Quantum effects and the effective action analysis in the massive two-dimensional CP(N-1) sigma model in the large N limit

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Introduction

QCD dynamics description is one of the main unsolved problems in theoretical physics. One approach to this problem is to consider an alternative theory, scalar QCD. There is a confinement-like mechanism, with its own non-Abelian strings. Two-dimensional sigma model,as the effective theory on the surface of such a string and with a mass deformation parameter m related to Z_N symmetry, is the object of our study. For $m = 0$ case this model was solved by Witten [1] in the large-N limit: massless photon interacts with N scalar fields n, "quarks", each of which carries a charge $\sim \frac{1}{\sqrt{N}}$ and which appear in the spectrum only in pairs n^*n, since between them there is a confining potential that grows linearly with distance. The purpose of our research is to generalize this result for the $m \neq 0$ case — there we can expect a phase transition which does not occur in its supersymmetric version.

Effective theory

The mass deformed action in the Euclidean formulation:

$$\mathcal{L} = |D_\mu n^\ell|^2 + \lambda\left(|n^\ell|^2 - r_0\right) + \sum_{\ell=1}^{N} |(\sigma - m_i)n^\ell|^2 - \tau_0 \sum_{\ell=1}^{N} |\sigma - m_\ell|^2. \qquad (0.1)$$

(action as in [2] with extra $\sim \tau_0$ term)
r_0 and τ_0 define scales of the theory, Λ andΛ_σ:

$$\Lambda^2 = M_{uv}^2 \exp\left(-\frac{4\pi r_0}{N}\right), \quad \Lambda_\sigma^2 = M_{uv}^2 \exp(-4\pi\tau_0).$$

The effective action can be obtained by integrating over $N-1$ n^ℓ fields:

$$V_{eff} = \left(\lambda + |\sigma - m_0|^2\right)|n|^2 + \frac{1}{4\pi}\sum_{\ell=1}^{N-1}|\sigma - m_\ell|^2 c$$
$$+ \frac{1}{4\pi}\sum_{\ell=1}^{N-1}\left(\lambda + |\sigma - m_\ell|^2\right)\left[1 - \ln\frac{\lambda + |\sigma - m_\ell|^2}{\Lambda^2}\right], c \equiv \ln\frac{\Lambda_\sigma^2}{\Lambda^2}.$$

From it we define renormalized r_0 constant:

$$r_{ren} = \frac{1}{4\pi}\sum_{\ell=1}^{N-1}\ln\frac{\lambda + |\sigma - m_\ell|^2}{\Lambda^2}.$$

The Higgs phase.

If $r_{ren} > 0$, then we are in the Higgs phase, vacuum equations from the effective action in this case are

$$|n|^2 = r_{ren}, \quad \lambda = -|\sigma - m_0|^2, \quad \sigma = \frac{m}{c}\left[\ln\left(\frac{\sigma m}{\Lambda^2}\right) + 1\right].$$

From these equations and the condition of the phase transition $r_{ren} = 0$ we find the phase transition point: $m^2 = c\Lambda^2$. Effective potential as a function of σ from these equations:

$$V_{eff}^{(Higgs)}(\sigma) = -\frac{m^2}{4\pi}N\left[2\frac{\sigma}{m}\left(\ln\frac{m^2}{\Lambda^2} + \ln\frac{\sigma}{m}\right) - c\left(\left(\frac{\sigma}{m}\right)^2 + 1\right)\right].$$

The Coulomb/confining phase.

In the Coulomb phase $r_{ren} = 0$. Then, vacuum equations are

$$n = 0, \quad \lambda = \Lambda^2 - m^2, \quad \sigma = 0.$$

Effective potential:

$$V_{eff}^{(Coulomb)}(\sigma) = \frac{N}{4\pi}\left[\Lambda^2 + cm^2 + c\sigma^2\left(1 - \frac{m^2}{c\Lambda^2}\right)\right].$$

Vacua and energy at the transition point.

It turns out that vacuum energy is a continuous function of m, but its first derivative has discontinuity at the transition point:

$$\frac{\partial}{\partial m}E^{(Coulomb)}\Big|_{m=\sqrt{c}\Lambda} - \frac{\partial}{\partial m}E^{(Higgs)}\Big|_{m=\sqrt{c}\Lambda} = \frac{N\Lambda}{2\pi\sqrt{c}}.$$

As for σ it has zero VEV in the Coulomb phase $\Rightarrow$ we are in Z_N symmetric phase. In the Higgs phase it has zon-zero VEV, i. e., it is Z_N asymmetric phase.

Correspondence to the classical solution

Since when for $m >> \Lambda, \Lambda_\sigma$ theory is at weak coupling, there must be correspondence with the classical solution:

$$\sigma = m_{\ell_0}\frac{r_0}{r_0 - N\tau_0}, \quad n^{\ell_0} = \sqrt{r_0}, \quad \text{and } n^\ell = 0 \text{ if } \ell \neq \ell_0.$$

It can be simply checked if one substitute the definition of r_0 and τ_0 into denominator and go to the renormalized value $r_0 \to r_{ren}$, then we get:

$$\sigma \approx \frac{m}{c}\ln\frac{m^2}{\Lambda^2},$$

and that is exactly the VEV of σ that can be obtained from vacuum equations for large m.

Dynamics in different phases.

We star with the Coulomb/confining phase. We restore the effective action. It consists of the effective potential and kinetic terms for the gauge and σ fields which induced at one loop of n^ℓ:

$$\mathcal{L}_{\text{Coulomb}} = -\frac{1}{4e_{\text{ren}}^2}F_{\alpha\beta}^2 + \frac{1}{e_\sigma^2}|\partial\sigma|^2 - V_{eff}^{(Coulomb)}(\sigma).$$

Coupling constant from loop calculations ([3],[4]):

$$e_{ren}^2 = \frac{12\pi\Lambda^2}{N}, \quad e_\sigma^2 = \frac{24\pi\Lambda^4}{Nm^2}.$$

From the Lagrangian we find field masses:

$$m_\gamma^2 = 2e_{ren}^2 r_{ren} = 0, \quad m_\sigma^2 = \frac{Nc}{4\pi}\left(1 - \frac{m^2}{c\Lambda^2}\right)e_\sigma^2 = \frac{6c\Lambda^4}{m^2}\left(1 - \frac{m^2}{c\Lambda^2}\right).$$

We can compare m_σ^2 with the lightest meson mass consisted of two n_1 quarks, $m_{meson} \approx 2m_{n_1} = 2\Lambda$, and find the gap of stability of the σ particle. It turns out it is

$$\frac{3}{5}c\Lambda^2 < m^2 < c\Lambda^2.$$

(upper boundary is the phase transition point)
As for the Higgs phase, σ and photon fields do not have any dynamics since coupling constant have bad N behaviour which causes infinite contributions from their kinetic terms.

Conclusions

In this work we generalized Witten's massless large-N analysis [1] on $m \neq 0$ case. We showed that in the original $U(1)$ gauge invariant formulation of $CP(N-1)$ arises an extra term required for the self-consistent renormalization procedure. We derived vacuum equations in an one-loop approximation and found the phase transition point, which distinguish Z_N asymmetric and symmetric phases. In each phase vacuum equations were solved, vacuum energy was calculated. It turned out that energy does not have a discontinuity, but its first derivative with respect to m do. It means we are dealing with the second order phase transition.
Also we tried to describe the dynamics in the Coulomb/confining and Higgs phases. For the first one we obtained the generalized result which coincides with the Witten's one[1] if we apply $m = 0$. Moreover, photon remains massless as should be expected in the Coulomb phase.

As we can see, photon and σ are massless in the transition point, therefore a further possible development of this work is to find some conformal theory in this point.

References

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