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Ising field theory in a magnetic field: analytic properties of the free energy

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Main idea

The project is based on the article [1] and the idea is to get more accurate results using Truncated Free Fermion Space Approach. We have implemented TFFSA method and got numerical results for lowest finite size energy levels $E_i(R)$ for some real η. We study the analytical properties of the scaling function associated with the 2D Ising model free energy in the critical domain $T \to T_c, H \to 0$.

Methods

The analysis is based on numerical data obtained through the Truncated Free Fermion Space Approach (TFFSA), which was firstly introduced in [1]. It is a modification of the well-known Truncated Conformal Space Approach (TCSA) [2,3]. This modification is designed specifically to treat the case of Ising Field Theory (while TCSA is applicable to a wide class of perturbed Conformal Field Theories (CFT). The Ising model free energy exhibits a singularity at the critical point $H = 0, T = T_c$. The singularity is described in terms of the Euclidian quantum field theory known as the Ising Field Theory. It can be defined as a perturbed conformal field theory through the action

$$A_{IFT} = A_{(c=1/2)} + \tau \int \varepsilon(x) d^2x + h \int \sigma(x) d^2x,$$

where $A_{(c=1/2)}$ stands for the action of c = ½ conformal field theory of free massless Majorana fermions, $\sigma(x)$ and $\varepsilon(x)$ are primary fields of conformal dimensions 1/16 and 1/2.

Results

(A) Implemented Truncated Free Fermion Space Approach.

The energies associated with the N-particle states, which can be obtained from the Neveu-Schwartz (NS) and Ramond (R) vacua by applying the corresponding canonical fermionic creation operators:

NS sector: $|k_1, \dots, k_N\rangle_{NS} = a^\dagger_{k_1} \dots a^\dagger_{k_N} |0\rangle_{NS}$ $\quad k_1, \dots, k_N \in \mathrm{Z} + 1/2$

R sector: $|n_1, \dots, n_N\rangle_R = a^\dagger_{n_1} \dots a^\dagger_{n_N} |0\rangle_R$ $\quad n_1, \dots, n_N \in \mathrm{Z}$

$$E_{N(NS)} = E_{0(NS)}(R) + \sum_{i=1}^{N} \omega_{k_i}(R),$$

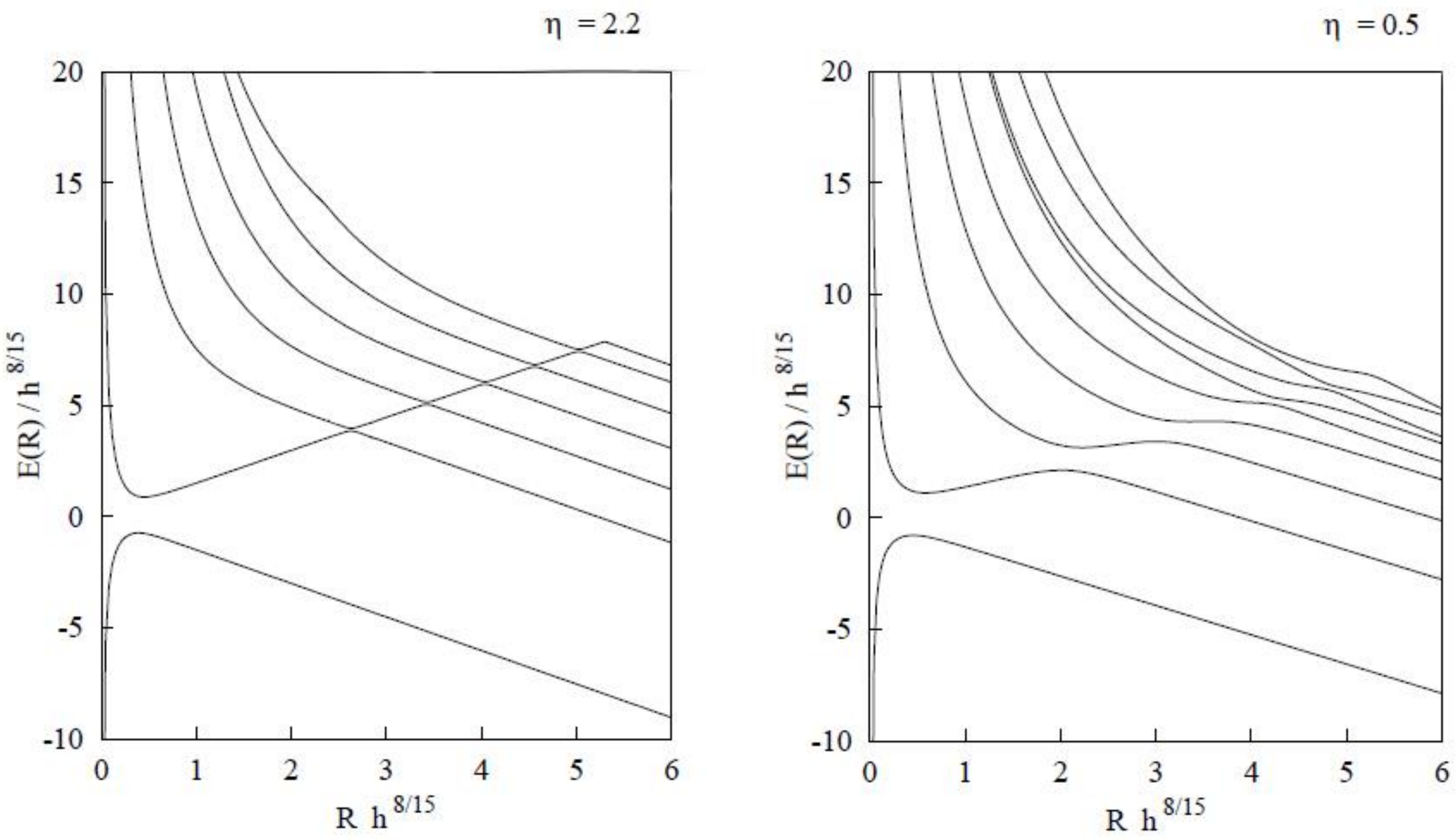
$$E_{N(R)} = E_{0(R)}(R) + \sum_{i=1}^{N} \omega_{n_i}(R),$$

where $\omega_k(R) = \sqrt{m^2 + (2\pi k/R)^2}$;
$\omega_n(R) = \sqrt{m^2 + (2\pi n/R)^2}$,
(with positive branch of the square root taken)
$E_{0(NS)}, E_{0(R)}$ – ground-states eigenvalues of

$$H_{IFT} = H_{FF} + hV, \qquad V = \int_o^R \sigma(x)\, dx$$

(B) Numerical analysis

Plots of several lowest energy levels $E_i(R), \; i = 0,1,2 \dots$ at different η.



(C) Analytical properties of a scaling function

The bulk energy density $F(m,h)$:

$$F(m,h) = \frac{m^2}{8\pi} \log m^2 + m^2 G(\xi)$$

We need two scaling functions $G(\xi)$ to describe free energy $F(m,h)$ (for positive and negative m): $G_{low}(\xi)$ for $m > 0$ and $G_{high}(\xi)$ for $m < 0$. They describe the free energy in low-T and high-T regime, respectively.

$G_{high}(0) = G_{low}(0) = 0$

- The function $G_{high}(\xi)$:
  $G_{high}(\xi) = G_{high}(-\xi)$. Around $\xi = 0$:
  $$G_{high}(\xi) = G_2 \xi^2 + G_4 \xi^4 + G_6 \xi^6 + \cdots$$
  $$G_{high}(\xi) = -\xi^2 \int_{\xi_0}^{\infty} \frac{2\Im m G_{imh}(t)}{t(t^2 + \xi^2)} \frac{dt}{\pi} \quad [4]$$
- The function $G_{low}(\xi)$:
  $G_{low}(\xi)$ is not an even function. Assimptotic expansion:
  $$G_{low}(\xi) \approx G_1 \xi + G_2 \xi^2 + \cdots$$
  $$G_{low}(\xi) = G_1 \xi - \xi^2 \int_{\xi_0}^{\infty} \frac{\Im m G_{meta}(t)}{t^2 (t + \xi)} \frac{dt}{\pi}$$

where $G_{meta}(\xi) \equiv G_{low}(-\xi + i0)$ [5],[6]

Conclusion and discussion

We have implemented TFFSA method to study Ising model free energy in its critical domain. Using numerical analysis, we plot several lowest energy levels. We studied analytical properties of the scaling function and also we are doing some correction work in numerical analysis of the scaling function $G(\xi)$.

Reference

[1] P. Fonseca, A. Zamolodchikov, Ising field theory in a magnetic field: analytic properties of the free energy
[2] V.P. Yurov, Al.B.Zamolodchikov, Truncated Conformal Space Approach to Scaling Lee-Yang Model
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[5] N.J.Günther, D.A. Nicole, D.J.Wallace, Goldstone model in vacuum decay and first-order phase transition
[6] M.J.Lowe, D.J.Wallace, Instantons and the Ising model below T_c.