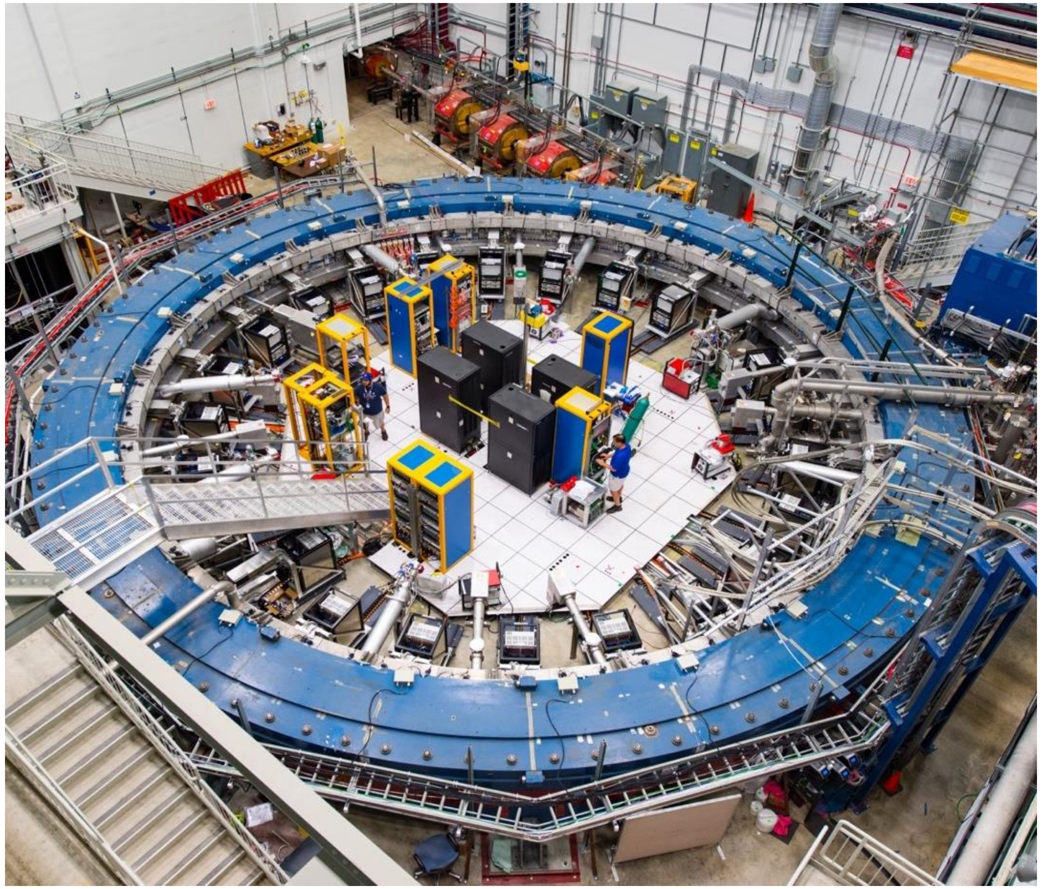
5000 km journey

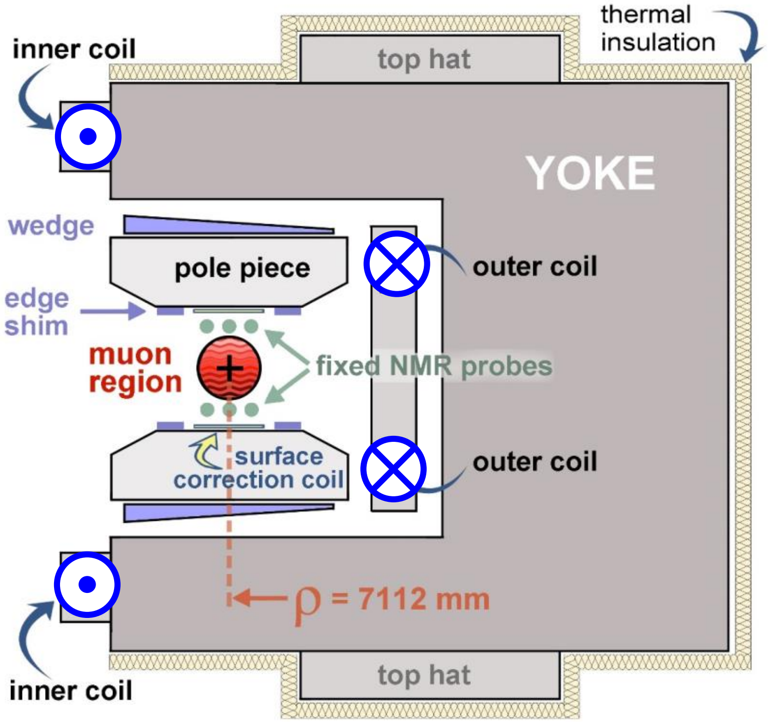


Arriving at FNAL

The ring magnet

The storage ring is a 14 m diameter, 1.45 T C-shaped magnet



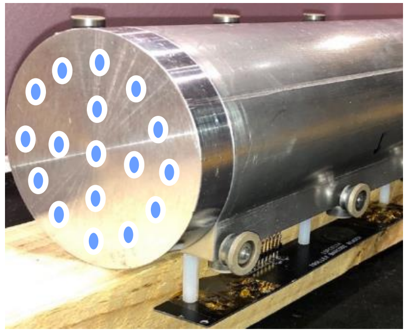


B field is measured in terms of proton NMR frequency ω_p

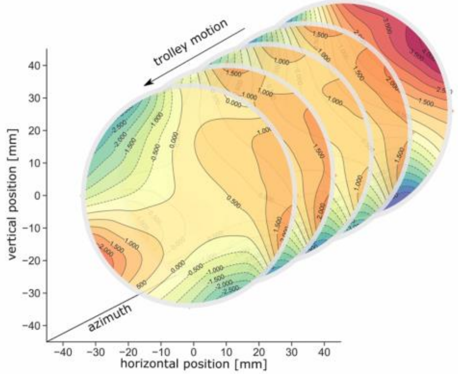
Monitoring B field

ω_p

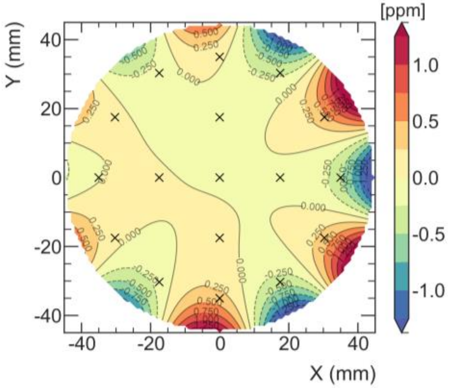
- In-vacuum NMR trolley **maps field every ~3 days**



17 petroleum jelly
NMR probes



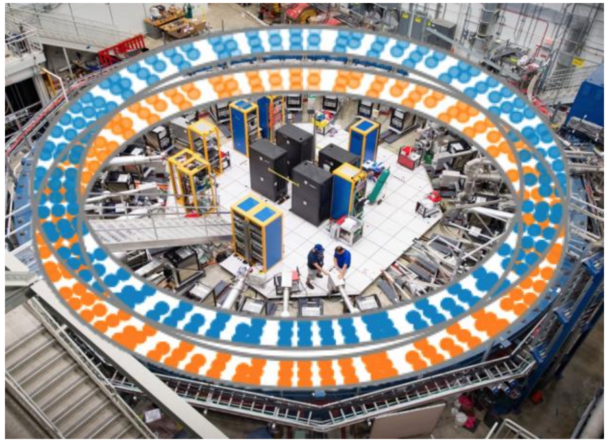
2D field maps
(~8000 points)

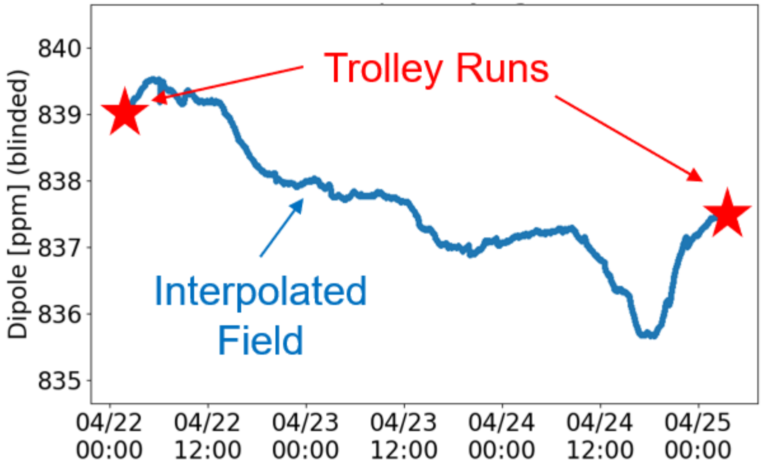


Azimuthally-Averaged
Variation < 1 ppm

- **378 fixed probes** monitor field during muon storage at 72 locations

Fixed probes
above/below muon
storage region

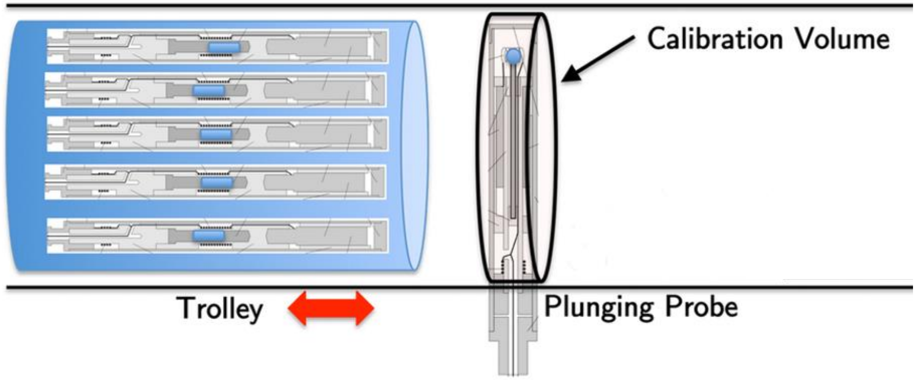


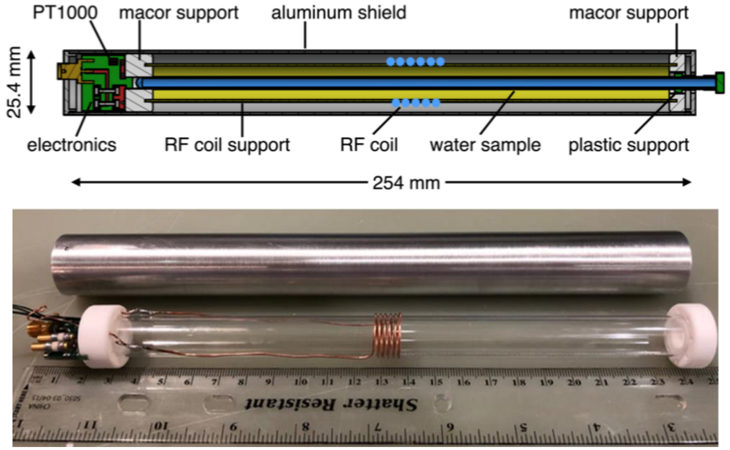


Field map is convoluted with muon spatial distribution to get an average field

Absolute calibration

- Cross-calibrate using a cylindrical **plunging H_2O probe** which repeatedly **changes places with trolley (petroleum jelly probes)**





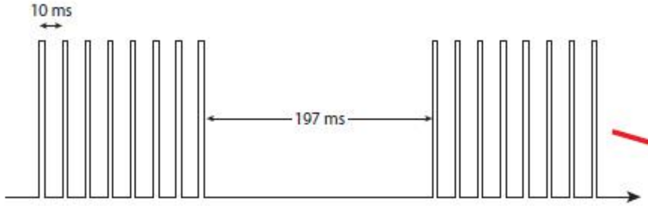
- This probe is **checked against a spherical probe** using an MRI magnet at ANL
- Both also cross-checked against a **^{3}He probe** (different systematics)

$$\Delta B / B \approx 5 \cdot 10^{-8}$$

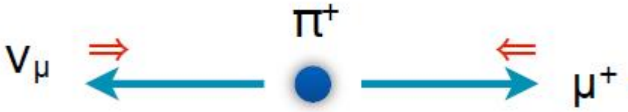
H_2O Probe

^{3}He Probe

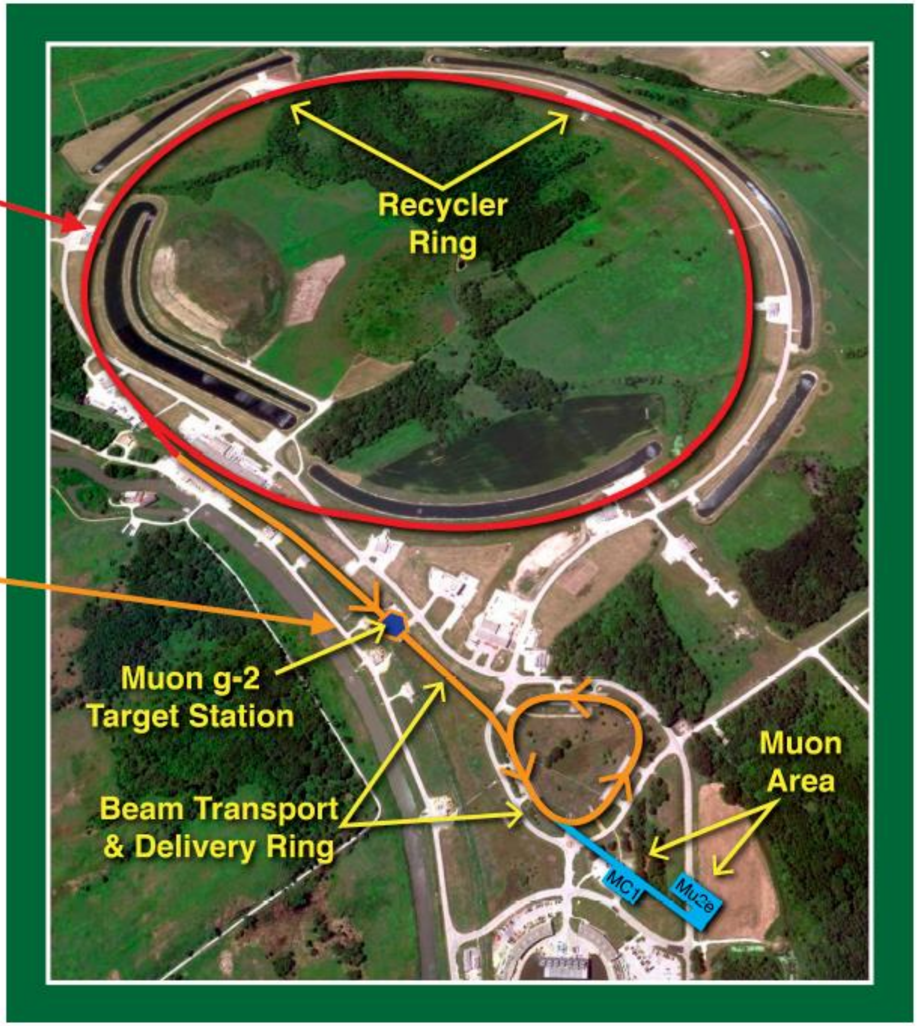
Generation of muons



4 Booster batchs → 16 muon fills
- 1.4 sec repetition rate

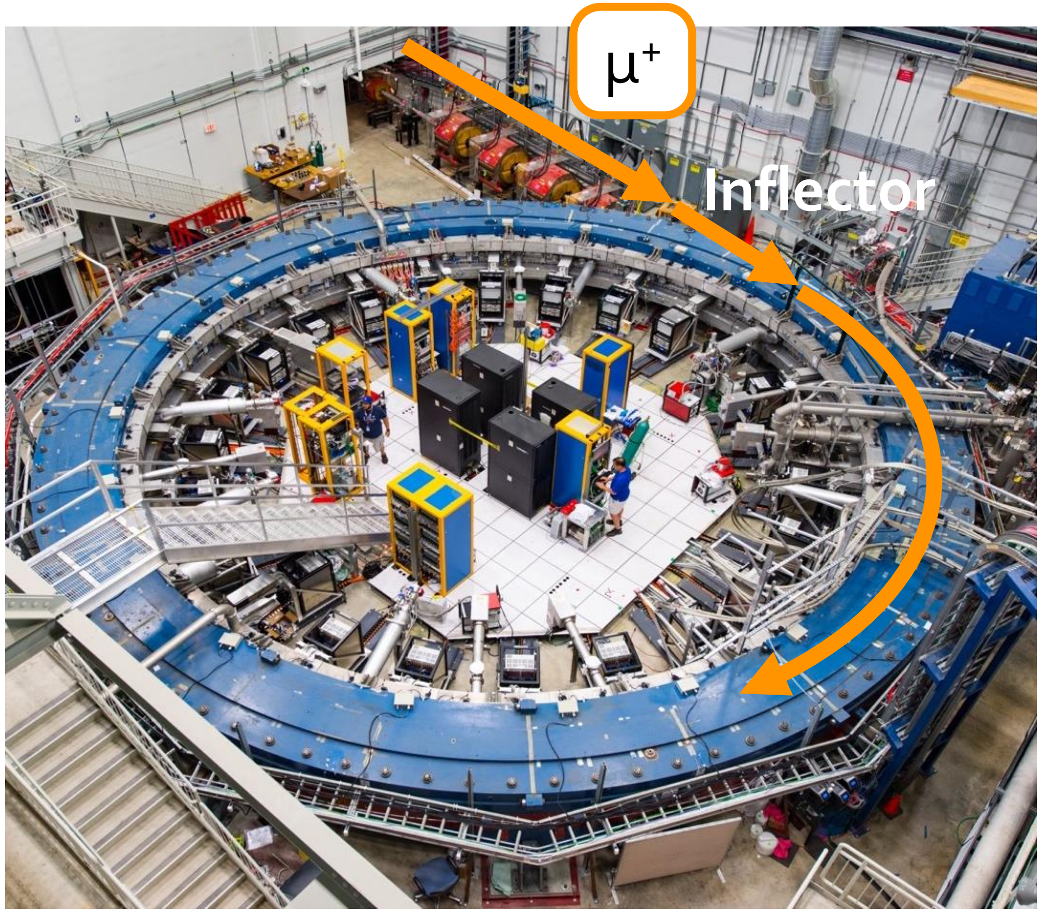


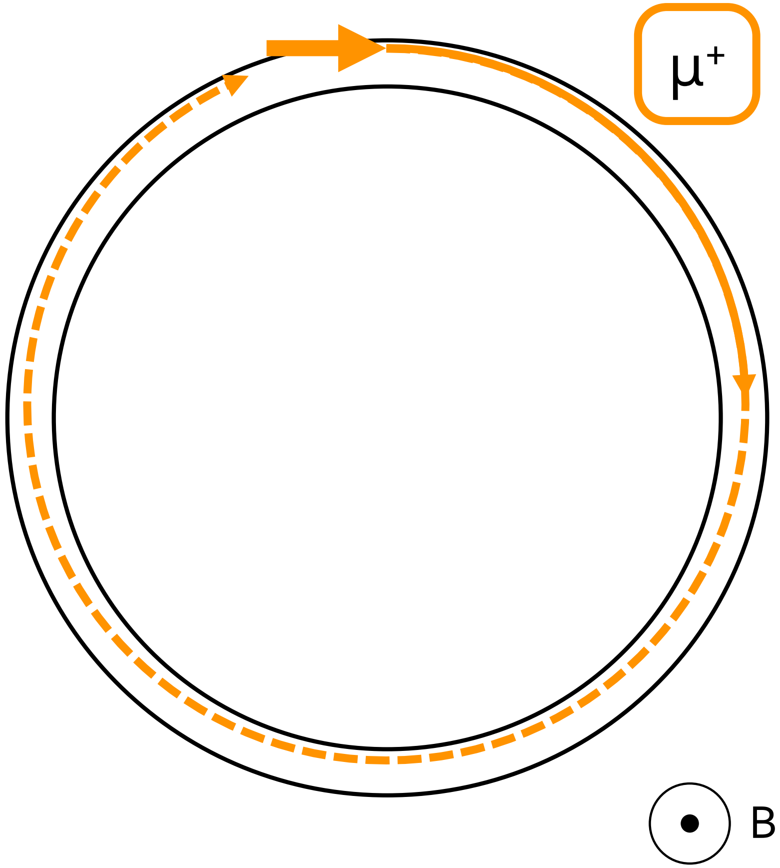
Select ~3.1 GeV π^+ (magic p)
- Parity violation → 95% polarized muons



Injection of muons

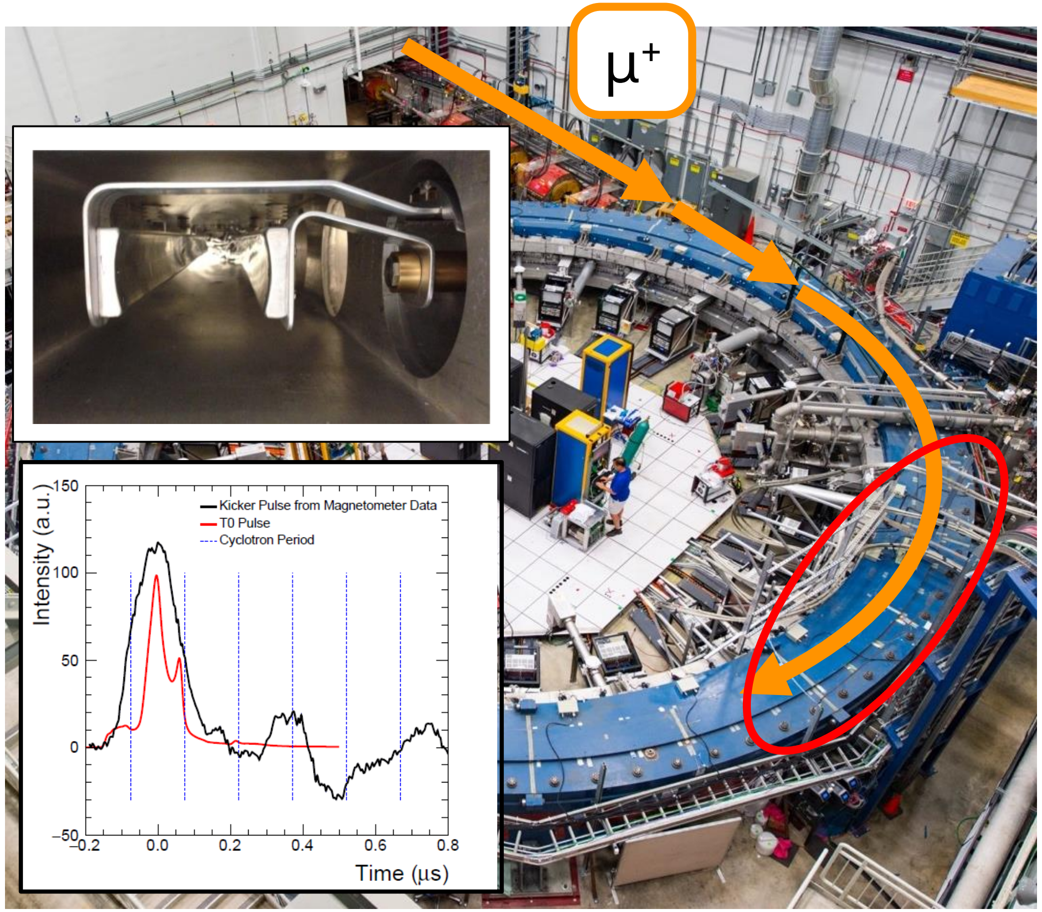
Muons are injected into the storage ring with uniform field. After one turn they hit the wall, unless…

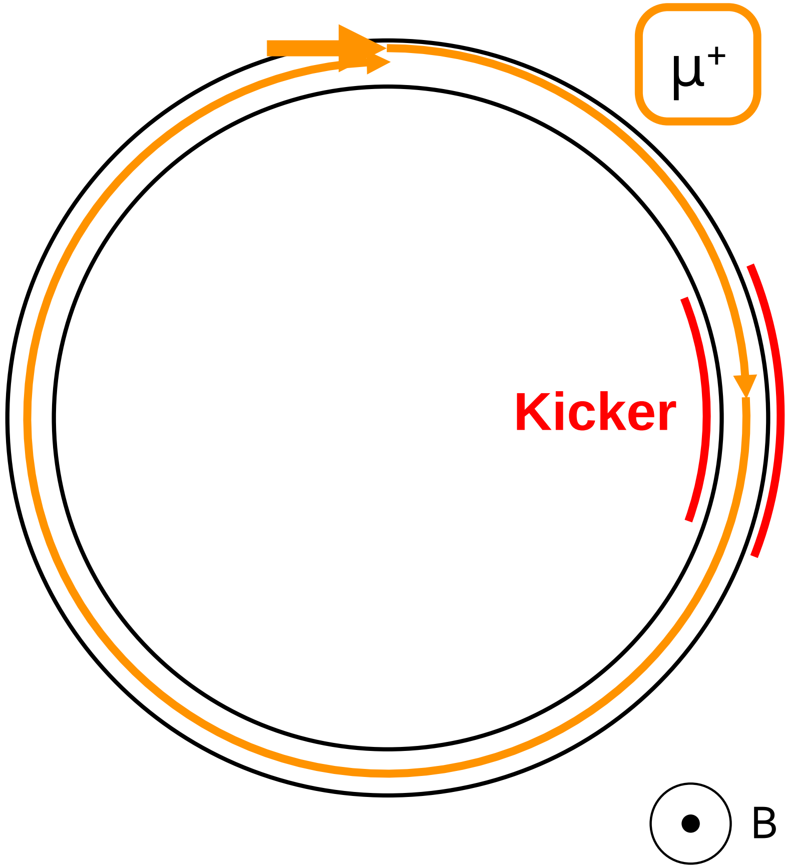




Kicker

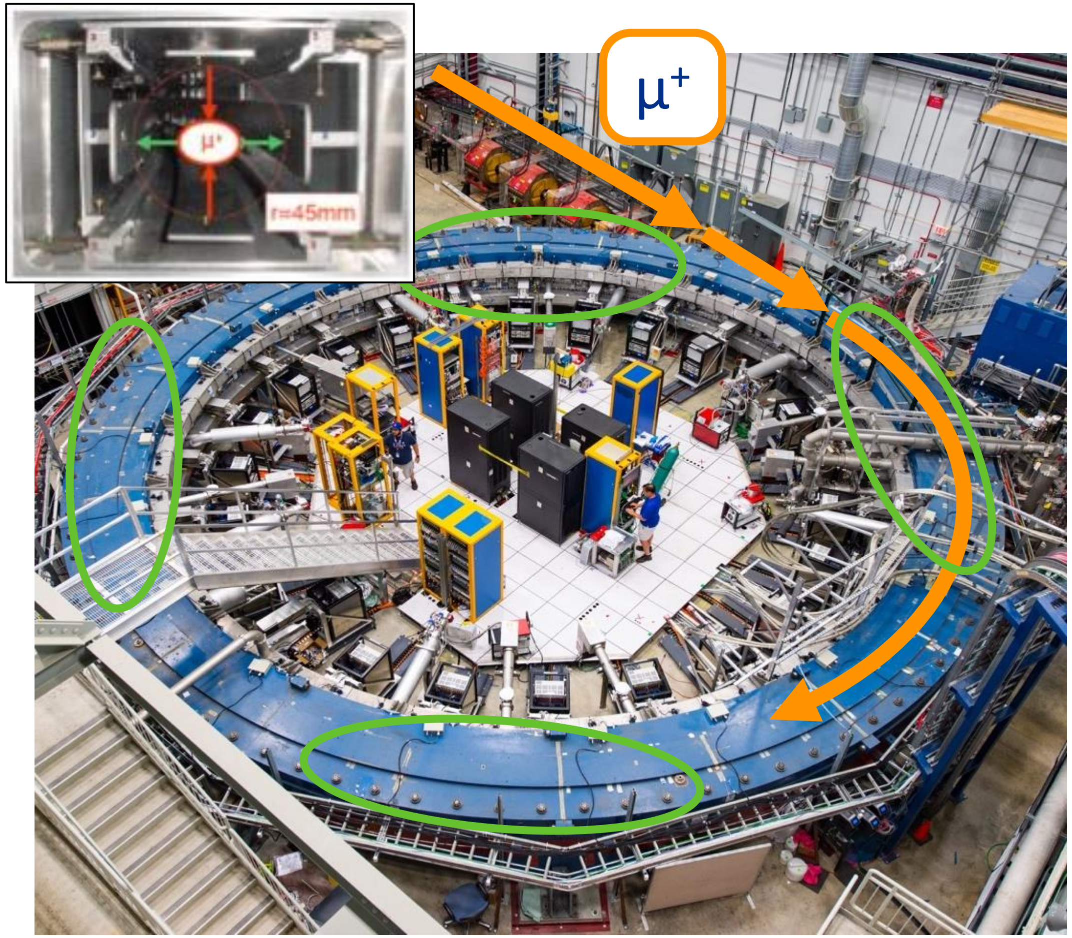
Fast kicker magnet briefly reduces field at 90° and puts beam to standard orbit

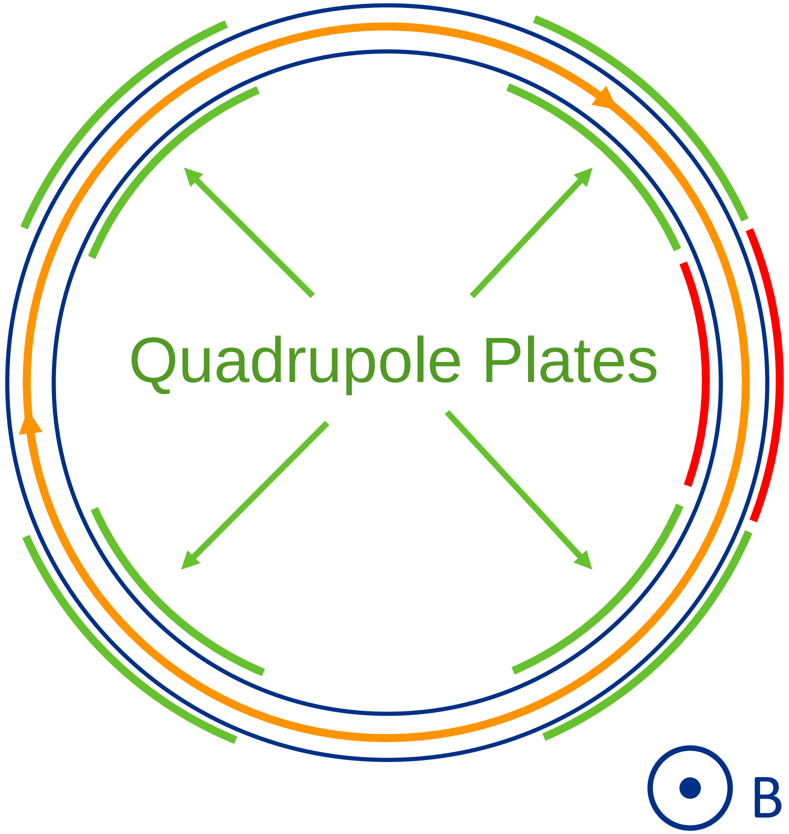




Quads

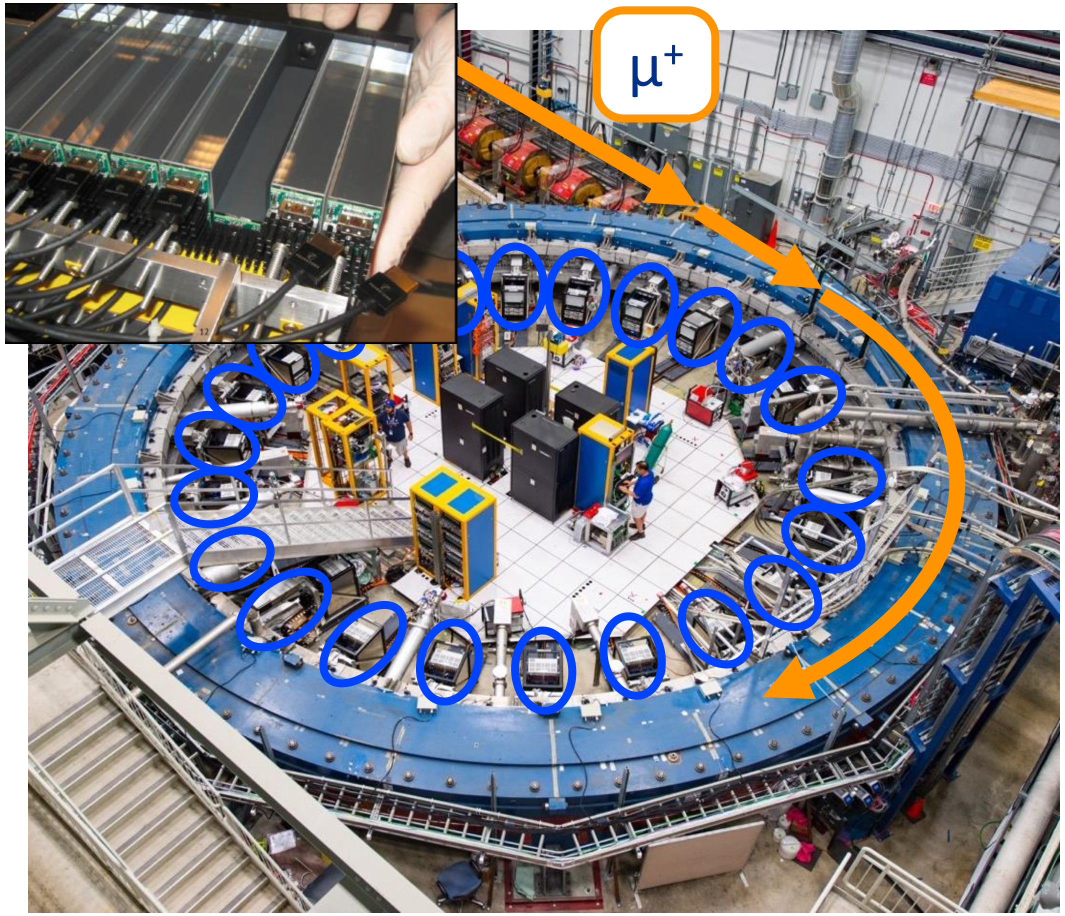
- Electrostatic quadrupoles vertically contain the beam

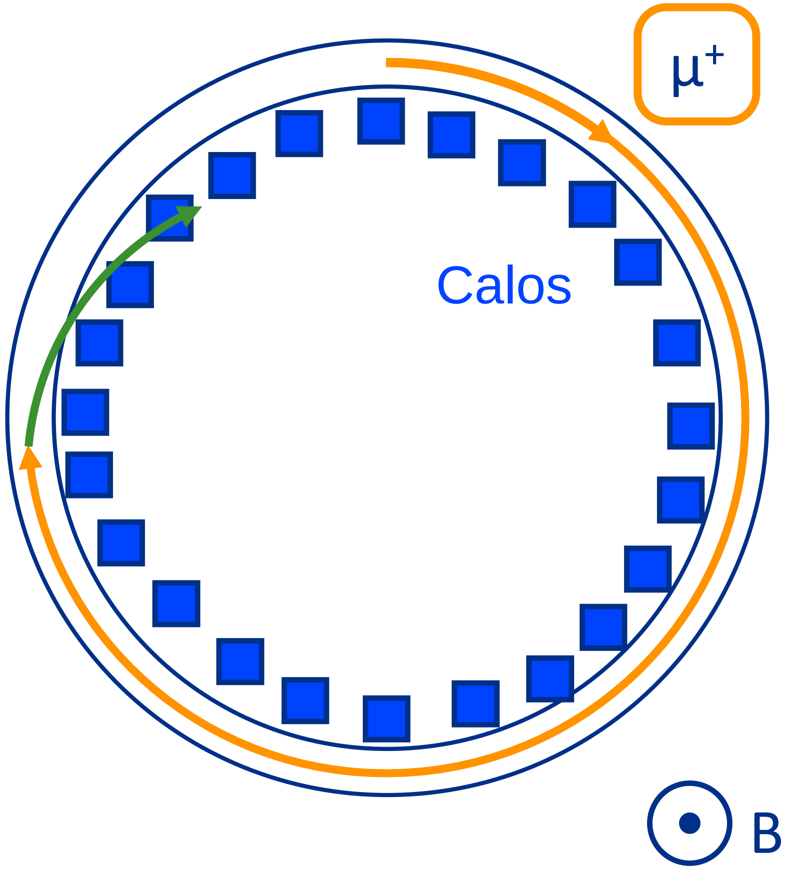




Calorimeters

- Time & energy of decay e^+ are measured by **24 calorimeters**

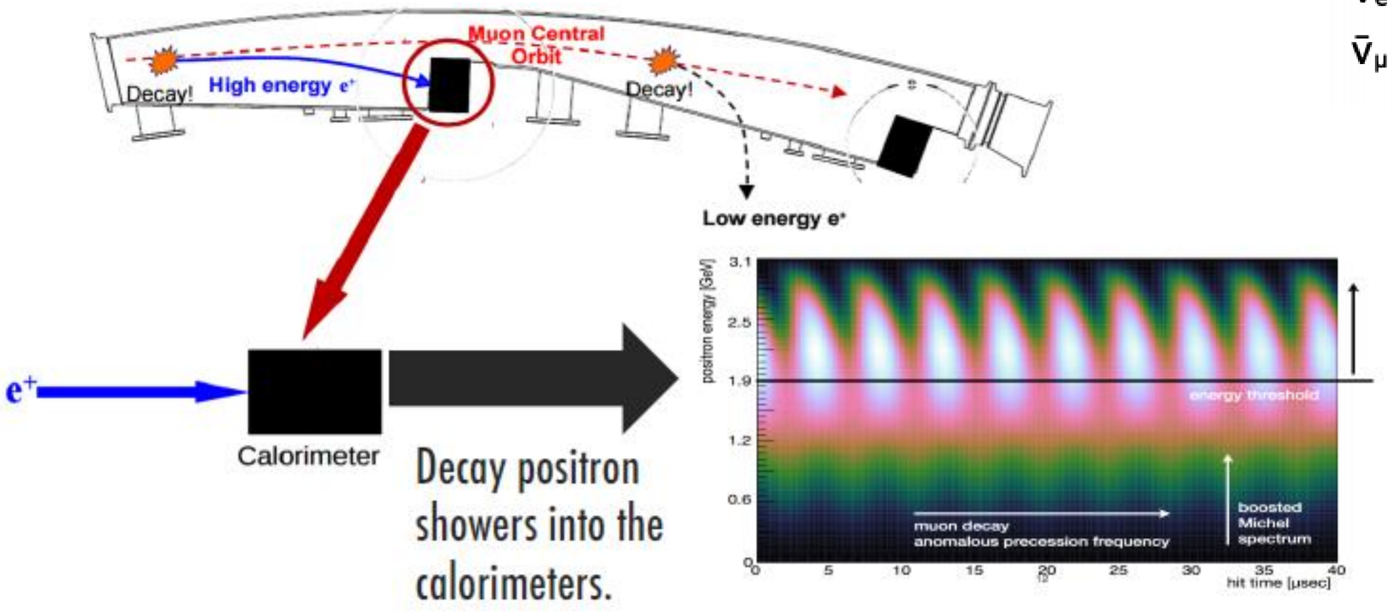


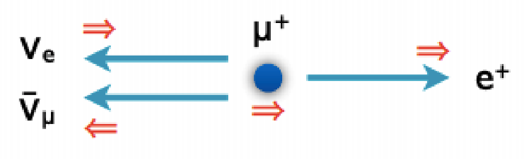


Each calorimeter: array of 9x6 PbF_2 crystals (2.5 x 2.5 cm^2 x 14 cm, $15X_0$), readout by SiPMs

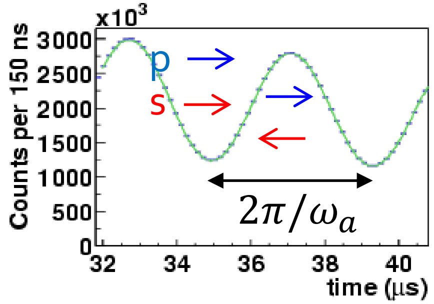
Measuring ω_a

The energy distribution of positrons depends on spin direction, thus number of high energy positrons is modulated by precession frequency





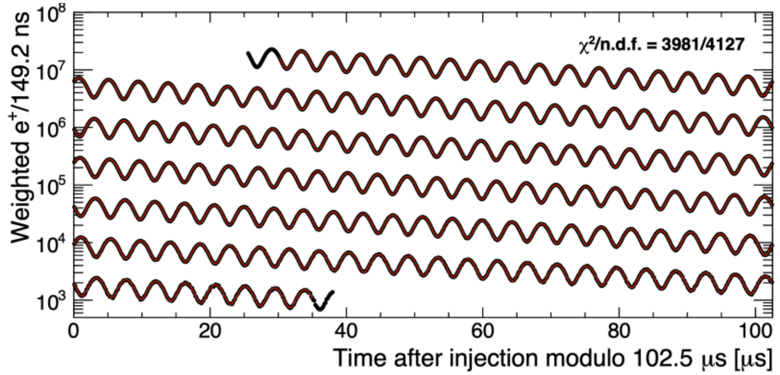
Positron energy is correlated to spin



Counting rate of high energy positrons

“Wiggle plot”

$$dN(e^+ > 1\ \mathrm{GeV})/dt$$



The potential nightmare:

Early-to-late systematics

$$\cos(\omega_a t + \varphi)$$

Leading systematics come from time dependence in the phase

Taylor expansion: $$\varphi(t) = \varphi_0 + \alpha t + \beta t^2 \cdots \approx \varphi_0 + \alpha t$$

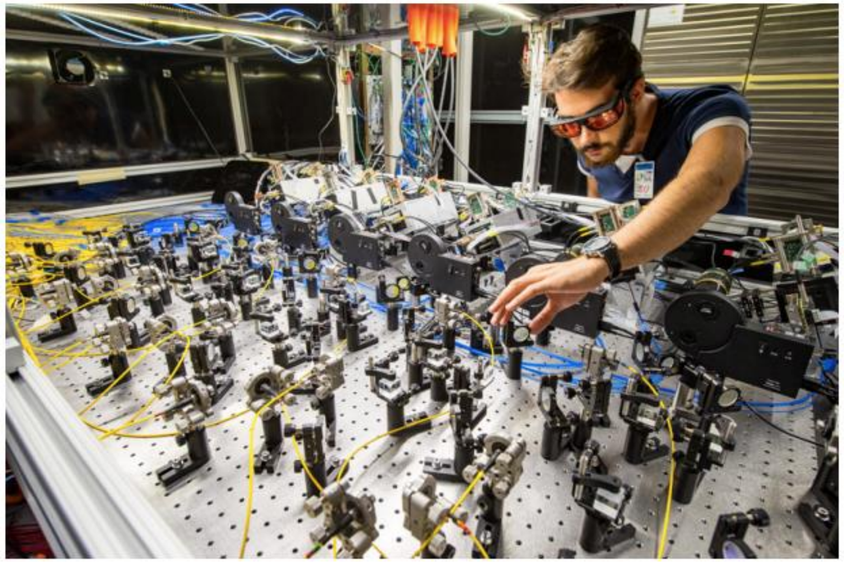
$$\cos(\omega_a t + \varphi(t)) \approx \cos((\omega_a + \alpha)t + \varphi_0)$$

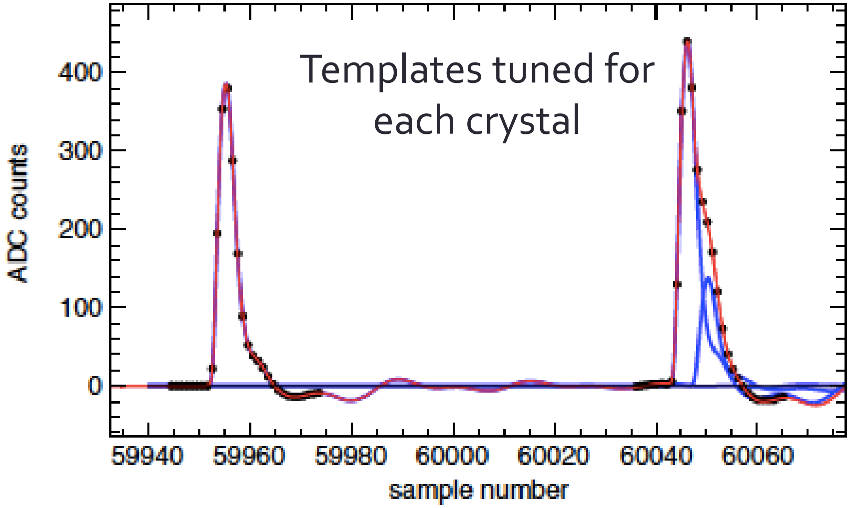
Things that change "early to late" in the fill typically lead to a time dependence in the phase of the accepted sample that directly biases the extracted value of ω_a

Muon G-2 measurement and data analysis is built to avoid or measure and compensate early-to-late effects

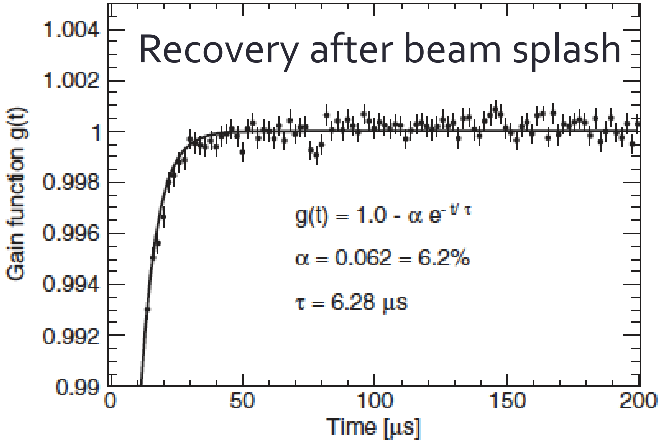
Laser calibration system

Highly tunable, precise laser system sends pulses to all crystals

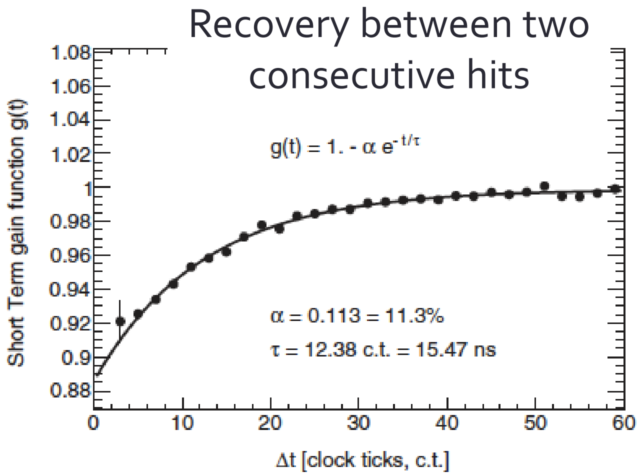




Templates tuned for each crystal



Recovery after beam splash

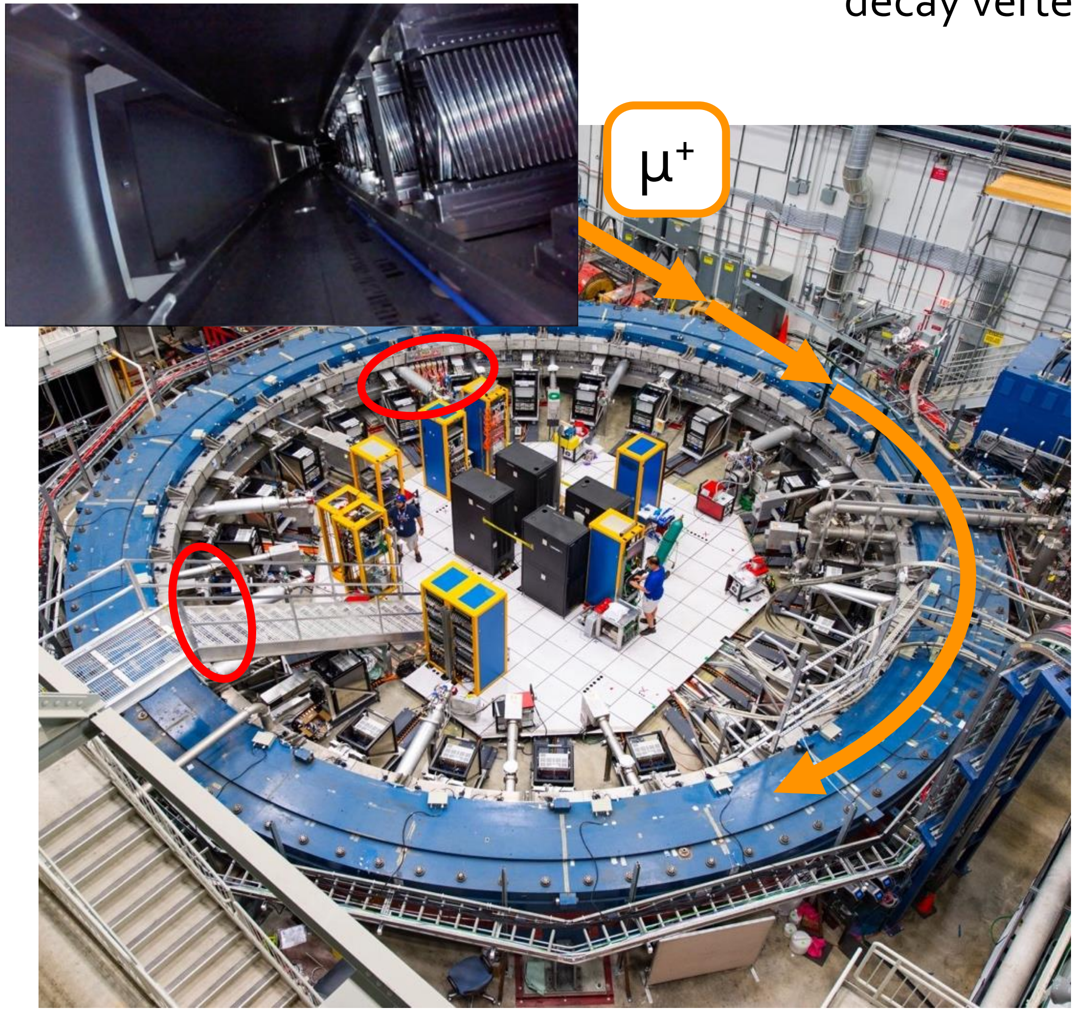


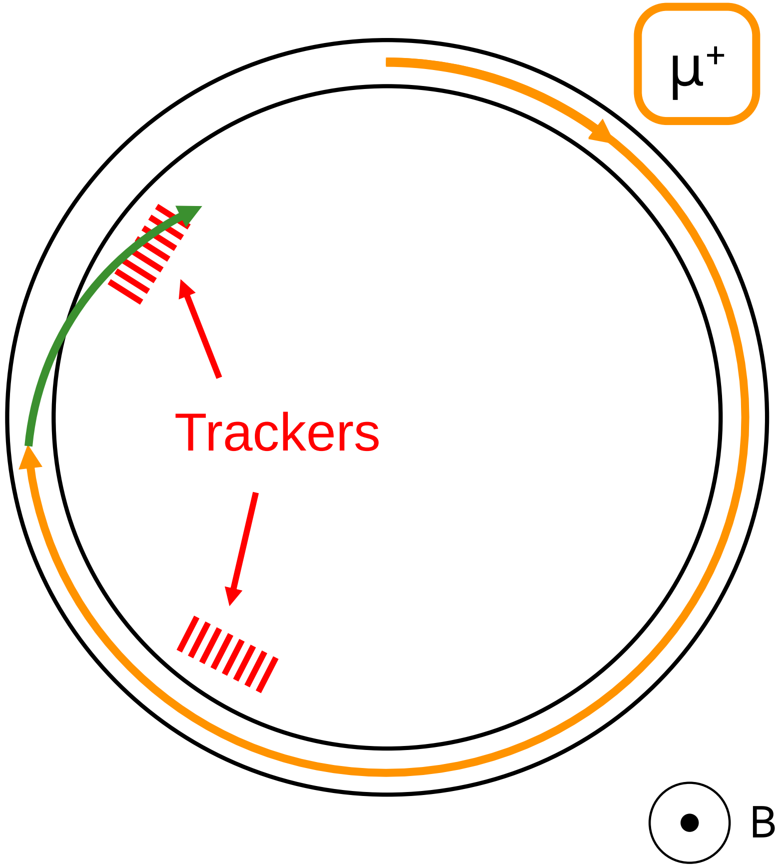
Recovery between two consecutive hits

Pileup and gain systematics reduced from 180 ppb at BNL to 41 ppb

Trackers

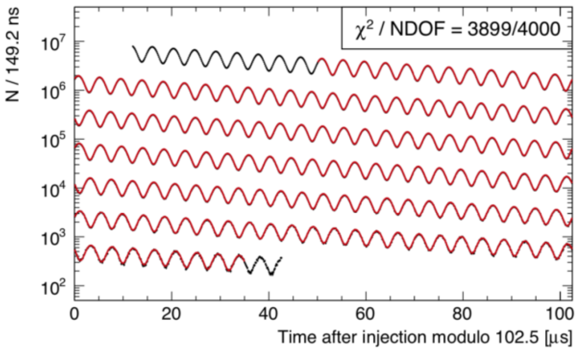
Two trackers allow to see muon beam dynamics in real time by reconstruction of muon decay vertex

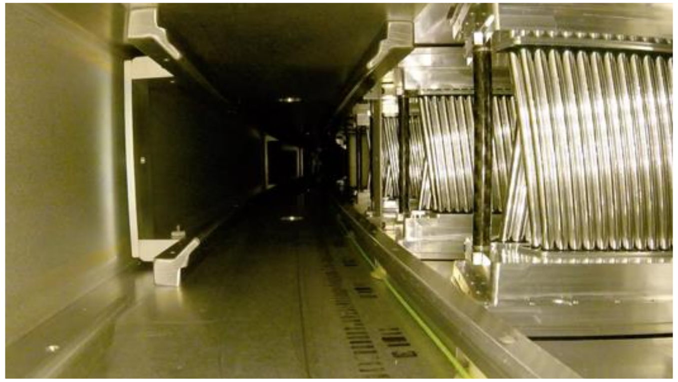


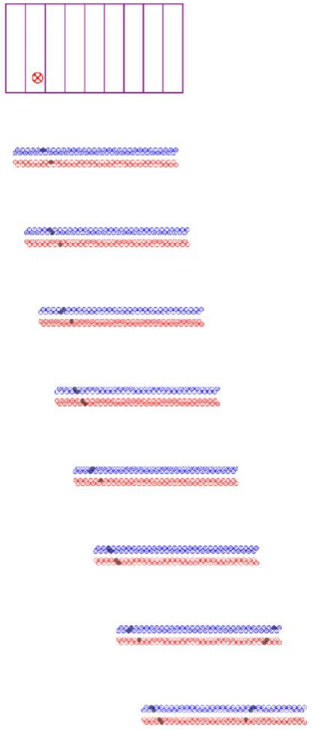


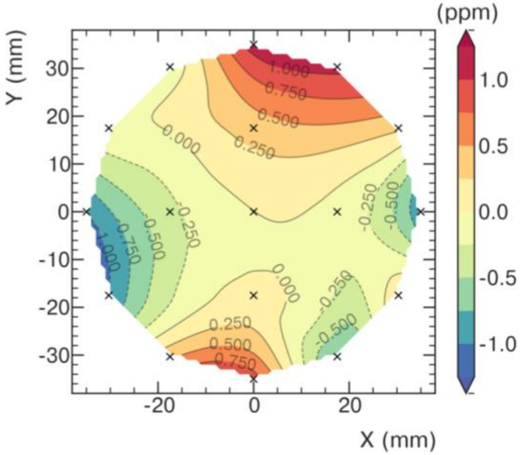
Distribution of stored muons

$$\frac{\omega_a}{\omega_p \otimes \rho(r)} \Rightarrow$$

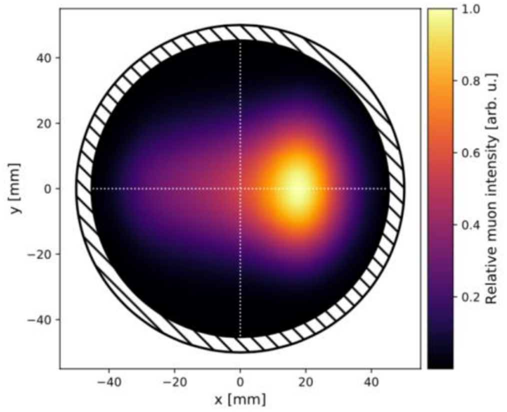








$\otimes$



*All plots actual Run 1 data

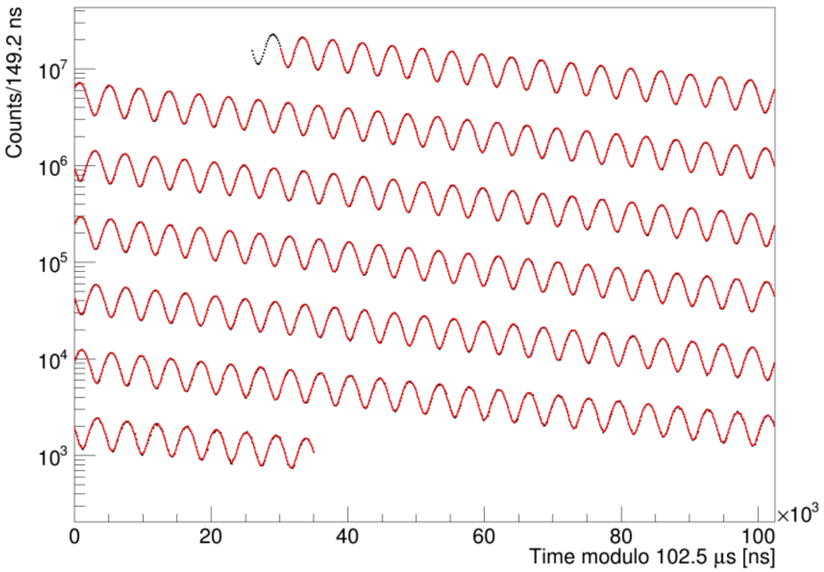
- *In vacuo* straw trackers tell us the spatial distribution and many other muon beam properties (CBO, p-dist)

5-par fit

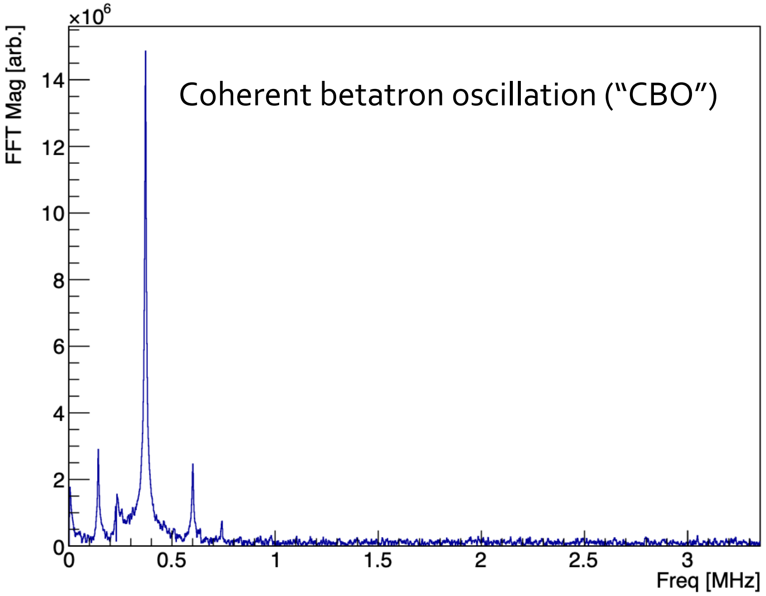
Simple model: exponential decay and precession

$$N(t) = \boldsymbol{N_0} e^{(-t/\boldsymbol{\tau})}[1 + \boldsymbol{A}\cos(\boldsymbol{\omega_a} t - \boldsymbol{\phi})]$$

χ^2/ndf = 51530/4150



Fourier transform of residuals



Coherent betatron oscillation ("CBO")

Realistic model must account for **detector effects**, **beam oscillations** that couple to acceptance, and **lost muons** that disrupt pure exponential

Full fit function

Fit function is extended to cover all extra effects

$$N_0 e^{-t/\tau}(1 + A\cos(\omega_a t + \phi))$$

$$\Downarrow$$

$$f(t) = N_0 e^{-t/\tau}\, \Lambda(t)\, N_{cbo}(t)\, N_{2cbo}(t)\, (1 + A_{cbo}(t) cos(\omega_a t + \phi_{cbo}(t))$$

- Muons that are lost from storage ring before they decay:

$$\Lambda(t) = 1 - \boldsymbol{\kappa_{loss}} \int_{t_0}^{t} L(t') e^{(t'/\tau)} dt'$$

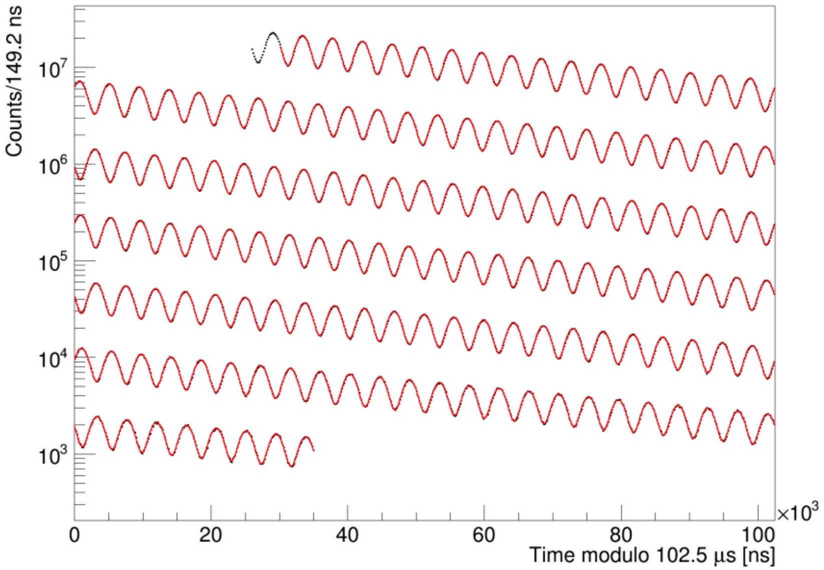
- Beam oscillations that modulate decay rate:

e.g. $N_{cbo}(t) = (1 + \boldsymbol{A_{cbo-N}} \cdot e^{-t/\boldsymbol{\tau_{cbo}}} \cdot \cos(\omega_{cbo}(t) \cdot t + \boldsymbol{\phi_{cbo-N}}))$

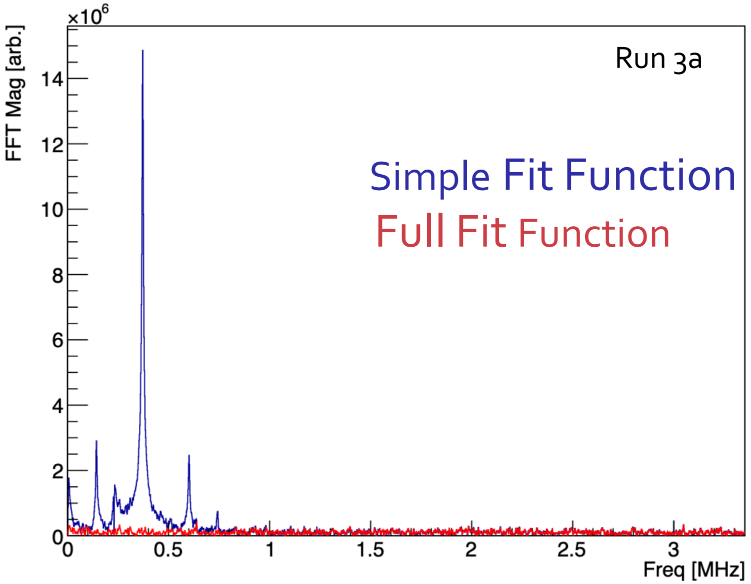
Full fit

Realistic model allows to reach good fit quality.
These effect are important! ω_a shifts by 1-2 ppm.

χ^2/ndf = 4086/4138



Fourier transform of residuals



Data from calorimeters and trackers are used to get parameters/confirm model

Obtaining a_μ

$$\frac{\omega_a}{\omega_p} = \frac{\omega_a^m}{\omega_p^m}\,\frac{1+\overbrace{C_e+C_p+C_{pa}+C_{dd}+C_{ml}}^{\text{Corrections due to beam dynamics}}}{1+\underbrace{B_k+B_q}_{\text{Corrections due to transient magnetic fields}}}$$

Measured Values

$$a_\mu = \boxed{\frac{\omega_a}{\omega_p}} \times \frac{\mu_p'(T_r)}{\mu_e(H)}\,\frac{\mu_e(H)}{\mu_e}\,\frac{m_\mu}{m_e}\,\frac{g_e}{2}$$

Metrological constants known to ~25 ppb

Total correction is about 622 ppb

Blind analysis

$$\frac{\omega_a}{\tilde{\omega}_p} = \left(\frac{f_{\mathrm{clock}}\, \omega_a \,(1 + C_e + C_p + C_{ml} + C_{pa})}{(1 + B_{QT} + B_{Eddy})\; f_{\mathrm{field}}\, \omega_p \otimes \rho(\mathbf{r})} \right)$$

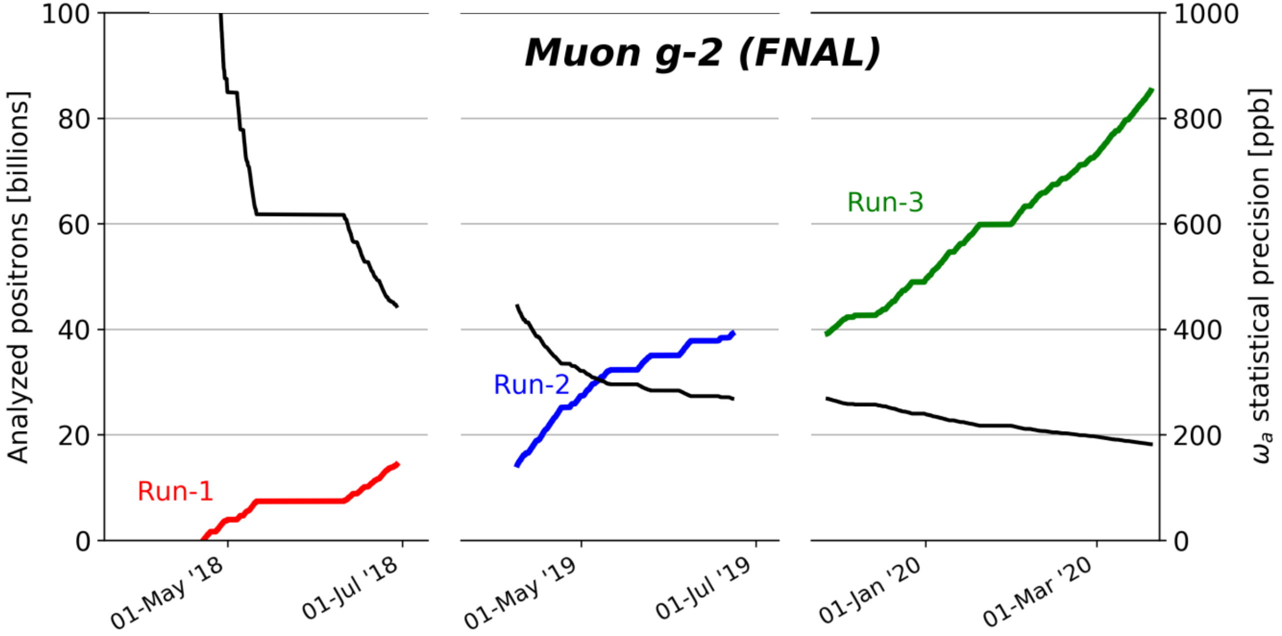
- f_{clock} is the frequency that our clock ticks
  - Precision timepiece, stable at ppt level
- Throughout the entire analysis the clock frequency is kept secret from all collaborators
  - Joe Lykken and Greg Bock (FNAL Directorate) stop in each week to check on the clock
  - Secret envelopes kept until physics analysis is complete and ready to be revealed

Run-1 vs Run-2/3

Statistics

Weighted e^+ in
our final fit after
quality control

E > 1 GeV
t > 30 us



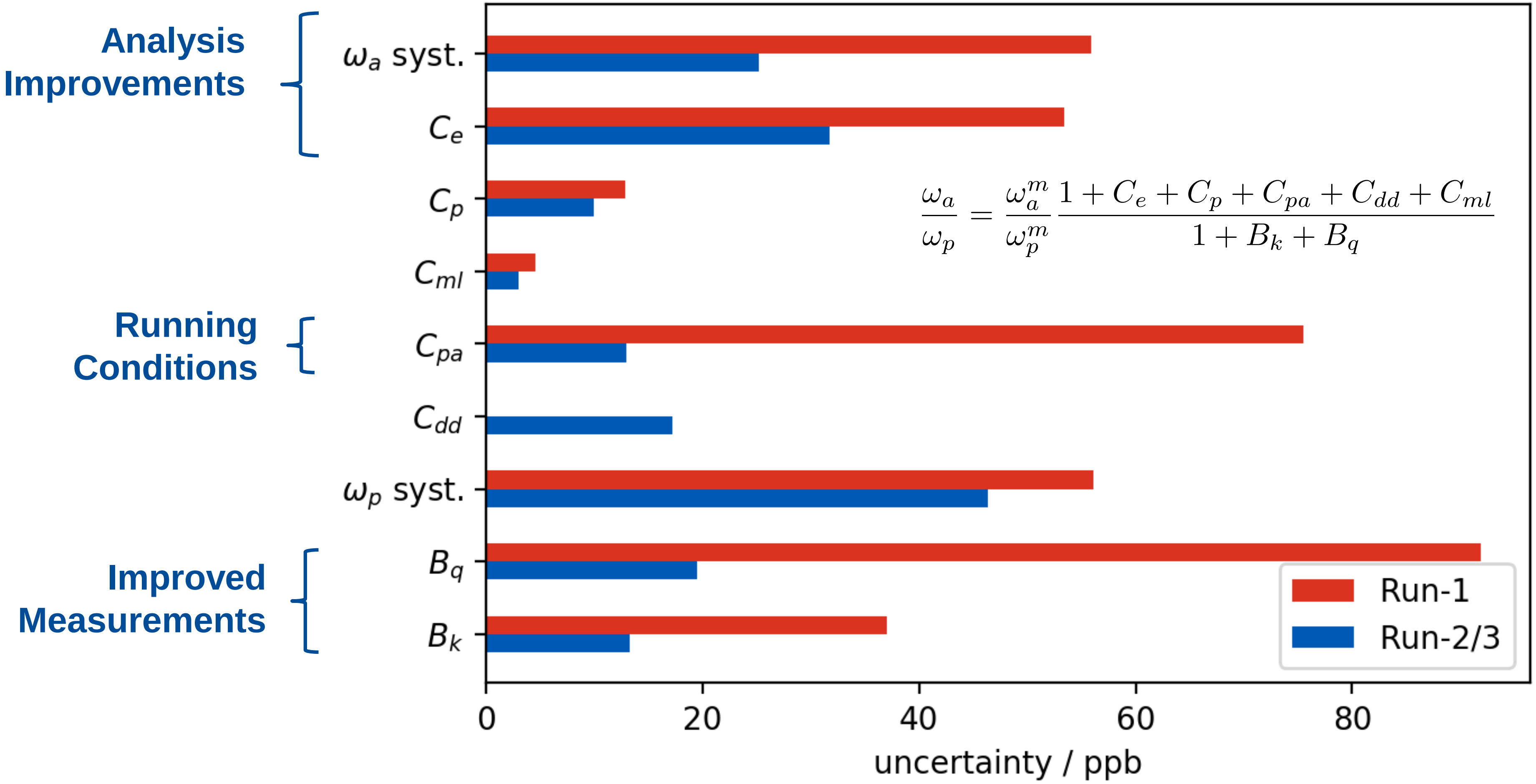
- **Factor 4.7** more data in Run-2/3 than Run-1

Dataset	Statistical Error [ppb]
Run-1	**434**
Run-2/3	**201**
Run-1 + Run-2/3	**185**

Improvement by factor 2.2

Run-1 vs Run-2/3

Systematics



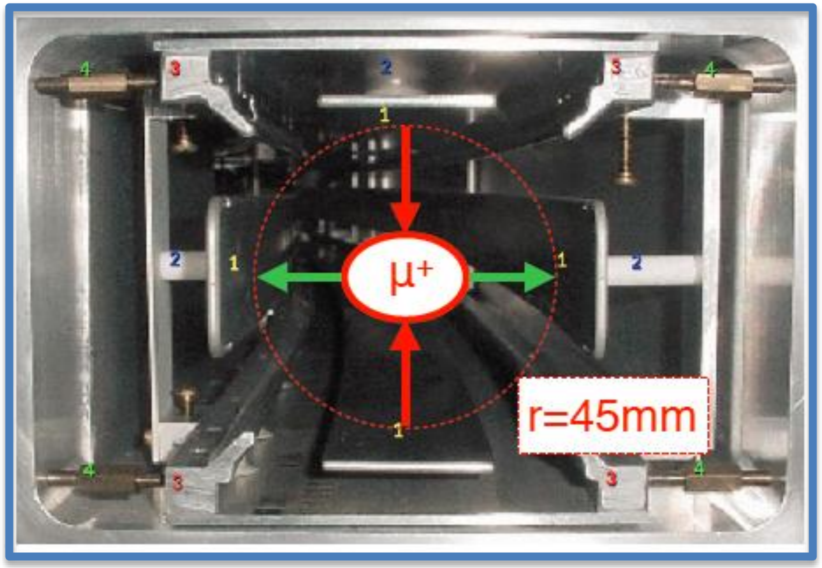
$$\frac{\omega_a}{\omega_p} = \frac{\omega_a^m}{\omega_p^m} \frac{1 + C_e + C_p + C_{pa} + C_{dd} + C_{ml}}{1 + B_k + B_q}$$

Overall improvement by factor 2.2

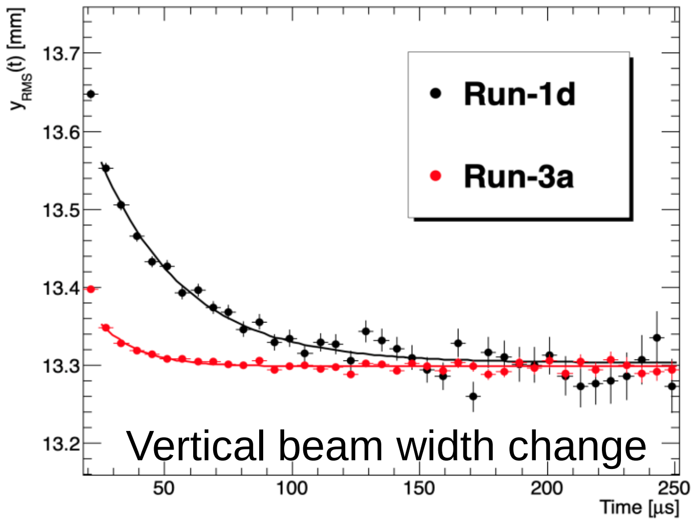
Improvements

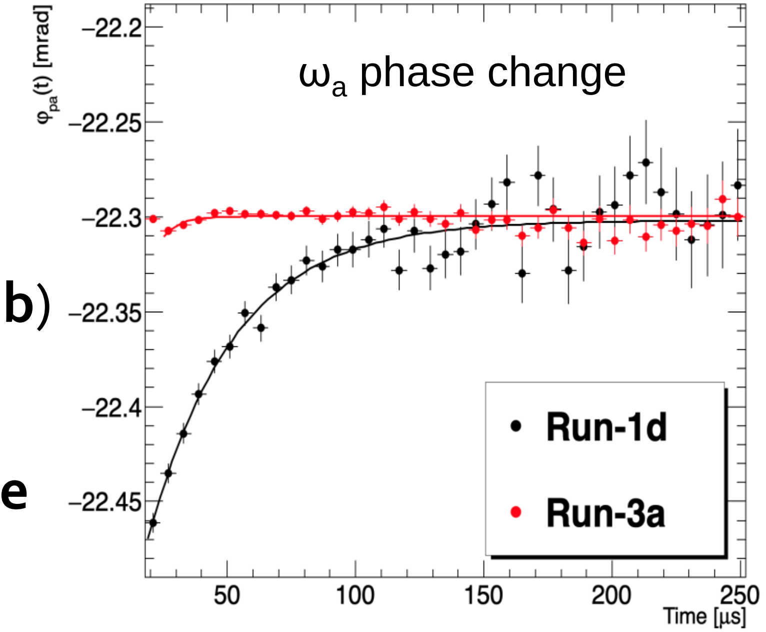
C_{pa}

- Run-1 had **damaged resistors** in 2/32 quad plates leading to **unstable beam storage**
- Resistors **replaced before Run-2**



- **C_{pa}** uncertainty is reduced (**75 ppb → 13 ppb**) thanks to a more stable beam
- Beam **oscillation frequencies** are also **more stable**

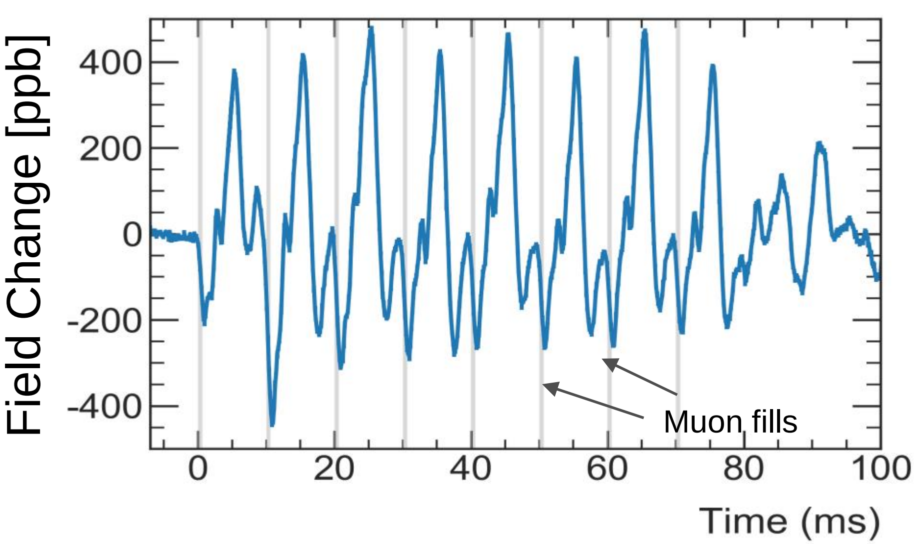


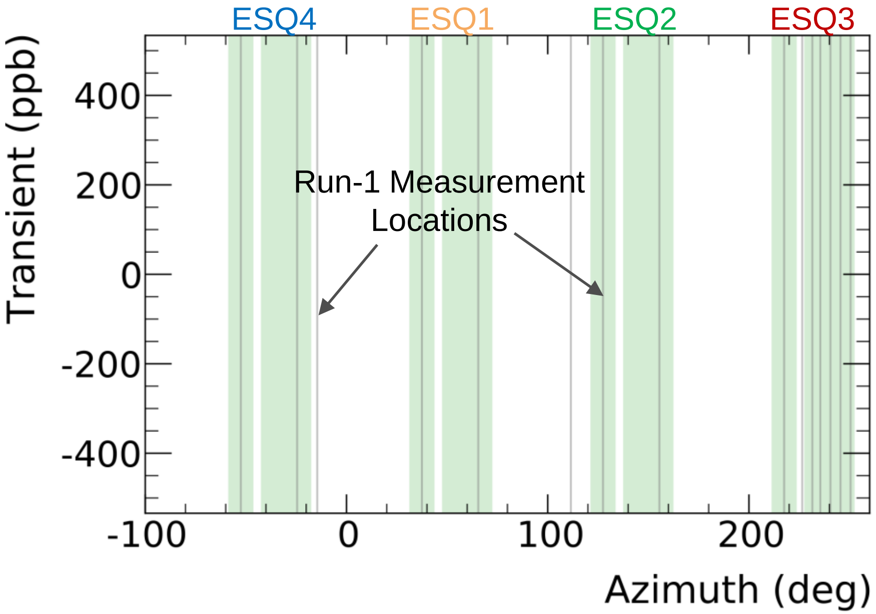


Improvements

B_q

- Pulsing **quads vibrate ⇒ oscillating magnetic fields**
- Measured with a **new NMR probe** housed in insulator



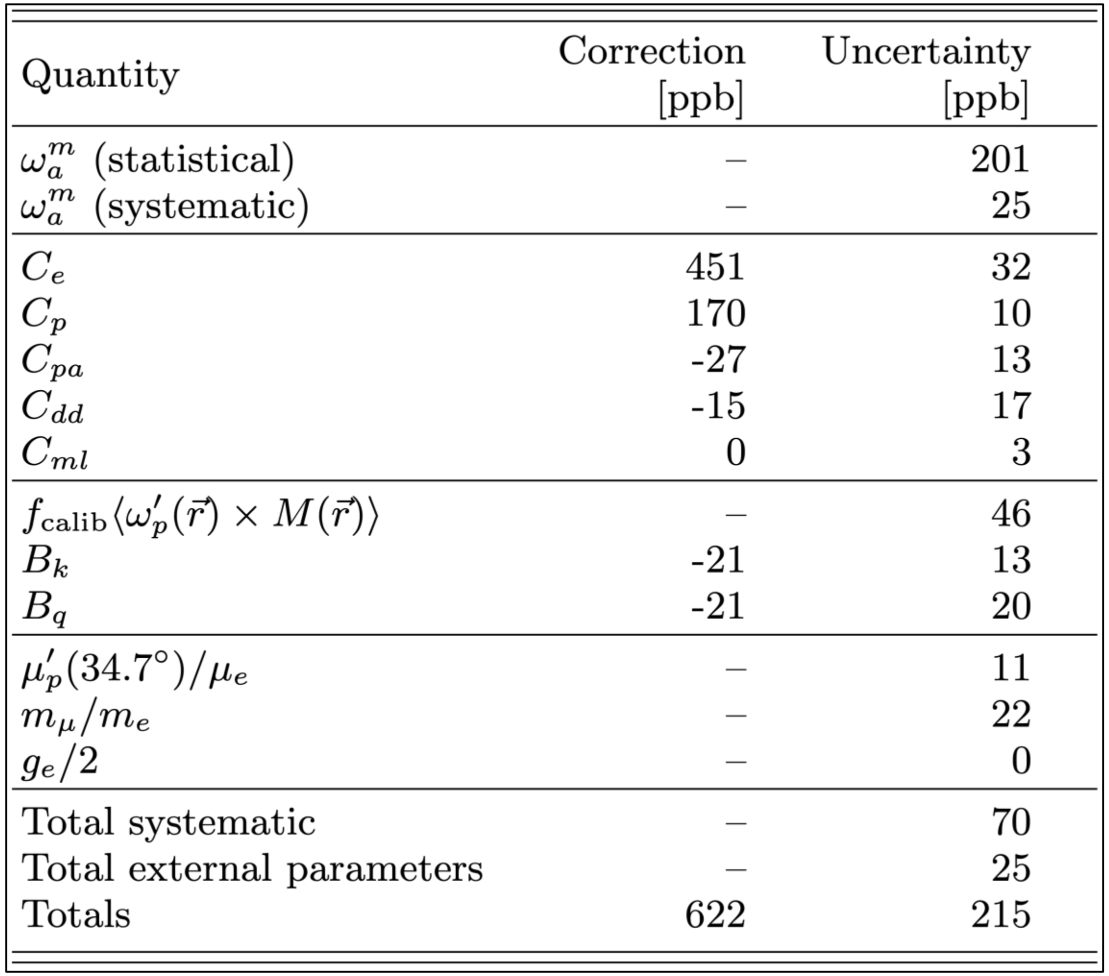


- For Run-1 analysis, we had **limited measurement positions**
- Largest Run-1 systematic: **92 ppb**
- For Run-2/3 the field was fully mapped and uncertainty is reduced to **20 ppb**

Other Improvements

- **Running conditions:**
  - Improved cooling of the hall and added insulation of the magnet which made the magnetic field more stable
  - Improved kicker strength which made the orbit more centered and reduced the E-field correction
- **Improved measurements:**
  - Reduced vibration noise for kicker transient field measurement
- **Analysis improvements:**
  - Improved treatment of the pileup for ω_a analysis
  - Improved analysis of E-field correction including correlations between momentum & time of injection.

Final error table

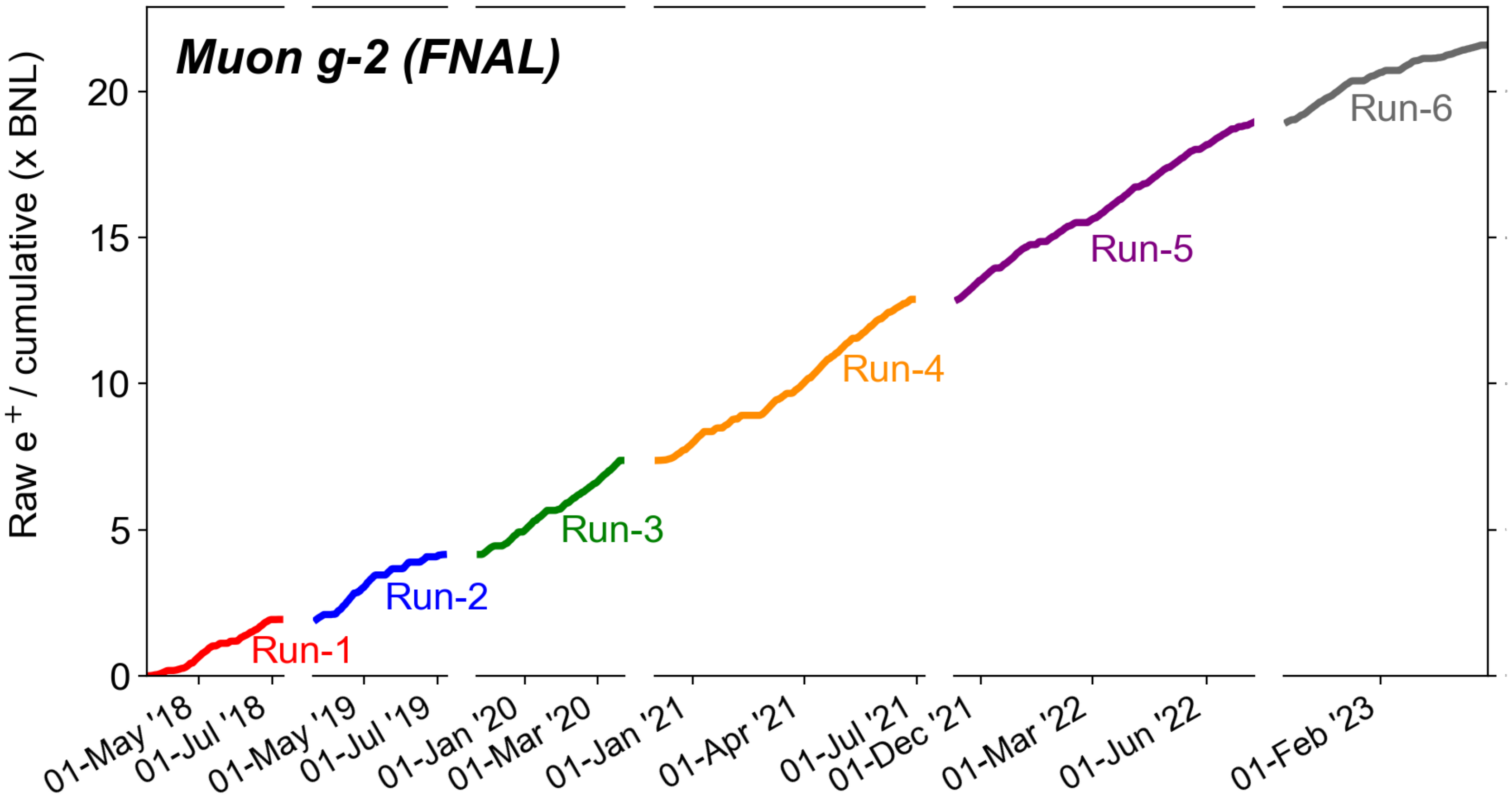


Quantity	Correction [ppb]	Uncertainty [ppb]
ω_a^m (statistical)	–	201
ω_a^m (systematic)	–	25
C_e	451	32
C_p	170	10
C_{pa}	-27	13
C_{dd}	-15	17
C_{ml}	0	3
$f_{\mathrm{calib}} \langle \omega'_p(\vec{r}) \times M(\vec{r}) \rangle$	–	46
B_k	-21	13
B_q	-21	20
$\mu'_p(34.7^\circ)/\mu_e$	–	11
m_μ/m_e	–	22
$g_e/2$	–	0
Total systematic	–	70
Total external parameters	–	25
Totals	622	215

The Run-2/3 result is statistically dominated

70 ppb systematic uncertainty surpasses the proposal goal of 100 ppb!

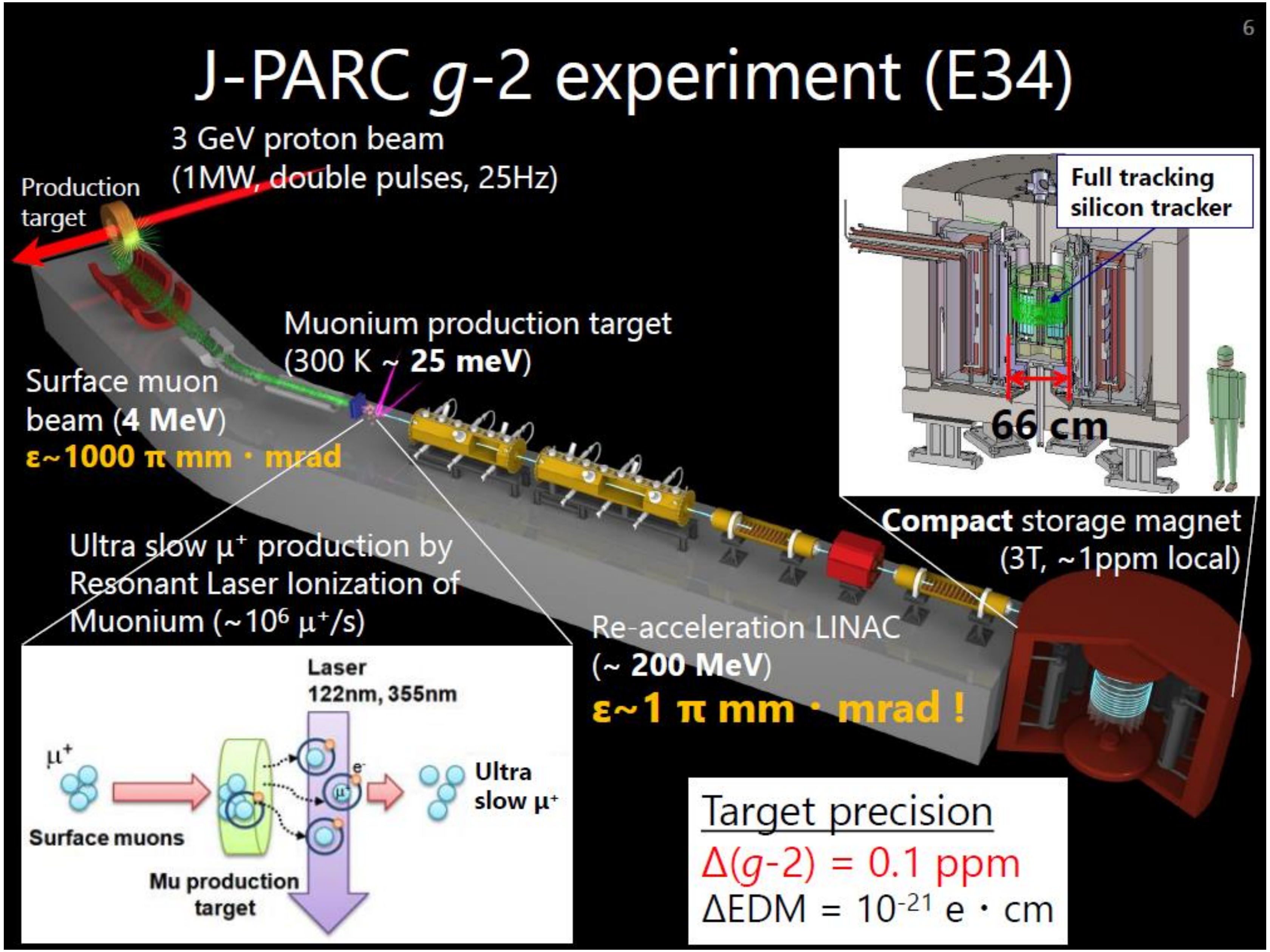
Total collected statistics



21.9 BNL datasets have been collected in FNAL (proposal – 21 BNL)

Run 4/5/6 statistics is x3 Run-1/2/3

J-PARC g-2



6

J-PARC *g*-2 experiment (E34)

3 GeV proton beam
(1MW, double pulses, 25Hz)

Production
target

Muonium production target
(300 K ~ **25 meV**)

Surface muon
beam (**4 MeV**)
$\varepsilon \sim 1000\ \pi$ mm · mrad

Full tracking
silicon tracker

66 cm

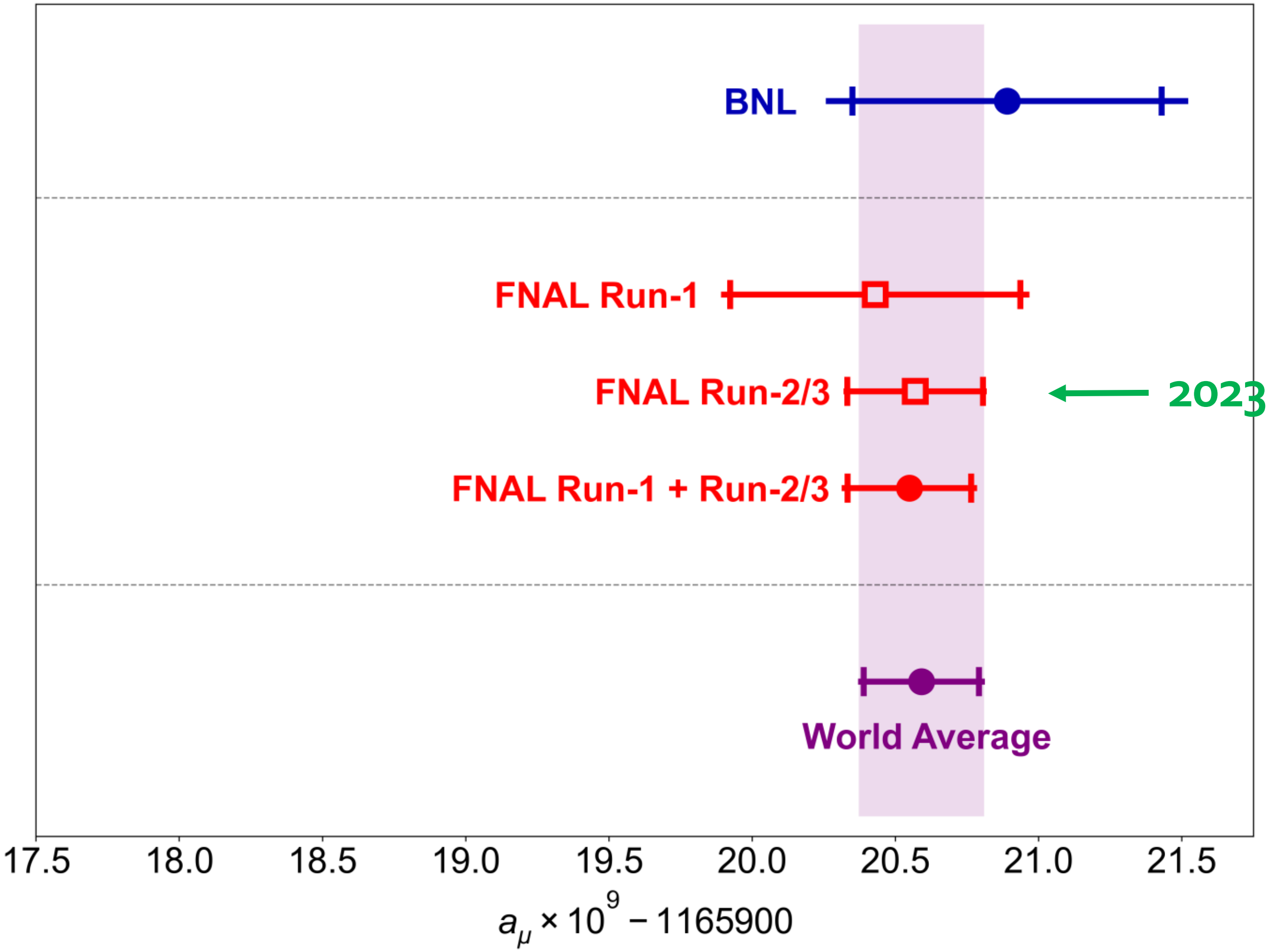
Compact storage magnet
(3T, ~1ppm local)

Ultra slow μ^+ production by
Resonant Laser Ionization of
Muonium ($\sim 10^6\ \mu^+/s$)

Re-acceleration LINAC
(~ **200 MeV**)
$\varepsilon \sim 1\ \pi$ mm · mrad !

Target precision
$\Delta(g\text{-}2) = 0.1$ ppm
$\Delta\text{EDM} = 10^{-21}$ e · cm

Muon G-2 2023 result



← 2023 result

What about theory?