Moscow International School of Physics
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Supernova neutrino oscillations and leptonic CP-violation

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Outline

- Dirac and Majorana neutrinos
- Neutrino mixing and CP-violation
- Electromagnetic properties of neutrinos
- Neutrinos in astrophysics
- Supernova neutrino oscillations and CP-violation

Majorana neutrinos mixing

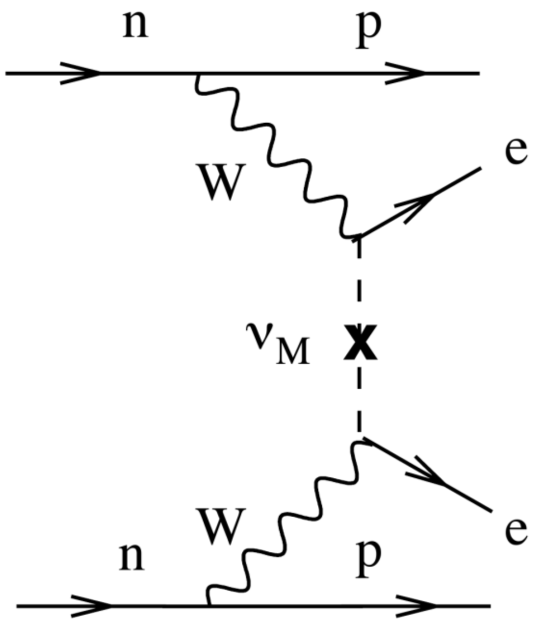
$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{i\alpha_1} & 0 & 0 \\ 0 & e^{i\alpha_2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$c_{ik} = \cos\theta_{ik}$$
$$s_{ik} = \sin\theta_{ik}$$

Dirac CP-violating phase

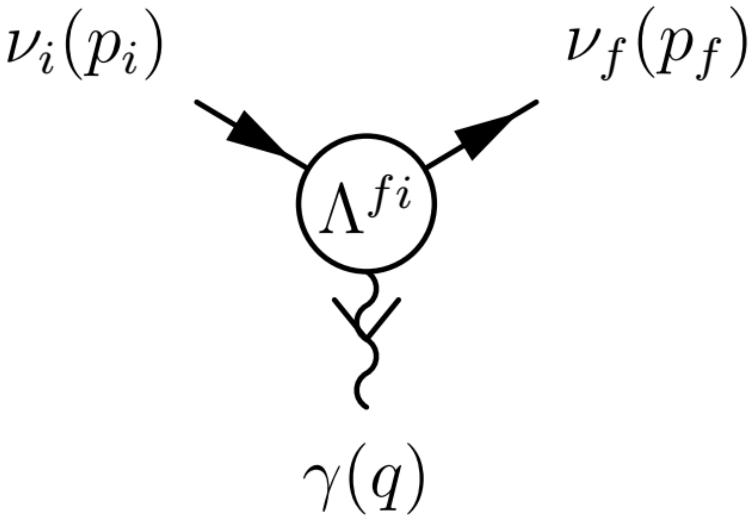
Majorana CP-violating phases

- **Dirac CP-violating phase** can be measured by oscillatory experiments.
- Neutrinoless double beta decay experiments are potentially sensitive to the values of **Majorana CP-violating phases.**
- Majorana phases can be potentially probed by e/m properties (this talk).



G.C. Branco, R.Gonzalez Felipe, F.R. Joaquim, Rev.Mod.Phys. 84 (2012) 515-565

Neutrino electromagnetic properties



$$\mathcal{H}^{(\nu)}_{\mathrm{em}}(x) = j^{(\nu)}_{\mu}(x) A^{\mu}(x) = \sum_{k,j=1}^{N} \overline{\nu_k}(x) \Lambda^{kj}_{\mu} \nu_j(x) A^{\mu}(x),$$

The vertex function is parametrized in terms of **charge, anapole, electric and magnetic form factors**:

$$\boxed{\Lambda_{\mu}(q) = \left(\gamma_{\mu} - q_{\mu} \not{q}/q^2\right) \left[\mathbb{f}_Q(q^2) + \mathbb{f}_A(q^2) q^2 \gamma_5\right] - i\sigma_{\mu\nu} q^{\nu} \left[\mathbb{f}_M(q^2) + i\mathbb{f}_E(q^2)\gamma_5\right]}$$

$\mathbb{f}^{fi}_{M}(0) = \mu_{fi}$ - neutrino magnetic moments

C.Giunti, A.Studenikin, Rev.Mod.Phys. 87 (2015) 531

Neutrino magnetic moments matrix

CPT-invariance + hermicity:

- Magnetic moments matrix for **Dirac** neutrinos is **real and symmetric**:

$$\mu^D = \begin{pmatrix} \mu_{11} & \mu_{12} & \mu_{13} \\ \mu_{12} & \mu_{22} & \mu_{23} \\ \mu_{13} & \mu_{23} & \mu_{33} \end{pmatrix}$$

- Magnetic moments matrix for **Majorana** neutrinos is **imaginary and asymmetric**:

$$\mu^M = \begin{pmatrix} 0 & i\mu_{12} & i\mu_{13} \\ -i\mu_{12} & 0 & i\mu_{23} \\ -i\mu_{13} & -i\mu_{23} & 0 \end{pmatrix}$$

- Thus, Dirac and Majorana neutrinos can be distinguished by their **electromagnetic properties**

Neutrino magnetic moments

Theory (Standard Model):

$$\mu_{ii}^{D} = \frac{3eG_F m_i}{8\sqrt{2}\pi^2} \approx 3.2 \times 10^{-19} \left(\frac{m_i}{1\ eV}\right) \mu_B$$

K.Fujikawa, R.Shrock,
Phys.Rev.Lett. 45 (1980) 963

Experiment:

$$\mu_{\nu} < 6.4 \times 10^{-12}\, \mu_{\mathrm{B}}$$

E.Aprile *et al.* [XENON collaboration],
Phys.Rev.Lett. 129 (2022) 16, 161805

Upper bounds from astrophysical neutrinos:
R.L. Workman et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2022, 083C01 (2022)

$$\mu_{\nu} \lesssim 10^{-12} \mu_B$$

Supernova neutrinos

- Supernova SN 1987A
  Kamiokande II,
  Baksan neutrino observatory,
  Irvine–Michigan–Brookhaven
  (≈20 events)
- Future neutrino experiments **JUNO, Hyper-Kamiokande, DUNE** and others: ≈ 50000 for galactic supernova explosion

[1] Fengpeng An *et al.* [JUNO collaboration], J.Phys.G 43 (2016) 3, 030401
[2] K.Abe *et al.* [Hyper-Kamiokande Collaboration], Astrophys.J. 916 (2021) 1, 15
[3] Abi Babak *et al.* [DUNE collaboration], JINST (2020) 15, 08

- Supernova magnetic fields are **10^{12} Gauss** or even more

Neutrino interaction with a magnetic field

Dirac neutrino:

$$\mathcal{L}^{D}_{mag} = \sum_{i,k} \mu_{ik} \left[\overline{\nu_i^R} \boldsymbol{\Sigma} \boldsymbol{B} \nu_k^L + \overline{\nu_i^L} \boldsymbol{\Sigma} \boldsymbol{B} \nu_k^R \right]$$

Majorana neutrino:

$$\mathcal{L}^{M}_{mag} = \sum_{i,k} \mu_{ik} \left[\overline{(\nu_i^L)^C} \boldsymbol{\Sigma} \boldsymbol{B} \nu_k^L + \overline{\nu_i^L} \boldsymbol{\Sigma} \boldsymbol{B} (\nu_k^L)^C \right]$$

$i, k = 1, 2, 3$

For the Majorana case magnetic field induces neutrino-antineutrino transitions

$$\nu_\alpha \to \bar{\nu}_\beta$$

Neutrino interaction with supernova matter

$$\mathcal{L}^{M}_{mat} = -\sum_{\alpha} V^{(f)}_{\alpha} \left[\overline{\nu^L_\alpha} \gamma_0 \nu^L_\alpha - \overline{(\nu^L_\alpha)^c} \gamma_0 (\nu^L_\alpha)^c \right]$$

$$\mathcal{L}^{D}_{mat} = -\sum_{\alpha} V^{(f)}_{\alpha} \overline{\nu^L_\alpha} \gamma_0 \nu^L_\alpha$$

$$\alpha, \beta = e, \mu, \tau$$

$$\underbrace{V^{(f)} = \mathrm{diag}\left(\frac{G_F n_e}{\sqrt{2}} - \frac{G_F n_n}{2\sqrt{2}}, -\frac{G_F n_n}{2\sqrt{2}}, -\frac{G_F n_n}{2\sqrt{2}} \right)}_{\text{Wolfenstein potential}}$$

n_n, n_e are neutron and electron number densities of supernova environment

Equation of motion

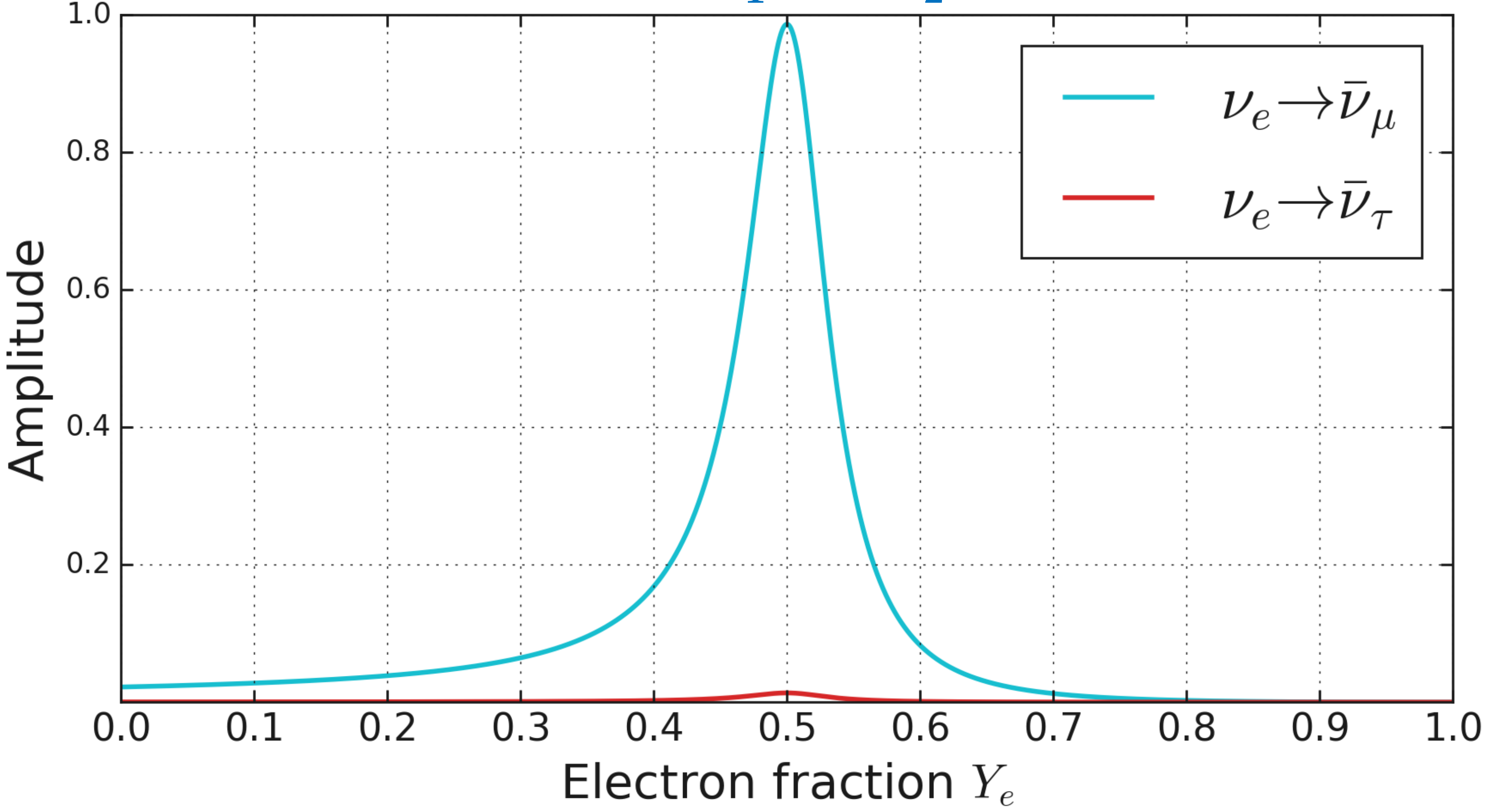
$$\left(i\gamma^{\mu}\partial_{\mu} - m_i\right)\nu_i(x) - \sum_{k}\left(\mu_{ik}\boldsymbol{\Sigma}\boldsymbol{B} + V_{ik}^{(m)}\gamma^{0}\gamma_{5}\right)\nu_k(x) = 0$$

A.Popov, A.Studenikin, Phys.Rev.D 103 (2021) 11, 115027

- We solve the equation numerically for both cases of Dirac and Majorana neutrinos.
- The possibility of resonant amplification of neutrino oscillations is investigated.
- Effects due to nonzero CP-violating phases are considered.

Resonant amplification of Majorana neutrino oscillations

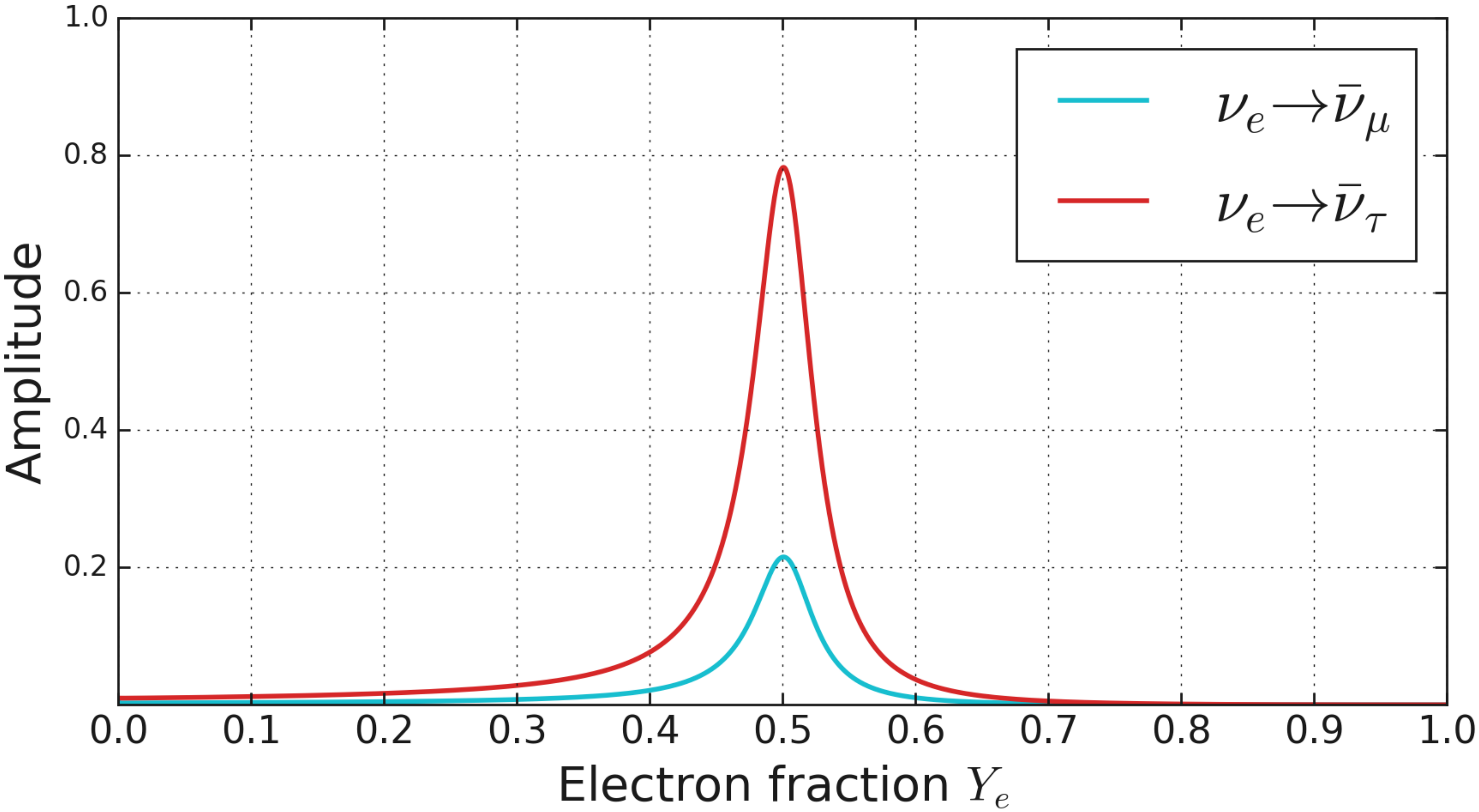
$\delta = 0, \alpha_1 = 0, \alpha_2 = 0$



[1] M.B.Voloshin, M.I.Vysotskii, L.B.Okun, Zh. Eksp. Teor. Fiz. 91 (1986), 754-765
[2] E.Akhmedov, Phys. Lett. B 213, 64 (1988)
[3] C.-S.Lim, W.Marciano, Phys.Rev.D37 (1988) 1368

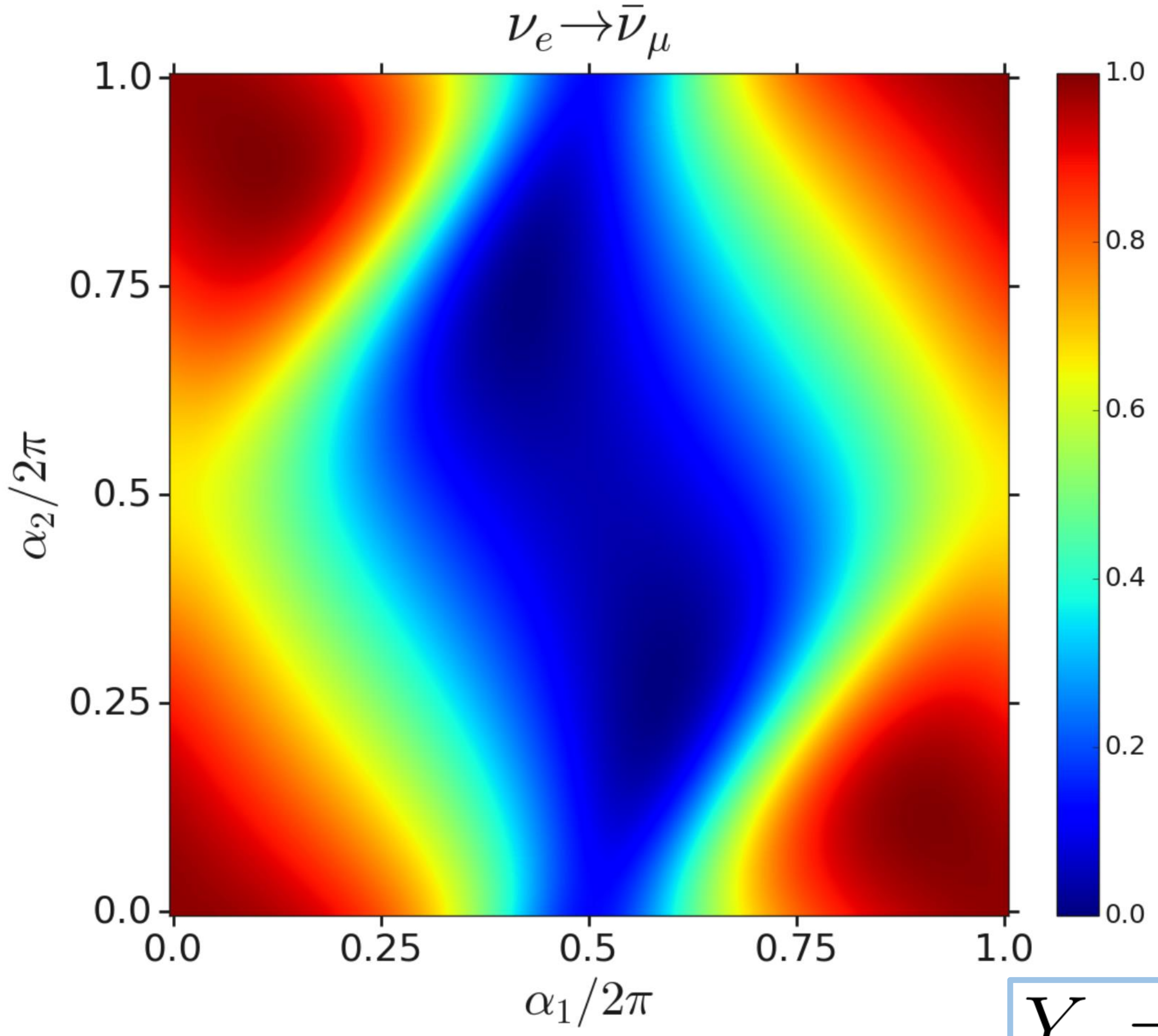
Resonant amplification of Majorana neutrino oscillations

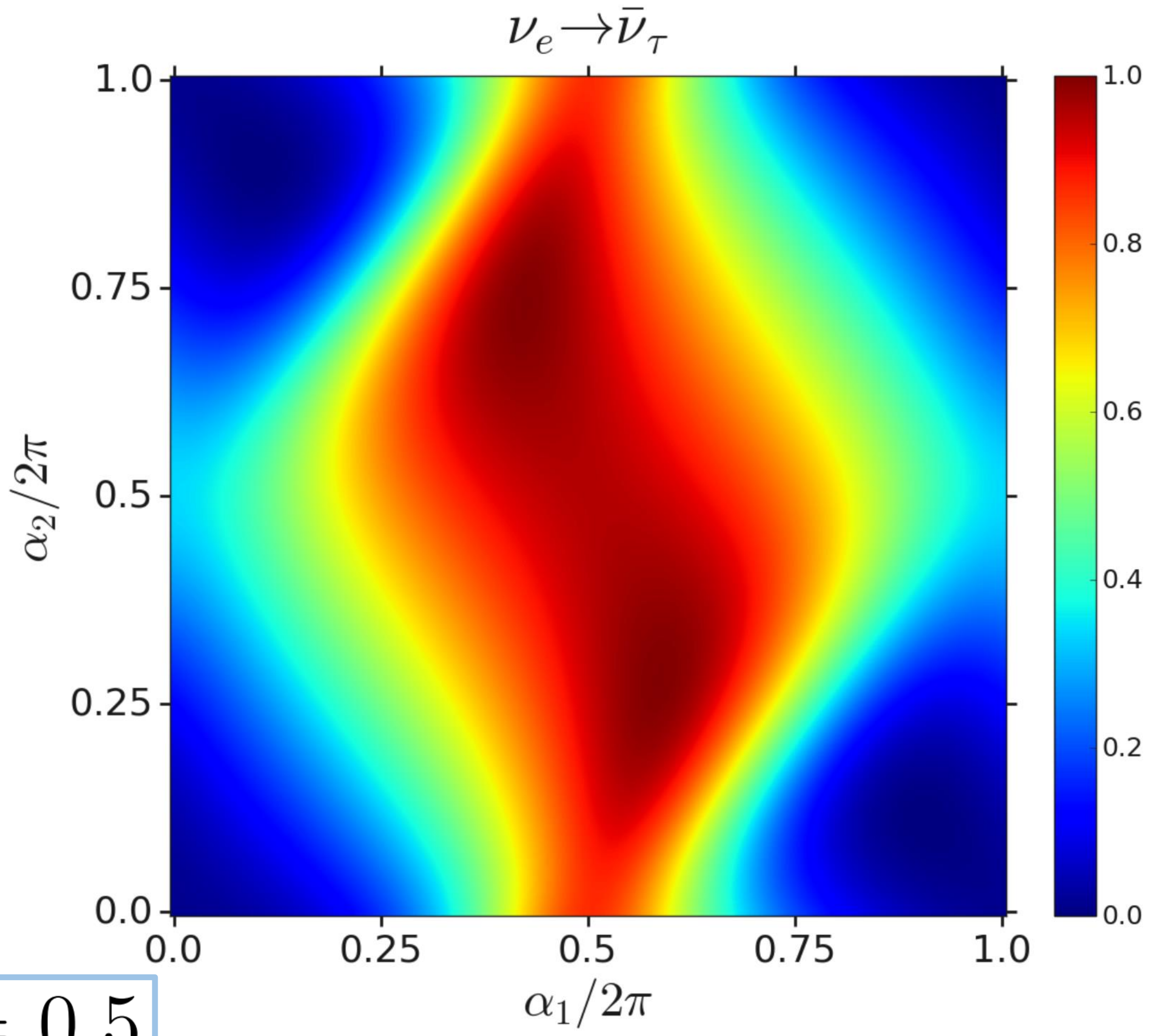
$\delta = 0, \alpha_1 = \pi, \alpha_2 = \pi$



A.Popov, A.Studenikin, Phys.Rev.D 103 (2021) 11, 115027.

Resonant amplification of Majorana neutrino oscillations





$$Y_e = 0.5$$

Discussion

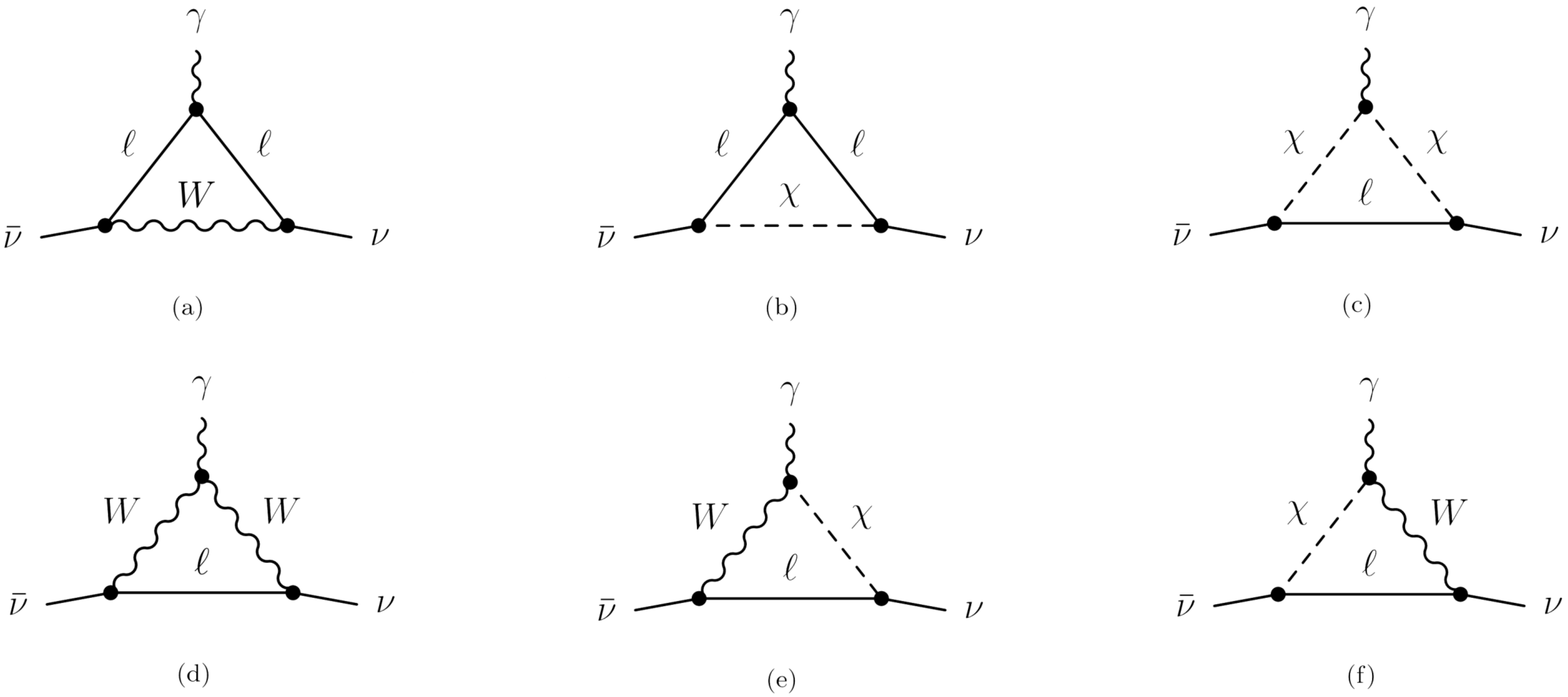
- During supernova neutronization stage (0.1…0.2 sec after the shock) neutrino emission mainly consists of electron neutrinos ν_e.
- Electron fraction Y_e reaches the resonant value of 0.5 at ≈100 km from the neutrinoshpere (see for example R.Buras, M.Rampp, H.-Th.Janka, K.Kifonidis, Astron. Astrophys. (2006) 447).
- Thus, for the case of Majorana neutrinos, a resonant conversion of electron neutrinos ν_e to antineutrinos $\bar{\nu}$ can occur (similar to Mikheev-Smirnov-Wolfenstein effect).
- $\frac{\bar{\nu}_e}{\bar{\nu}_e+\nu_e}$ ratio depends on the values of Majorana CP-violating phases α_1 and α_2.

Summary

- We find new resonances in neutrino-antineutrino oscillations in a magnetic field in the case of nonzero Majorana CP-violating phases.
- The resonances appear at Y_e = 0.5.
- New resonances possibly can alter evolution of supernova neutrino fluxes and affect the flavour composition, in particular $\frac{\bar{\nu}_e}{\bar{\nu}_e+\nu_e}$ ratio.
- Thus, we conclude that astrophysical neutrino experiments potentially can be used to probe neutrino magnetic moments, the nature of neutrino mass and the presence of leptonic CP-violation.

Backup

Neutrino magnetic moment



M.Dvornikov, A.Studenikin, Phys.Rev.D. (2004)

Dirac and Majorana neutrinos

Dirac fermion

$$\Psi_D = \Psi_L + \Psi_R$$

Majorana fermion

$$\Psi_R = \Psi_L^c$$

A Majorana field can be written as $\Psi_M = \Psi_L + \Psi_L^c$

$\boxed{\Psi_M^c = \Psi_M}$ is satisfied for a Majorana field

Majorana mass term violates total lepton number by 2

$$m_i \overline{\nu_i} \nu_i = m_i \overline{(\nu_i^L)^c} \nu_i^L + m_i \overline{\nu_i^L} (\nu_i^L)^c$$

Neutrinos in astrophysics

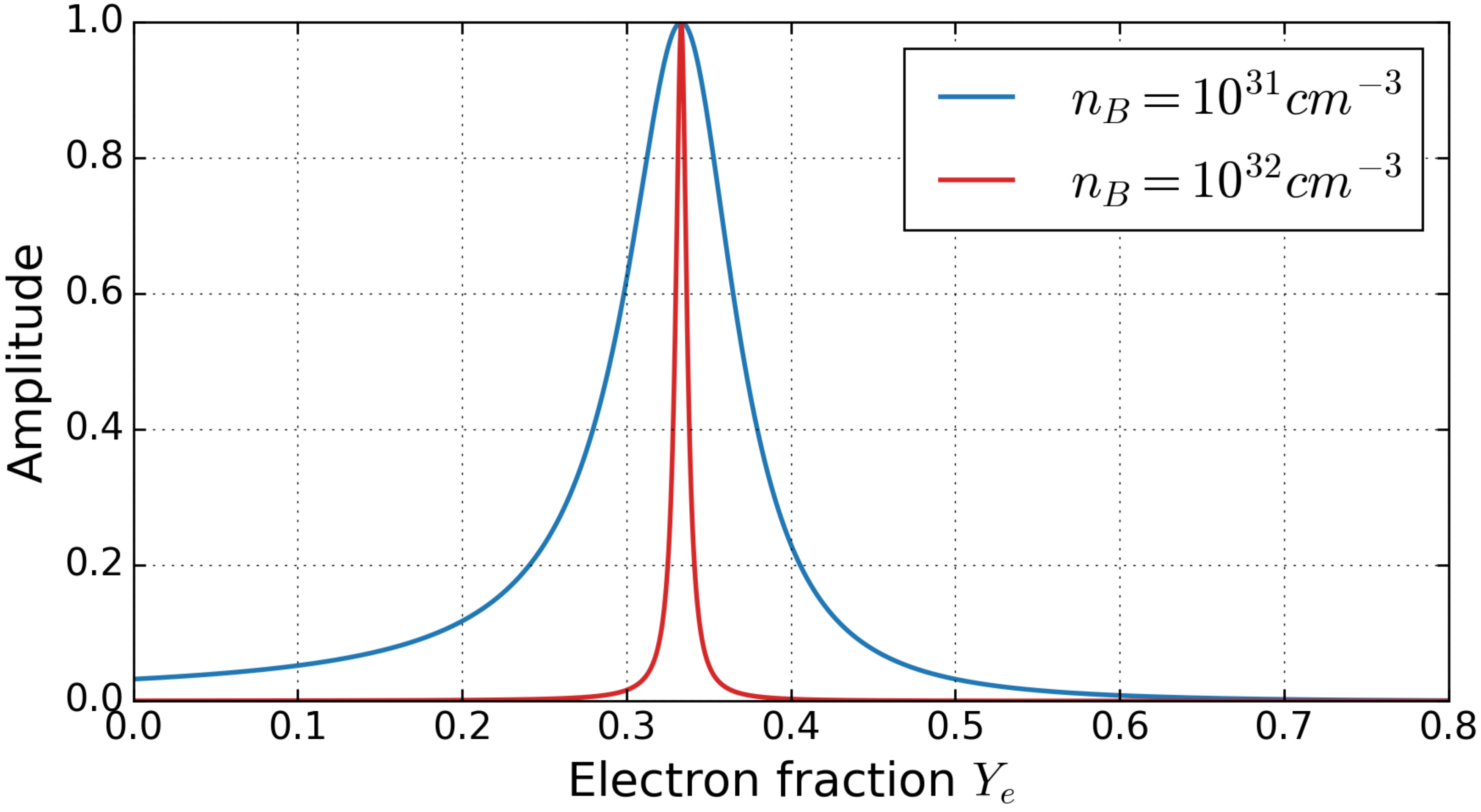
Known types:

- Solar neutrinos
- Supernova neutrinos
- High-energy neutrinos

Hypothetical sources:

- Diffuse Supernova Neutrino Background
- Gamma-ray bursts
- Active Galactic Nuclei
- Pulsars, magnetars
- Cosmogenic neutrinos
- Relic neutrinos

Resonant amplification of Dirac neutrino oscillations



- $|\mu_{12}| = |\mu_{13}| = |\mu_{23}| = 10^{-12}\mu_B$
- $n_n = 10^{31}\ \mathrm{cm}^{-3}$
- $B = 10^{12}\ \mathrm{G}$
- $E = 10\ \mathrm{MeV}$
- $\underbrace{Y_e = n_e/n_B}_{\text{Electron fraction}},\ \underbrace{n_B = n_n + n_p}_{\text{Baryon number density}}$

$$\boxed{Y_e \approx 1/3}$$

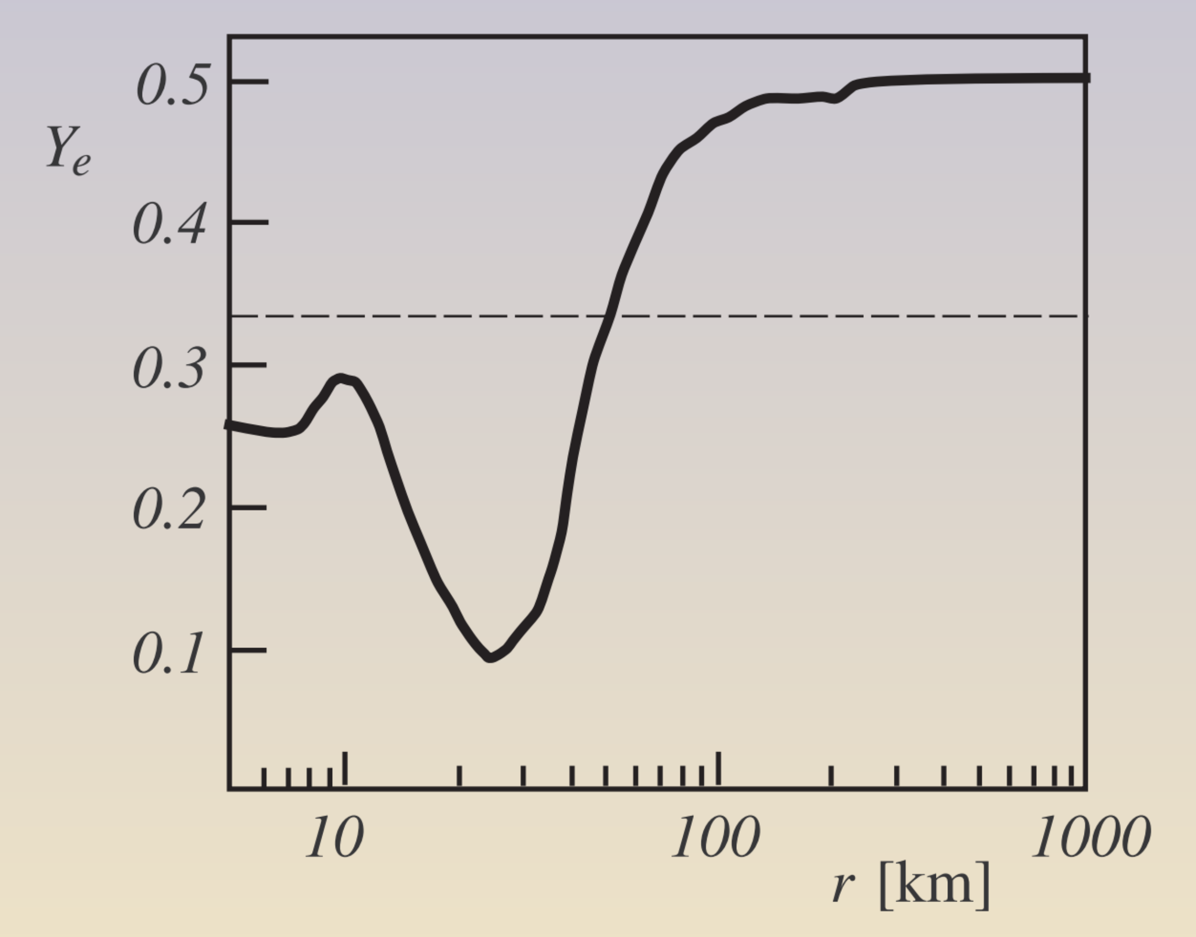
$$\Downarrow$$

Resonant enhancement of

$$\boxed{\nu_e^L \to \nu_e^R}$$

[1] M.B.Voloshin, M.I.Vysotskii, L.B.Okun, Zh. Eksp. Teor. Fiz. 91 (1986), 754-765
[2] E.Akhmedov, Phys. Lett. B 213, 64 (1988)
[3] C.-S.Lim, W.Marciano, Phys.Rev.D37 (1988) 1368

Supernova electron fraction profile (t = 0.1...0.2 s after the shock)



R.Buras, M.Rampp, H.-Th.Janka, K.Kifonidis, Astron. Astrophys. (2006) 447

CP-violation in neutrino oscillations

$$\begin{tikzcd}
\nu_\alpha \rightarrow \nu_\beta \arrow[r, leftrightarrow, "\text{CP}"] \arrow[d, leftrightarrow, "\text{T}"'] \arrow[dr, leftrightarrow, "\text{CPT}"] & \bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta \arrow[d, leftrightarrow, "\text{T}"] \\
\nu_\beta \rightarrow \nu_\alpha \arrow[r, leftrightarrow, "\text{CP}"] \arrow[ur, leftrightarrow] & \bar{\nu}_\beta \rightarrow \bar{\nu}_\alpha
\end{tikzcd}$$

For a review of leptonic CP-violation see
G.C. Branco, R.Gonzalez Felipe, F.R. Joaquim, *“Leptonic CP Violation”*, Rev.Mod.Phys. 84 (2012) 515-565

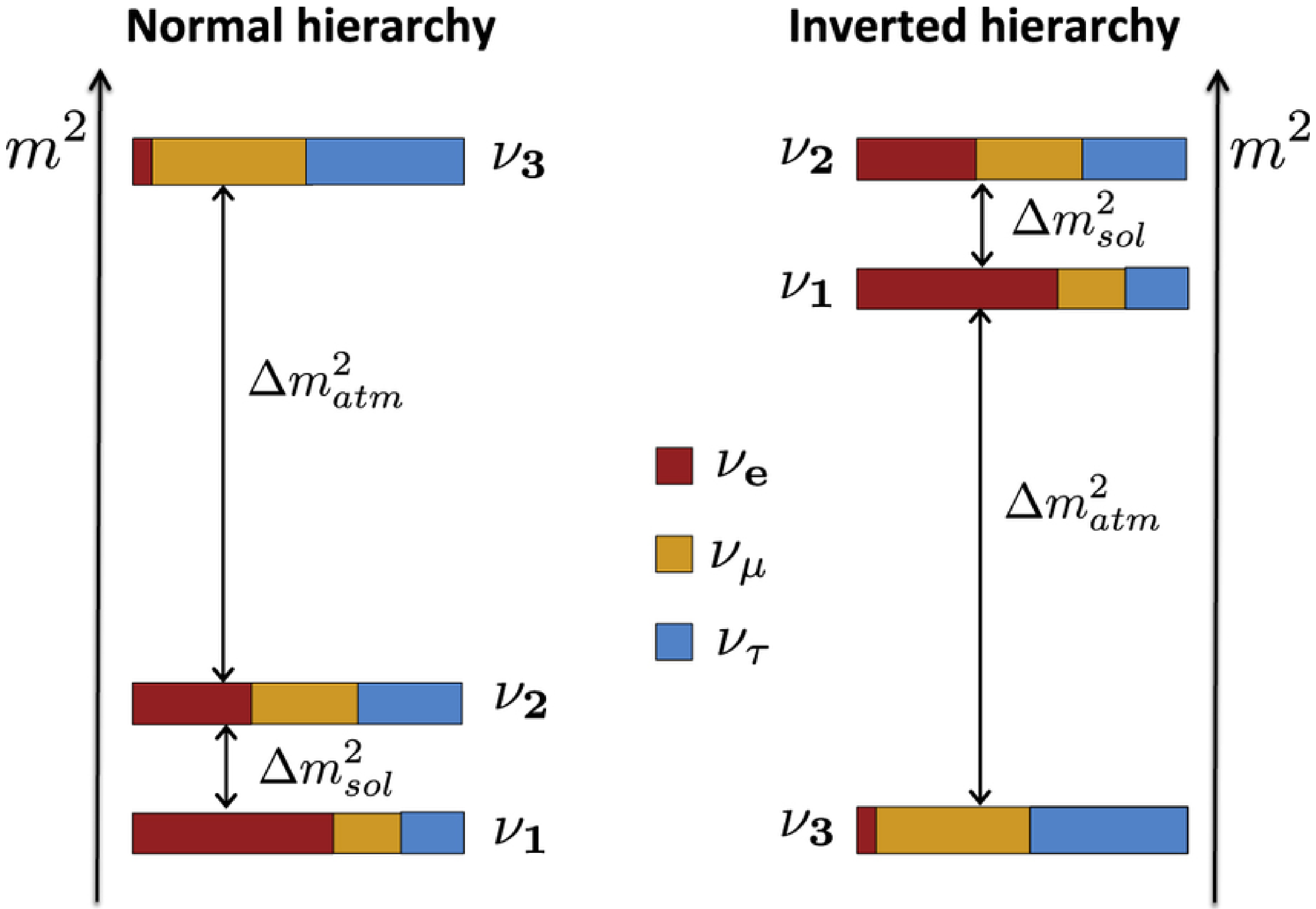
Mixing angles and phases

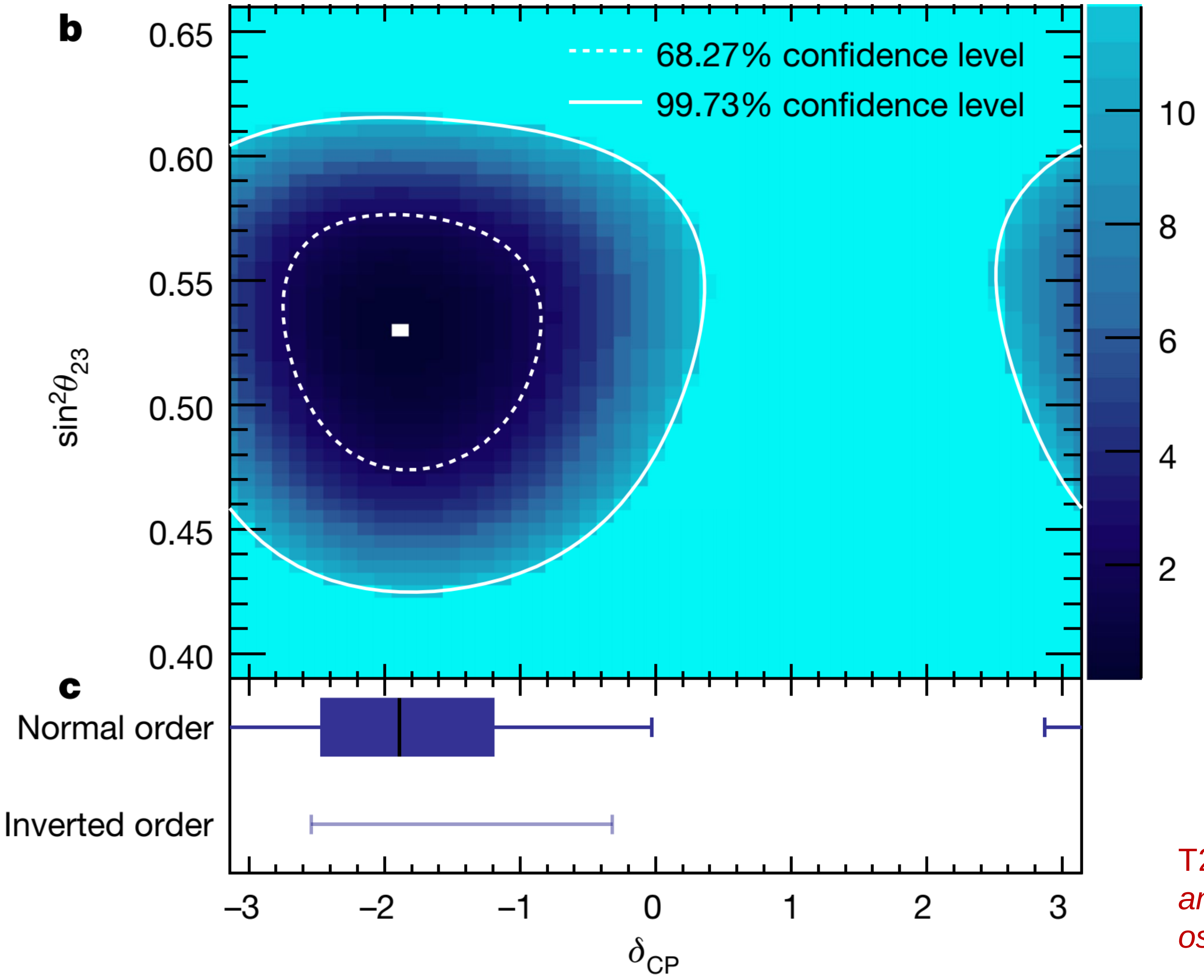
- $\dim SU(n) = n^2 = \underbrace{\frac{n(n-1)}{2}}_{n_{angles}} + \underbrace{\frac{n(n+1)}{2}}_{n_{phases}}$
- The number of *physical phases* is smaller than n_{phases} and depends on the nature of neutrino mass: $n_{phases} = 1$ (Dirac case) and $n_{phases} = 3$ (Majorana case)

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \underbrace{\begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix}}_{\text{Dirac CP-violating phase}} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \underbrace{\begin{pmatrix} e^{i\alpha_1} & 0 & 0 \\ 0 & e^{i\alpha_2} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{Majorana CP-violating phases}}$$

$c_{ik} = \cos\theta_{ik}$
$s_{ik} = \sin\theta_{ik}$

Neutrino mass hierarchy

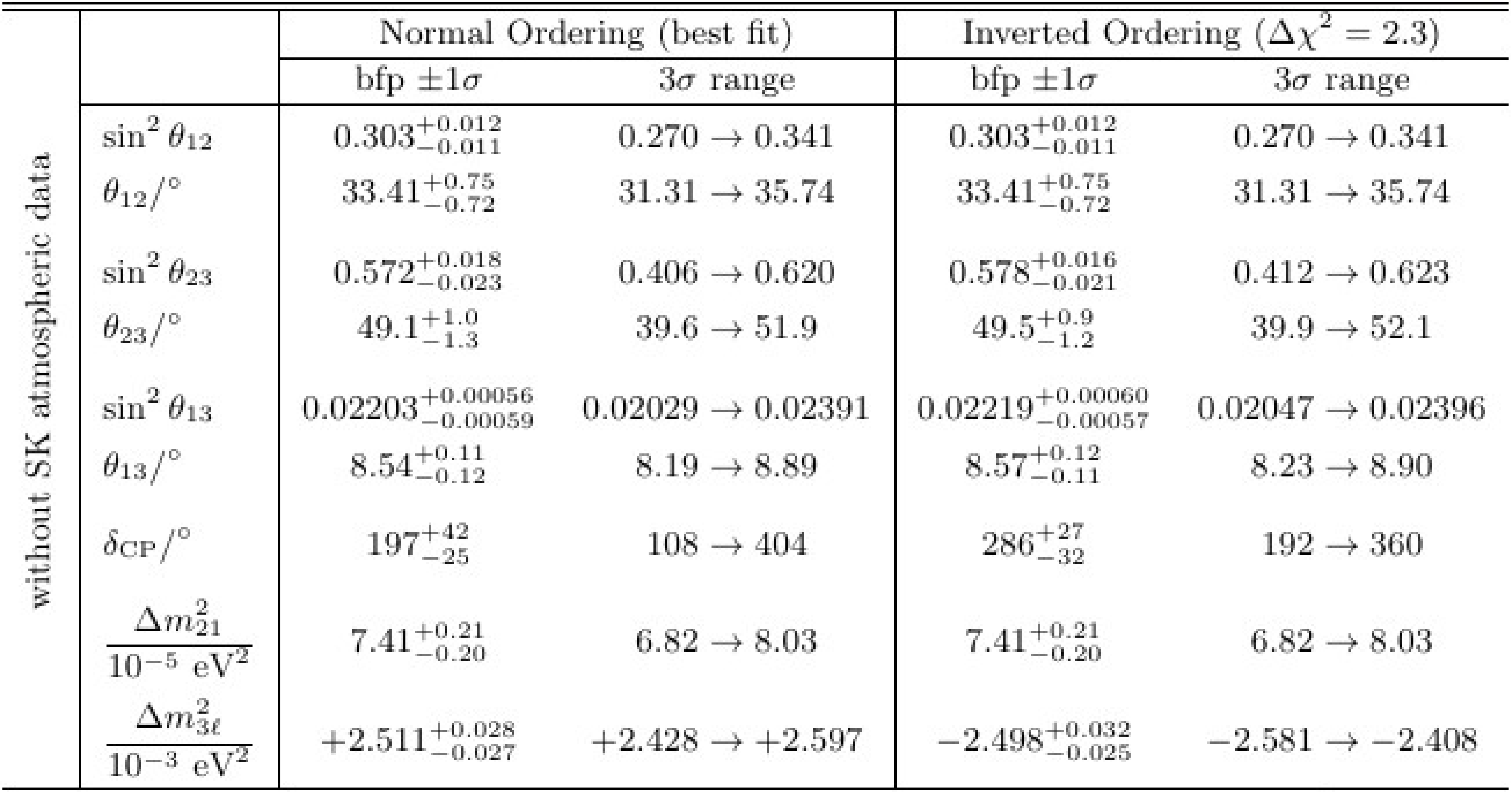




T2K Collaboration, *"Constraint on the matter–antimatter symmetry-violating phase in neutrino oscillations"*, Nature 580 (2020) 7803, 339-344

I.Esteban, M.C.Gonzalez-Garcia, M.Maltoni, T.Schwetz, A.Zhou , *"The fate of hints: updated global analysis of three-flavor neutrino oscillations"*, JHEP 09 (2020) 178; NuFIT 5.2 (2022), www.nu-fit.org

NuFIT 5.2 (2022)



		Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 2.3$)	
		bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
without SK atmospheric data	$\sin^2\theta_{12}$	$0.303^{+0.012}_{-0.011}$	$0.270 \to 0.341$	$0.303^{+0.012}_{-0.011}$	$0.270 \to 0.341$
	$\theta_{12}/^\circ$	$33.41^{+0.75}_{-0.72}$	$31.31 \to 35.74$	$33.41^{+0.75}_{-0.72}$	$31.31 \to 35.74$
	$\sin^2\theta_{23}$	$0.572^{+0.018}_{-0.023}$	$0.406 \to 0.620$	$0.578^{+0.016}_{-0.021}$	$0.412 \to 0.623$
	$\theta_{23}/^\circ$	$49.1^{+1.0}_{-1.3}$	$39.6 \to 51.9$	$49.5^{+0.9}_{-1.2}$	$39.9 \to 52.1$
	$\sin^2\theta_{13}$	$0.02203^{+0.00056}_{-0.00059}$	$0.02029 \to 0.02391$	$0.02219^{+0.00060}_{-0.00057}$	$0.02047 \to 0.02396$
	$\theta_{13}/^\circ$	$8.54^{+0.11}_{-0.12}$	$8.19 \to 8.89$	$8.57^{+0.12}_{-0.11}$	$8.23 \to 8.90$
	$\delta_{\mathrm{CP}}/^\circ$	197^{+42}_{-25}	$108 \to 404$	286^{+27}_{-32}	$192 \to 360$
	$\frac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$	$7.41^{+0.21}_{-0.20}$	$6.82 \to 8.03$	$7.41^{+0.21}_{-0.20}$	$6.82 \to 8.03$
	$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \mathrm{eV}^2}$	$+2.511^{+0.028}_{-0.027}$	$+2.428 \to +2.597$	$-2.498^{+0.032}_{-0.025}$	$-2.581 \to -2.408$

Recent studies of supernova neutrino oscillations

[1] A.Ahriche, J.Mimouni, *"Supernova neutrino spectrum with matter and spin flavor precession effects"*, JCAP 11 (2003) 004
[2] J.Gava, C.Volpe, *"Collective neutrinos oscillation in matter and CP-violation"*, Phys.Rev.D 78 (2008) 083007
[3] B.Balantekin, J.Gava, C.Volpe, *"Possible CP-Violation effects in core-collapse Supernovae"*, Phys.Lett.B 662 (2008) 396-404
[4] A. de Gouvea, S.Shalgar, *"Effect of Transition Magnetic Moments on Collective Supernova Neutrino Oscillations"*, JCAP 10 (2012) 027
[5] A. de Gouvea, S.Shalgar, *"Transition Magnetic Moments and Collective Neutrino Oscillations:Three-Flavor Effects and Detectability"*, JCAP 04 (2013) 018
[6] O.Kharlanov, P.Shustov, *“Effects of nonstandard neutrino self-interactions and magnetic moment on collective Majorana neutrino oscillations”*, Phys.Rev.D 103 (2021) 9, 095004