Black holes

Part 3. Hawking radiation and the Information paradox

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Penrose diagrams

Where can you go? $\leftarrow$ causal structure!

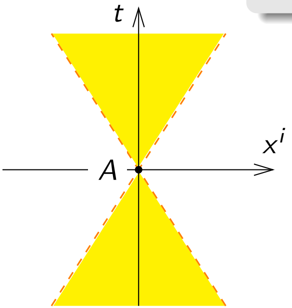
Draw a diagram:

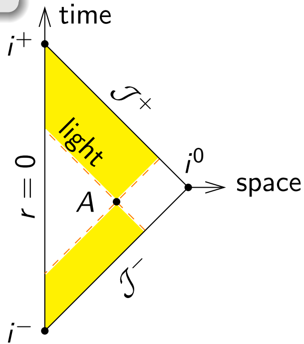
- $\rightarrow$ ignore θ and φ;
- $\rightarrow$ keep time & radius;
- $\rightarrow$ light rays are diagonal (45°)!
- $\rightarrow$ draw infinity as a finite box.

Notations

$i^{\pm}$ — past/future infinities
i^{0} — space infinity
$\mathcal{J}^{\pm}$ — null infinities

flat spacetime





Penrose diagrams

Where can you go? $\leftarrow$ <u>causal structure</u>!

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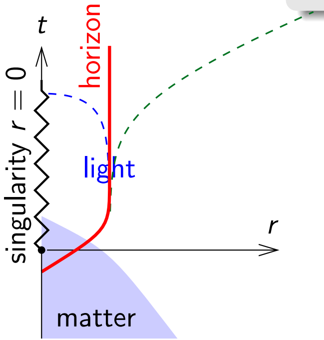
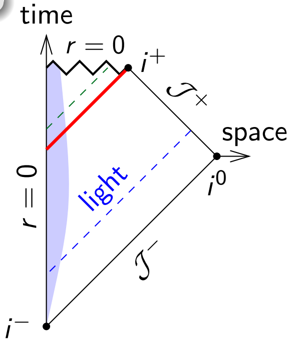
matter collapse





Definition of a black hole

- Black hole = spacetime region
  from where you cannot go to infinity

  Nonlocal: depends on the future!

- Horizon = black hole boundary

  → also nonlocal in time

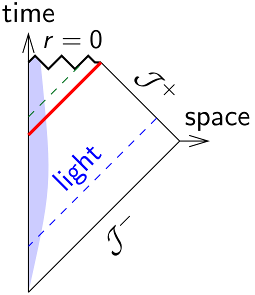
  → fictitious surface

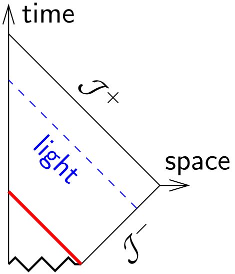
  → may have small curvature

- White hole = spacetime region
  where you cannot enter

  → time reflection $t \to -t$

  → singular spacetime — does not exist?

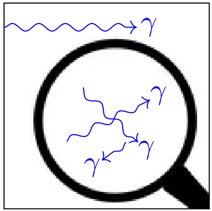




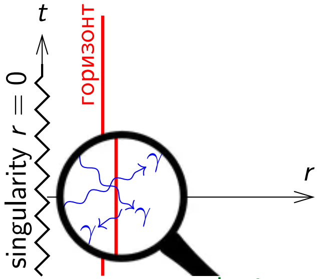
Black holes are defined by causal structure!

Hawking radiation: main idea

Uncertainty relation: $\boxed{\Delta p \gtrsim L^{-1}}$ $\Rightarrow$ **vacuum fluctuations (virtual particles)**



vacuum in QFT



vacuum near horizon

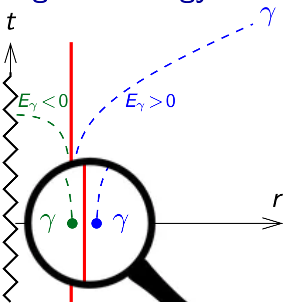
Flat spacetime — **not real particles!**

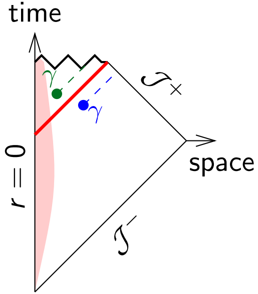
→ Energy conservation: $E_{tot} = 0$

→ Do not exist for long: $\Delta E_\gamma \Delta t \gtrsim 1$

Hawking radiation: main idea

But: **negative energy states under the horizon!**





Pairs of photons $|E_\gamma, -E_\gamma\rangle$:

$\left.\begin{array}{l} \rightarrow \text{born in vacuum: } 2p_\gamma \gtrsim \Delta x^{-1} \\ \rightarrow \text{but conserve energy: } E_{tot} = 0 \end{array}\right\} \Rightarrow$ **real photons!**

Any energy, any particle type!

$$|U\rangle \approx \sum_{E>0} c(E)\, |\underbrace{E}_{\text{Hawking particle}},\ \underbrace{-E}_{\text{falls into singularity}}\rangle$$

$\Rightarrow$ **Ever-lasting radiation!**

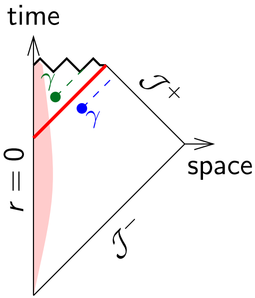
Density matrix

$$\boxed{|U\rangle \approx \sum_{E>0} c(E)\,|E, -E\rangle}$$

- $|U\rangle$ — pure quantum state
- But we cannot register particles under horizon

$\Rightarrow$ $\hat{\rho}_{\mathrm{rad}} = \mathrm{tr}_{\mathrm{bh}}\,|U\rangle\langle U| = \sum_{E>0} |c(E)|^2\,|E\rangle\langle E|$

$\Rightarrow$ $\hat{\rho}_{\mathrm{bh}} = \mathrm{tr}_{\mathrm{rad}}\,|U\rangle\langle U| = \sum_{E>0} |c(E)|^2\,|-E\rangle\langle E|$



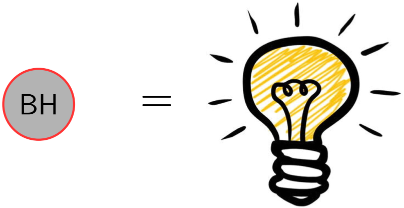
- Hawking radiation is characterized by a density matrix!

$$\langle U|\hat{A}_{\mathrm{rad}}|U\rangle \equiv tr_{rad}\left(\hat{\rho}_{\mathrm{rad}}\hat{A}_{rad}\right)$$

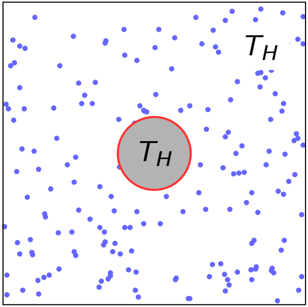
- Actual calculation: $c(E) = Z^{-1}\,\mathrm{e}^{-E/2T_H}$

$\Rightarrow$ **Thermal density matrix** $\boxed{\hat{\rho}_{rad} = Z^{-1}\mathrm{e}^{-\hat{H}_{rad}/T_H}}$ **!**

Black hole shines like a true thermal state!



black hole = lamp at temperature T_H



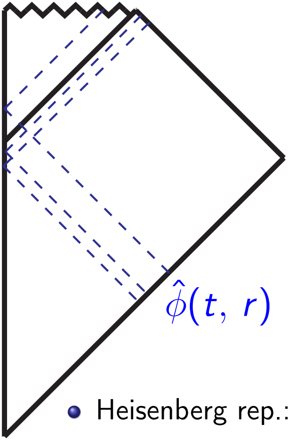
thermal equilibrium!

But:

- Changes chemical composition of the gas
- E.g., breaks $B - L$!

Hawking radiation: the actual calculation

$\boxed{\Box \hat{\phi} = 0}$



- Background is fixed and time–dependent!
- Solution: $\hat{\phi} = \sum_{\omega} \Big[\hat{a}_\omega f_\omega(t,r) + \hat{a}^\dagger_\omega f^*_\omega(t,r) \Big]$

  annihilation/creation operators

  solutions
- Ignore f's falling into BH $\leftrightarrow$ $\mathrm{tr}_{\mathrm{BH}}$
- $f_\omega \propto \begin{cases} \mathrm{e}^{-i\omega t} & \text{as } t \to -\infty \\ \alpha_\omega \, \mathrm{e}^{-i\omega t} + \underbrace{\beta^*_\omega \, \mathrm{e}^{i\omega t}}_{\text{positive–freq.!}} & \text{as } t \to +\infty \end{cases}$

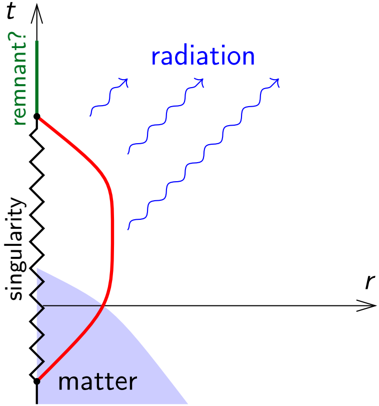
  $\Rightarrow$ True cre/ann operators as $t \to +\infty$:

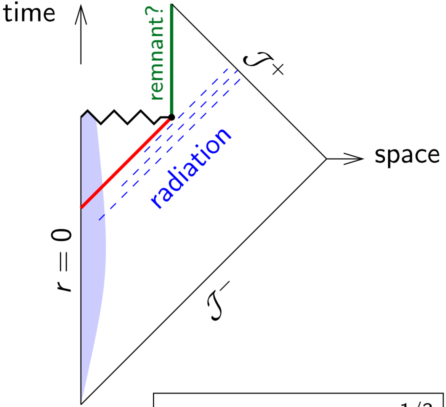
  $\hat{b}_\omega = \alpha_\omega \hat{a}_\omega + \beta_\omega \hat{a}^\dagger_\omega$ and $\hat{b}^\dagger_\omega$
- Heisenberg rep.: $|U\rangle = |0_{\mathrm{in}}\rangle$ — time–independent
- Occupation numbers: $\langle U | \hat{b}^\dagger_\omega \hat{b}_\omega | U \rangle = |\beta_\omega|^2 = \underbrace{\frac{1}{\mathrm{e}^{\omega/T_H} - 1}}$

$\boxed{\hat{\rho}_{rad} = Z^{-1} \mathrm{e}^{-\hat{H}_{rad}/T_H}}$ $\Longleftrightarrow$ **thermal result!**

Black hole lifetime

Radiation + energy conservation ⇒ evaporating BHs!





Stefan–Boltzmann law: $\dfrac{dM}{dt} = -\dfrac{\pi^2}{60} A_h T_H^4 \Rightarrow \boxed{M = M_i \left[1 - \dfrac{t}{t_{BH}}\right]^{1/3}}$

Lifetime: $t_{BH} = 5120\pi G^2 M_i^3$

Primordial BHs: $\boxed{M_i > 10^{14}\ g \sim 10^{-20}\ M_\odot \quad \Leftrightarrow \quad t_{BH} > 10^{10}\ \mathrm{yr}}$

Generalized black hole thermodynamics

law I: Energy conservation $S_B dT_H = dM$

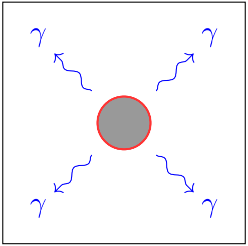
law II: Total entropy cannot decrease

$$S_{\rm tot} = \frac{A_h}{4 l_{pl}^2} + S_{\rm rad}$$

law 0: Thermal states = maxima of $S_{\rm tot}$!

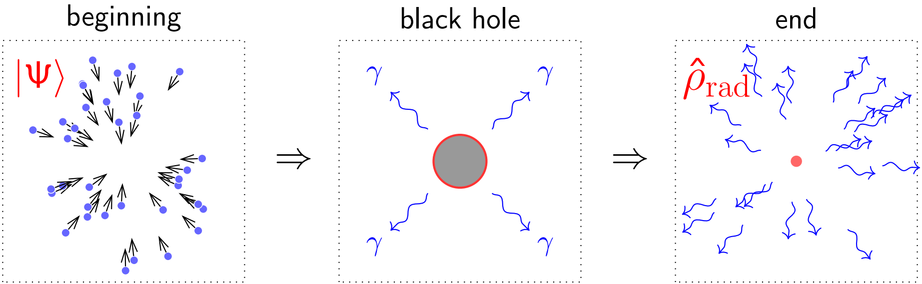
law III: Critical BHs cannot be reached in finite time

black hole



Now, we have a consistent picture!

Ha!



$$|\Psi\rangle \xrightarrow{\text{evolution}} \underbrace{\hat{\rho}_{\mathrm{rad}}}_{\text{depends \underline{only} on } T_H}$$

Impossible in a unitary quantum theory!

- Complete loss of information on $|\Psi\rangle$!
- This is the <u>information paradox</u>

$|\Psi\rangle \to \hat{\rho}_{\rm rad}$: what is wrong?

- Errors in calculation $\leftrightarrow$ pure Hawking radiation?
- Quantum theory is wrong?
- General relativity should be modified?
- Remnant will save the day?

$|\Psi\rangle \to \hat{\rho}_{\rm rad}$: what is wrong?

- Errors in calculation $\leftrightarrow$ pure Hawking radiation?

  $\to$ Corrections to Hawking result: series in $\boxed{M_{pl}/M \ll 1}$

  $\to$ Can they purify radiation: $\hat{\rho}_{\rm rad} = |\Psi_{\rm rad}\rangle\langle\Psi_{\rm rad}| \approx Z^{-1}{\rm e}^{-\hat{H}/T_H}$?
- ~~Quantum theory is wrong?~~

  $\to$ Should be non–unitary!

  $\to$ Loss of predictability & ~~CPT~~

  Black hole: cannot predict the past

  Time reversal: cannot predict the future
- ~~General relativity should be modified?~~

  $\to$ Definition of horizon depends on future

  $\Rightarrow$ Such GR modifications should be strongly nonlocal!

  $\Rightarrow$ Consistent modifications were not found ...
- Remnant will save the day?

  $\to$ Expected to have $M \sim M_{pl}$

  $\Rightarrow$ Too small to contain all information?

Cloning paradox

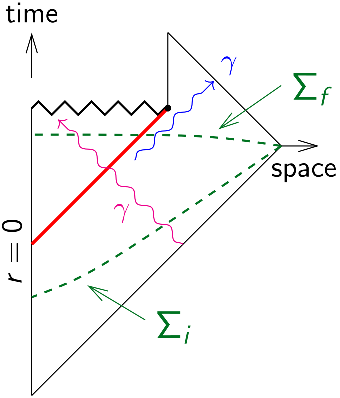
Suppose Hawking radiation is pure ← error in calculations

⇒ Hawking photons γ contain all info about fallen photons γ

- Nice slices $\Sigma_{i,\,f}$ — smooth 3d spaces
- Initial state $|\Psi_\gamma\rangle$ — lives in Σ_i

  Final state $|\Psi_\gamma,\, \Psi_\gamma\rangle$ — lives in Σ_f
- $\boxed{|\Psi_\gamma\rangle \xrightarrow{\text{evolution}} |\Psi_\gamma,\, \Psi_\gamma\rangle}$

  cloning



We are in trouble!

Cloning is forbidden in quantum theory

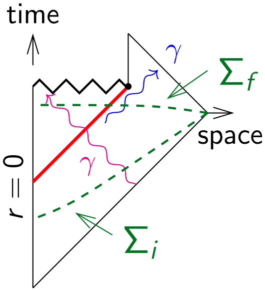
$\left.\begin{array}{l}\text{measure } x_\gamma \text{ of one photon}\\ \text{measure } p_\gamma \text{ of another photon}\end{array}\right\}$ breaks uncertainty

We cannot pretend that the paradox does not exist!

Black hole complementarity

Complementarity in QM: cannot measure $\hat{p}$ and $\hat{x}$ simultaneously

Observers outside never meet observers inside



Black hole complementarity

Cannot compare measurements
inside and outside of BH

$\Rightarrow$ Cannot define final state on Σ_f

$\rightarrow$ Not clear, where it lives

$\Rightarrow$ **Falling and outside observers**
see different worlds!

... like the BH insides mirror the BH outsides

Black holes are strange!

Cloning paradox returns!

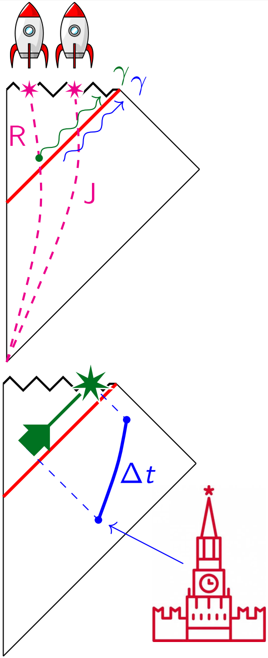
Kamikaze: R = Romeo, J = Josephine

Paradox:

- Romeo jumps into a BH & sends γ
- Josephine waits & jumps
- Meets both R-photon γ
  and Hawking photon γ!

Scrambling time = max time to reach $r = 0$

$$\boxed{t_{\rm scr} = 2r_h \ln(r_h/l_{pl})}$$



Resolution of the paradox

Assume BHs detain information for

$$\Delta t \geq t_{scr}$$

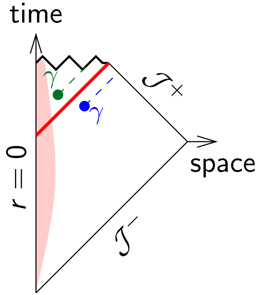
Looks strange!

Maybe, remnant saves the day?

Recall Hawking calculation:

- Particles are created pairs
- Pairs are in pure state $|U\rangle$

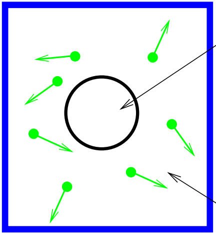
$\Rightarrow$ BH + radiation is in pure state!



Remnant with $M = M_{pl}$ contains all information?

No, it does not!

Entanglement entropy



Black hole, $\Gamma_{BH} = e^{A_h/4l_{pl}^2}$

$$\hat{\rho}_{BH} = \mathrm{tr}_{\mathrm{rad}} |\Psi_{\mathrm{tot}}\rangle\langle\Psi_{\mathrm{tot}}|$$

Radiation, $\Gamma_{\mathrm{rad}} = e^{S_{\mathrm{rad}}}$

$$\hat{\rho}_{\mathrm{rad}} = \mathrm{tr}_{\mathrm{BH}} |\Psi_{\mathrm{tot}}\rangle\langle\Psi_{\mathrm{tot}}|$$

Entanglement entropy: $\boxed{E_i = -\mathrm{tr}_i(\hat{\rho}_i \ln \hat{\rho}_i)}$

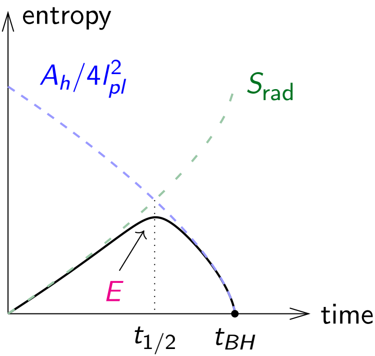
Algebraic theorems:

- $|\Psi\rangle$ — pure state $\Rightarrow E_{\mathrm{rad}} = E_{\mathrm{BH}}$
- $\max E_i = \ln \Gamma_i$ — reached at thermal $\hat{\rho}_i$
- $\min E_i = 0$ — reached at pure $\hat{\rho}_i = |\Psi_i\rangle\langle\Psi_i|$

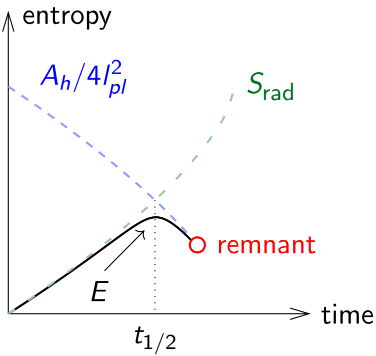
Conclusion

- $E_{\mathrm{rad\ or\ BH}} \leq \min\left(\ln \Gamma_{\mathrm{rad}}, \ln \Gamma_{\mathrm{BH}}\right) = \min\left(A_h/4l_{pl}^2,\ S_{\mathrm{rad}}\right)$

Page time



Page curve: no remnant



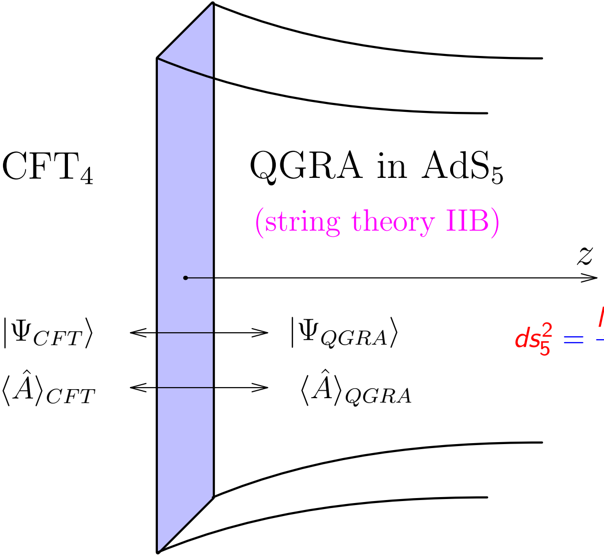
Page curve: remnant

Planckian remnant does not help!

AdS/CFT correspondence

A formulation of quantum gravity

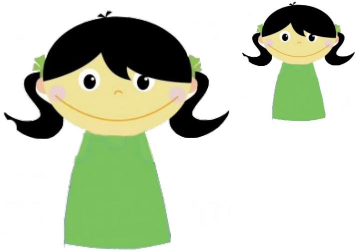
Maldacena, 1997



$$ds_5^2 = \frac{l_{\mathrm{AdS}}^2}{l^2}\left(\eta_{\mu\nu}dx^\mu dx^\nu + dl^2\right)$$

Conformal Field Theories: theories without scale

CFT:

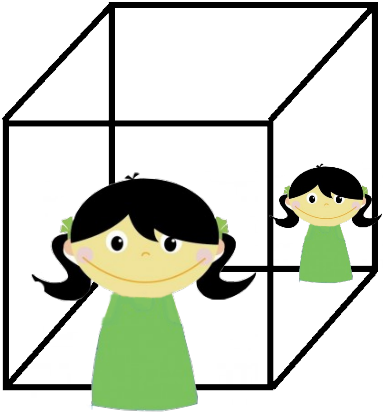


The second girl is smaller

All sizes are equivalent

The idea of the AdS/CFT correspondence

AdS$_5$:



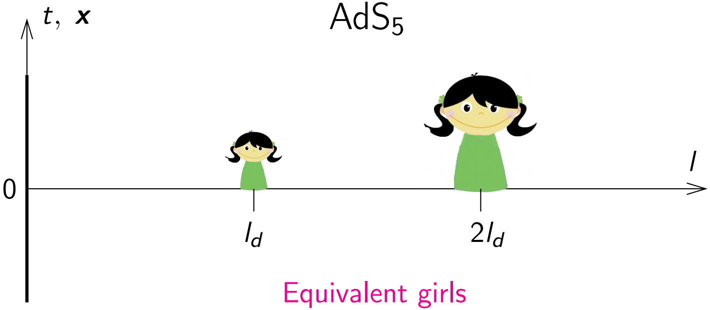
The second girl is further away

The girls are equal in fifth dimension

AdS_5 as a book–keeper of the sizes

$$\boxed{ds_5^2 = \frac{l_{\mathrm{AdS}}^2}{l^2}\left(\eta_{\mu\nu}dx^\mu dx^\nu + dl^2\right)}$$

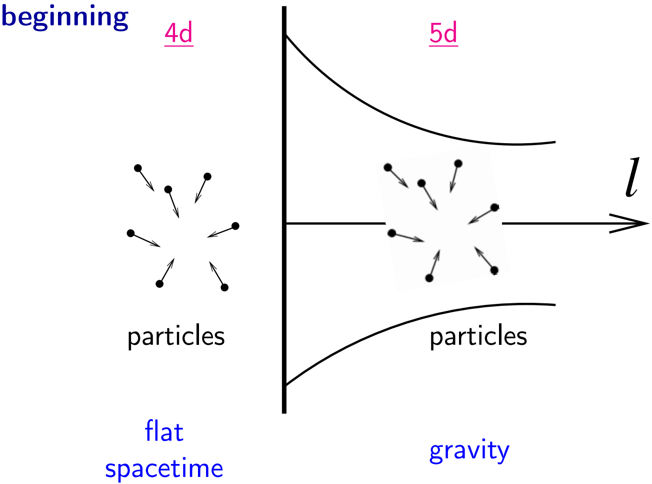
Symmetry: $x^\mu \to a x^\mu$, $l \to a l$



Equivalent girls

The paradox is absent!

CFT is a unitary theory in flat spacetime!



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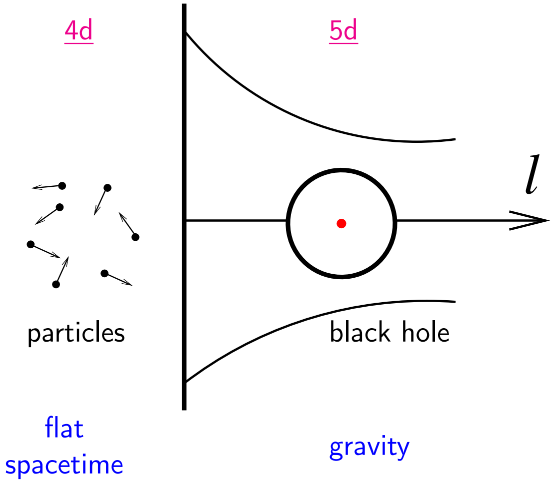
CFT is a unitary theory in flat spacetime!

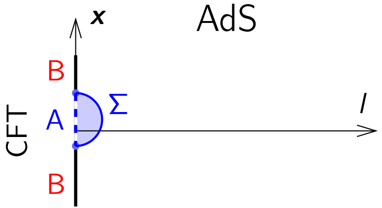
end



Search better: information is conserved!

Ryu–Takayanagi formula for the entanglement entropy

- Divide space into $A \cup B = \mathbb{R}^3$
- Σ — hypersurface with boundary A



Entanglement entropy:

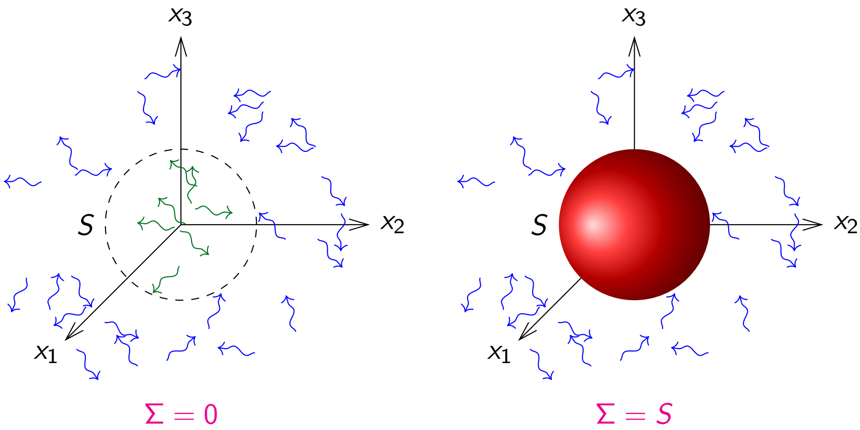
- CFT: $E_A^{\mathrm{CFT}} = \mathrm{tr}_B\left(\hat{\rho} \ln \hat{\rho}\right)$
- AdS$_5$ $E_A^{\mathrm{AdS}} = \min_\Sigma \left\{ \dfrac{A_\Sigma}{4 l_{pl}^3} + E(\text{matter inside } \Sigma) \right\}$

AdS/CFT: $\boxed{E_A^{\mathrm{CFT}} = E_A^{\mathrm{AdS}}}$

Application to black holes

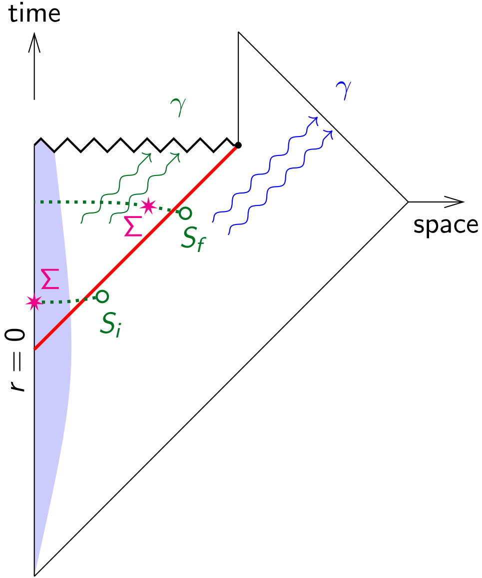
Penington '19

$$\boxed{E = \min_{\Sigma} \left\{ \frac{A_\Sigma}{4 l_{pl}^3} + E(\text{matter inside } S\text{, but outside } \Sigma) \right\}}$$



Application to black holes

Penington '19



Summary

Black holes shine

- As black bodies with $T_H = (8\pi G)^{-1}$
- Live $t_{BH} = 5120\pi G^2 M^3$
- This is consistent with BH thermodynamics

No information paradox

- How to get this information? — Nobody knows

Black holes are strange!

- The internal space is a copy of the space outside
- Hold information for scrambling time
- Start to release it after Page time
- Strange formula for the entanglement entropy

Thank you!