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Relic gravitational wave conversion into photons in the intergalactic magnetic field

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Outline

1. **Relic gravitational waves**
2. **Intergalactic magnetic field**
3. **Gersenstein, Zeldovich effect**
4. **The goal of the work**
5. **Method**
6. **Results and conclusions**

Relic gravitational waves

Inflation theory: exponential expanding of the early Universe

Helps to solve *problems of the hot big bang theory*:

1. Homogeneity and isotropy
2. Flatness problem
3. Entropy problem
4. Primordial inhomogeneities problem

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- **Scalar** perturbations
- **Tensor** perturbations

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Affected CMB in the range $\sim (10^{-18} - 10^{-16}$ Hz)

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Tensor to scalar intensity ratio:

$$\boldsymbol{r < 0.028\ (95\%\ \mathrm{CL})}$$

BICEP/KeckArray/Planck/LIGO-Virgo-KAGRA; arXiv:2208.00188 (2023)

Inflaton: $r \approx 0.13 - 0.16$ при $n = 2$

$r \approx 0.27 - 0.32$ при $n = 4$

Intergalactic magnetic field

- fills intergalactic space and space between clusters
- limits from theory and observations:

$$\mathbf{10^{-16} - 10^{-18}}\ \text{Гс} < \mathbf{B} \leq \mathbf{10^{-9} - 10^{-12}}\ \text{Гс}$$

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halos of blazars (in gamma-range)

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- Amplification of the primary magnetic field by the *MHD dynamo* or by *adiabatic compression* – galaxies and clusters

<u>Primary magnetic field</u>

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$$\partial_\lambda F_{\mu\nu} + \partial_\nu F_{\lambda\mu} + \partial_\mu F_{\nu\lambda} = 0$$

No electric field

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$$F_i^{\cdot\, j} = g^{jk}F_{ik} = -F_{jk}/a^2$$

$$\boxed{\boldsymbol{B} = \frac{\boldsymbol{B_0}}{\boldsymbol{a^2}}}$$

$B \uparrow$ when $\downarrow a(t)$

Gersenstine, Zeldovich effect

1961, Gertsenshtein – the effect of photon and graviton mixing under the influence of an external magnetic field

analogy with neutrino oscillations

Gertsenshtein effect: $\gamma \rightarrow \mathrm{g}$

Zeldovich effect: $\mathrm{g} \rightarrow \gamma$

inverse Gertsenshtein effect

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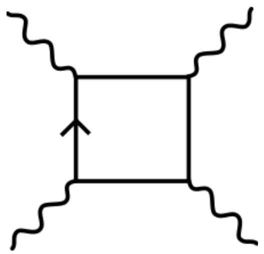
$$\mathcal{A}_{HE} = \int d^4x\sqrt{-g}\,C_0\left[(F_{\mu\nu}F^{\mu\nu})^2 + \frac{7}{4}(\tilde{F}^{\mu\nu}F_{\mu\nu})^2\right]$$

where dual electromagnetic tensor

$$\tilde{F}_{\alpha\beta} = \frac{\sqrt{-g}}{2}\epsilon_{\alpha\beta\mu\nu}F^{\mu\nu}, \quad \tilde{F}^{\alpha\beta} = \frac{1}{2\sqrt{-g}}\epsilon^{\alpha\beta\mu\nu}F_{\mu\nu}$$

$$C_0 = \alpha^2/(90m_e^4)$$



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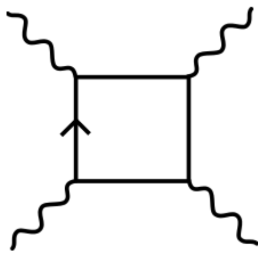

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Magnetic field magnitude for RD era

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for $a \in [10^{-9}, 10^{-4}]$ $\boxed{B_{max} = 10^9 \text{ Gs}}$

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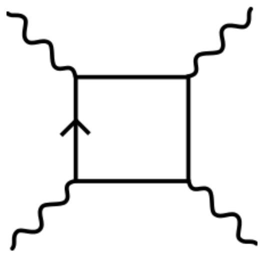

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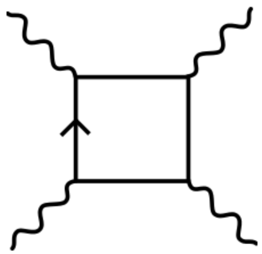

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$$CB^2 = \frac{1}{90 * 137^2}\frac{(10^9 * 1.95 * 10^{-14}MeV^2)^2}{(0.5\ MeV)^4}$$

$$\boxed{CB^2 \sim 10^{-15}}$$

We can neglect it

The goal of the work

The goal

To evaluate the influence of the inverse Gertsenshtein effect

on the amplitude of relic gravitational waves

Motivation

1. No imprint of relic gravitational waves on CMB
2. $\mathrm{B} \sim 1/\mathrm{a}^2$
3. GW propagation in relatively strong magnetic field for a long time (RD era ~80 000 years)

Taking the effect into account may be important for the inflation models verification

Method

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- electromagnetic potential $\bar{A}^{\mu}$

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Expansion of the Einstein equation up to the first perturbation order
+ corrections to the EMT from the electromagnetic field action

$$\boxed{\bar{R}_{\mu\nu} - \frac{1}{2}\bar{g}_{\mu\nu}\bar{R} = \frac{8\pi}{m_{pl}^2}\bar{T}_{\mu\nu}}$$

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no current

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$$\bar{F}^{\mu\nu} = \partial^{\mu}\bar{A}^{\nu} - \partial^{\nu}\bar{A}^{\mu} \equiv F^{\mu\nu} + f^{\mu\nu} \Rightarrow \bar{F}_{\mu\nu} = \bar{g}_{\mu\alpha}\,\bar{g}_{\nu\beta}\bar{F}^{\alpha\beta} = F_{\mu\nu} + f_{\mu\nu} + h_{\mu\alpha}F^{\alpha}_{\cdot\nu} + h_{\nu\beta}F_{\mu}^{\cdot\beta}$$

we define $f^{\mu\nu} = \partial^{\mu}f^{\nu} - \partial^{\nu}f^{\mu}$

we define $f_{\mu\nu} = g_{\mu\alpha}g_{\nu\beta}f^{\alpha\beta}$

II. Derive the eq. of motion for metric perturbations:

Expansion of the Einstein equation up to the first perturbation order + corrections to the EMT from the electromagnetic field action

$$\boxed{\bar{R}_{\mu\nu} - \frac{1}{2}\bar{g}_{\mu\nu}\bar{R} = \frac{8\pi}{m_{pl}^2}\bar{T}_{\mu\nu}}$$

$$\bar{\mathcal{A}}_{Maxwell} = -\frac{1}{4}\int d^4x\,\sqrt{-\bar{g}}\left(\bar{F}^{\alpha\beta}\bar{F}_{\alpha\beta} + \cancel{\bar{A}_{\alpha}J^{\alpha}}\right)$$

no current

$$\bar{T}_{\mu\nu} = \frac{2}{\sqrt{-\bar{g}}}\frac{\delta\bar{\mathcal{A}}_{Maxwell}}{\delta\bar{g}_{\mu\nu}} = \frac{1}{4}\bar{g}_{\mu\nu}\bar{F}^2 - \bar{F}_{\mu\alpha}\bar{F}_{\nu}^{\cdot\alpha}$$

FLRW metric:

$$\left[\partial_t^2 + 3H\partial_t - \frac{\Delta}{a^2}\right]h^i_j = -16\pi G T_j^{i(EM\,1)}$$

Method

II. Derive the eq. of motion for metric perturbations:

$$\bar{T}^{Max}_{\mu\nu} = \frac{1}{4}\bar{g}_{\mu\nu}\bar{F}^2 - \bar{F}_{\mu\alpha}\bar{F}_{\nu}^{\cdot\alpha}$$

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Method

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$$\bar{T}_{\mu\nu}^{Max} = \frac{1}{4}\bar{g}_{\mu\nu}\bar{F}^2 - \bar{F}_{\mu\alpha}\bar{F}_{\nu}^{\cdot\alpha} \Rightarrow$$

$$\bar{T}_{\mu\nu}^{Max} = \frac{1}{4}(g_{\mu\nu} + h_{\mu\nu})(F^{\alpha\beta} + f^{\alpha\beta})(h_{\alpha\lambda}F^{\lambda}_{\cdot\beta} + h_{\beta\lambda}F_{\alpha}^{\cdot\lambda} + F_{\alpha\beta} + f_{\alpha\beta}) + \\ + (h_{\mu\lambda}F^{\lambda}_{\cdot\alpha} + h_{\alpha\lambda}F_{\mu}^{\cdot\lambda} + F_{\mu\alpha} + f_{\mu\alpha})(h_{\nu\lambda}F^{\alpha\lambda} + F^{\alpha}_{\cdot\nu} + f^{\alpha}_{\cdot\nu})$$

$$\boxed{\left[\partial_t^2 + 3H\partial_t - \frac{\Delta}{a^2}\right]h_j^i = -16\pi G T_j^{i(Max\ 1)}}$$

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$$+(h_{\mu\lambda}F^{\lambda}_{\cdot\alpha} + h_{\alpha\lambda}F_{\mu}^{\cdot\lambda} + F_{\mu\alpha} + f_{\mu\alpha})(h_{\nu\lambda}F^{\alpha\lambda} + F^{\alpha}_{\cdot\nu} + f^{\alpha}_{\cdot\nu})$$

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$$+F_{\mu\alpha}f^{\alpha}_{\cdot\nu} + f_{\mu\alpha}F^{\alpha}_{\cdot\nu} + (F_{\mu\alpha}h_{\lambda\nu} + h_{\mu\alpha}F_{\lambda\nu})F^{\alpha\lambda} + F_{\mu\alpha}F_{\lambda\nu}h^{\alpha\lambda}$$

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We need eq. of motion for f^{ν} => expand the action up to the first perturbation order vary it by δf^{ν}

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$$\mathcal{A}_{Maxwell} = -\frac{1}{4}\int d^4x\sqrt{-g}\left(1 + \frac{h}{2}\right)(F^{\alpha\beta} + f^{\alpha\beta})(F_{\alpha\beta} + f_{\alpha\beta} + h_{\alpha\lambda}F^{\lambda}_{\cdot\beta} + h_{\lambda\beta}F_{\alpha}^{\cdot\lambda})$$

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$$D_{\alpha}\underbrace{\left[f^{\alpha\nu} + h^{\nu}_{\lambda}F^{\alpha\lambda} + h^{\alpha}_{\lambda}F^{\lambda\nu} + \frac{h}{2}F^{\alpha\nu}\right]}_{\text{antisymmetric tensor}} = 0 \quad \Rightarrow \quad \frac{\partial_{\alpha}\left(\sqrt{-g}\left[f^{\alpha\nu} + h^{\nu}_{\lambda}F^{\alpha\lambda} + h^{\alpha}_{\lambda}F^{\lambda\nu} + \frac{h}{2}F^{\alpha\nu}\right]\right)}{\sqrt{-g}} = 0$$

Method

II. Derive the eq. of motion for metric perturbations:

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FLRW:

$$\left[\partial_t^2 - \frac{\Delta}{a^2} + 3H\partial_t\right]f^j + \mathrm{F}^{i\lambda}\partial_i h^j_\lambda + F^{ij}\partial_i h = 0$$

Method

IV. Assumptions:

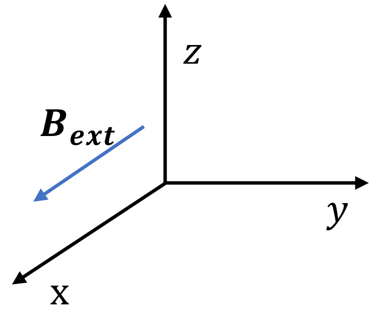
- $\boldsymbol{B}$ is homogeneous and directed along x axis
- Isotropic background space-time (neglect gravity from $\boldsymbol{B}$)

Nonzero components:

$$F^{y}_{\cdot z} = -F^{z}_{\cdot y} = B_x$$

$$F^{yz} = -F^{zy} = -\frac{B_x}{a^2}$$

$$F_{yz} = -F_{zy} = -B_x a^2$$



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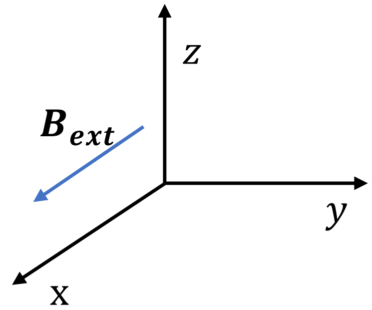

V. Simplification:

$$\begin{cases} \left[\partial_t^2 - \frac{\Delta}{a^2} + 3H\partial_t\right] f^{j} + \mathrm{F}^{i\lambda}\partial_i h^{j}_{\lambda} + F^{ij}\partial_i h = 0 \\ \left[\partial_t^2 + 3H\partial_t - \frac{\Delta}{a^2}\right] h^{i}_{j} = -16\pi G T^{i(Max\ 1)}_{j} \end{cases}$$

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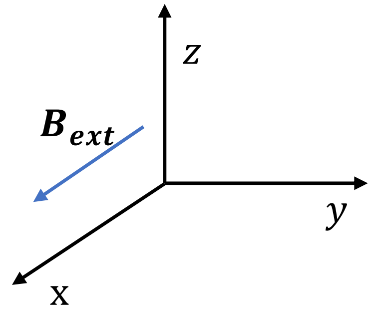

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$$T_{\mu\nu}^{Max(1)} = \frac{1}{4} h_{\mu\nu}F^2 + \frac{1}{2} g_{\mu\nu}(Ff + FFh) +$$
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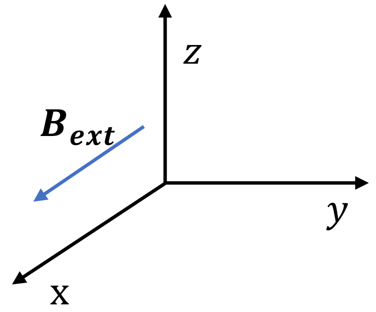

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Simplifying of convolutions

$$F^2 = 2B^2$$

$$Ff = F_{\alpha}^{\cdot\,\beta} f^{\alpha}_{.\,\beta} = F_y^{\cdot\,z} f^{y}_{.\,z} + F_z^{\cdot\,y} f^{z}_{.\,y} = B\left(f^{y}_{.\,z} - f^{z}_{.\,y}\right) = 2Bf^{y}_{.\,z}$$

$$FFh = h^{\alpha}_{\sigma} F_{\beta\alpha} F^{\beta\sigma} = B^2\left(h^y_y + h^z_z\right)$$

…

Method

VI. Decomposition: $\boldsymbol{k} = \boldsymbol{k}_{||} + \boldsymbol{k}_{\perp} = \boldsymbol{k}_x + \boldsymbol{k}_z$

$\boldsymbol{k} \parallel \boldsymbol{B}$	$\boldsymbol{k} \perp \boldsymbol{B}$
$h^\mu_\nu(t=0,x) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & h^0_+ & h^0_\times \\ 0 & 0 & h^0_\times & -h^0_+ \end{bmatrix} e^{ik_x x}$	$h^\mu_\nu(t=0,z) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & h^0_+ & h^0_\times & 0 \\ 0 & h^0_\times & -h^0_+ & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} e^{ik_z z}$

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$$\left[\partial_t^2 + 3H\partial_t + \left(\frac{k^2}{a^2} - 8\pi G B^2\right)\right] h_+ = 0$$

$$\left[\partial_t^2 + 3H\partial_t + \left(\frac{k^2}{a^2} - 8\pi G B^2\right)\right] h_\times = 0$$

no mixing with a photon

effective frequency is changed

$\Downarrow$

amplification of GW amplitude

$$h^\mu_\nu(t=0,z) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & h^0_+ & h^0_\times & 0 \\ 0 & h^0_\times & -h^0_+ & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} e^{ik_z z}$$

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$\boldsymbol{h}_\times$

$$\begin{cases} \left[\partial_t^2 + 3H\partial_t + \left(\frac{k^2}{a^2} - \underbrace{8\pi G B^2}_{\text{amplification}}\right)\right] h_\times = \underbrace{-ik16\pi G B f^x}_{\text{suppression}} \\ \left[\partial_t^2 + 3H\partial_t + \frac{k^2}{a^2}\right] f^x = -\frac{ikB}{a^2} h_\times \end{cases}$$

Method

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$$h^{\mu}_{\nu}(t=0,z) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & h^0_+ & h^0_\times & 0 \\ 0 & h^0_\times & -h^0_+ & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} e^{ik_z z}$$

$\boldsymbol{h}_\times$

$$\begin{cases} \left[\partial_t^2 + 3H\partial_t + \left(\frac{k^2}{a^2} - 8\pi G B^2\right)\right] h_\times = -ik16\pi G B f^x \\ \left[\partial_t^2 + 3H\partial_t + \frac{k^2}{a^2}\right] f^x = -\frac{ikB}{a^2} h_\times \end{cases}$$

amplification *suppression*

$\boldsymbol{h}_+$

mixing with scalar metric perturbations

$$\{\Phi, \Psi, h_+, f^y\}$$

Method

$$h^\mu_\nu(t=0,z) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & h^0_+ & h^0_\times & 0 \\ 0 & h^0_\times & -h^0_+ & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} e^{ik_z z} \quad \Rightarrow \quad h^\mu_\nu(t,z) = \begin{bmatrix} 2\Phi(t) & 0 & 0 & 0 \\ 0 & -2\Psi(t)+h_+(t) & h_\times(t) & 0 \\ 0 & h_\times(t) & -2\Psi(t)-h_+(t) & 0 \\ 0 & 0 & 0 & -2\Psi(t) \end{bmatrix} e^{ik_z z}$$

Method

The eq. for scalar modes:

$$\boxed{\partial^2 h + 4h^{\alpha\beta}R_{\alpha\beta} - hR = 16\pi G T_\alpha^{\alpha(1)}}$$

$$T_\alpha^{Max(1)\alpha} = \frac{B^2}{2}h - B^2\left(h_y^y + h_z^z\right) = B^2(\Phi + \Psi + h_+)$$

$$h^\mu_\nu(t=0,z) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & h_+^0 & h_\times^0 & 0 \\ 0 & h_\times^0 & -h_+^0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} e^{ik_z z} \quad \Rightarrow \quad h^\mu_\nu(t,z) = \begin{bmatrix} 2\Phi(t) & 0 & 0 & 0 \\ 0 & -2\Psi(t) + h_+(t) & h_\times(t) & 0 \\ 0 & h_\times(t) & -2\Psi(t) - h_+(t) & 0 \\ 0 & 0 & 0 & -2\Psi(t) \end{bmatrix} e^{ik_z z}$$

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$$h_\nu^\mu(t,z) = \begin{bmatrix} 2\Phi(t) & 0 & 0 & 0 \\ 0 & -2\Psi(t) + h_+(t) & h_\times(t) & 0 \\ 0 & h_\times(t) & -2\Psi(t) - h_+(t) & 0 \\ 0 & 0 & 0 & -2\Psi(t) \end{bmatrix} e^{ik_z z}$$

$\boldsymbol{k \perp B, h_+}$:

$$\begin{cases} \left[\partial_t^2 + 3H\partial_t + \left(\frac{k^2}{a^2} - 8\pi G B^2\right)\right] h_+ = ik16\pi G B f^y + 8\pi G B^2 6\Psi \\ \left[\partial_t^2 + 3H\partial_t + \frac{k^2}{a^2}\right] f^y = -\frac{ikB}{a^2}\left[2\Phi - 8\Psi - \mathrm{h}_+\right] \\ 4ikH(\Phi + \Psi) = -16\pi G B \dot{f}^y \\ B^2\Psi = -\frac{1}{3} ikBf^y \end{cases}$$

Method

The eq. for scalar modes:

$$\boxed{\partial^2 h + 4h^{\alpha\beta}R_{\alpha\beta} - hR = 16\pi G T_{\alpha}^{\alpha(1)}}$$

$$T_{\alpha}^{Max(1)\alpha} = \frac{B^2}{2}h - B^2\left(h_y^y + h_z^z\right) = B^2(\Phi + \Psi + h_+)$$

$$h_\nu^\mu(t,z) = \begin{bmatrix} 2\Phi(t) & 0 & 0 & 0 \\ 0 & -2\Psi(t) + h_+(t) & h_\times(t) & 0 \\ 0 & h_\times(t) & -2\Psi(t) - h_+(t) & 0 \\ 0 & 0 & 0 & -2\Psi(t) \end{bmatrix} e^{ik_z z}$$

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Summary

$\boldsymbol{k} \parallel \boldsymbol{B}$

$\boldsymbol{k} \perp \boldsymbol{B}$

$$\left[\partial_t^2 + 3H\partial_t + \left(\frac{k^2}{a^2} - 8\pi G B^2\right)\right] h_+ = 0$$

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$B_0 = 1$ nGs

$ka \lesssim 5 * 10^{-24}$ Hz

$\boldsymbol{k} \perp \boldsymbol{B}$

Summary

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$a \in [10^{-9}, 10^{-4}]$

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$\boldsymbol{h}_\times$

$$\left[\partial_t^2 + 3H\partial_t + \left(\frac{k^2}{a^2} - 8\pi G B^2\right)\right] h_\times = -ik16\pi G B f^x$$

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the term is more significant than the term if

$$\frac{8\pi G B_0^2}{a^4} \lesssim \frac{k16\pi G B_0 m_{pl}}{a^2}$$

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$k_1 \gtrsim 10^{-5}$ Hz

$k_2 \gtrsim 10^{-16}$ Hz

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- $k = 10^{-16}$- 10^{-18} Hz => 0.01 % ***amplification*** of GW amplitude (end of RD era)
- $k = 10^{-3}$ Hz => 10 % ***attenuation***

Summary

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  with interaction with the primary plasma

$\boldsymbol{h}_+$

$$\left[\partial_t^2 + 3H\partial_t + \left(\frac{k^2}{a^2} - 8\pi G B^2\right)\right] h_+ = 0$$

+ generation of f^y and Φ, Ψ

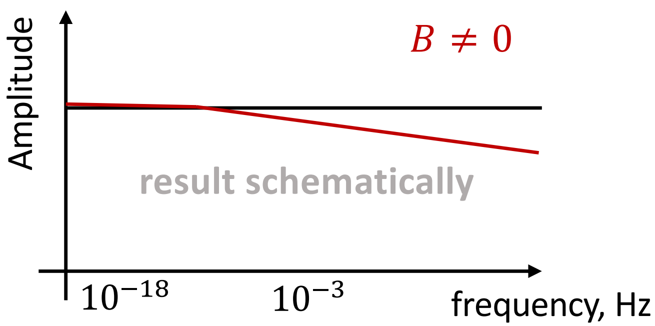
Results and conclusions

Results

1. Equation system for the inverse Gertsenshtein effect is derived in FLRW spacetime
2. Two cases of $\boldsymbol{k}$ direction respectively $\boldsymbol{B}$ are considered
3. Qualitative conclusions are done for each polarization $h_+, h_\times$ in cases $\boldsymbol{k} \parallel \boldsymbol{B}$ and $\boldsymbol{k} \perp \boldsymbol{B}$

Conclusions

1. The influence of the effect on relic GW with frequencies $10^{-16} - 10^{-18}$ Hz (which have an imprint on CMB) is negligible
2. The result can be more significant for the higher frequencies => $\boldsymbol{n_T}$ can be changed



More details in the article:

A.D. Dolgov, L. A. Panasenko, V. A. Bochko
Graviton to photon conversion in curved space-time and external magnetic field
Universe 10 (2024) 1, 7
e-Print: 2310.19838 [gr-qc]

l.vetoshkina@g.nsu.ru

Other effects

Конформная аномалия

Квантовые поправки к ТЭИ электромагнитного поля в искривленном пространстве-времени => ненулевой след

$$T_{\mu}^{\mu(anom)} = \frac{\alpha\beta}{8\pi} G_{\mu\nu}^{(a)} G^{\mu\nu(a)}$$

где β – первый коэффициент разложения бета-функции для калибровочной группы ранга *N*

$$\beta = \frac{11}{3}N - \frac{2}{3}N_F$$

N_F – число сортов фермионов.
После преобразования Фурье:

$$T_{\mu\nu}^{(anom)} \sim \frac{q_\mu q_\nu - g_{\mu\nu}q^2}{q^2} F_{\alpha\beta}F^{\alpha\beta}$$

где *q* – передача 4-импульса гравитационному полю

$$\boxed{\alpha\beta\left(\partial_\mu F^{\mu}_{.\nu} \ln a - HF^{t}_{.\nu}\right)}$$

Взаимодействие с первичной плазмой

1) Дисперсионное соотношение

$$\omega^2 - k^2 = \Omega_{pl}^2,$$

для релятивистских частиц с $m < T$

$$\Omega_{pl}^{rel} = \frac{2T^2}{9}\sum_j e_j^2$$

волны с частотой $\omega < \Omega_{pl}$ не распространяются

$$\boxed{\Omega_{pl}^2 f_\nu \sim \alpha T^2 f_\nu}$$

2) Потеря когерентности => затухание

$$\boxed{\Gamma \dot{f}_\nu = v\sigma n \dot{f}_\nu \sim \alpha^2 T}$$

для релятивистских частиц с $m < T$

$n = 0.1\ g_* T^3$- плотность заряженных частиц в плазме,
$g_* = 10 - 100$ – число сортов заряженных частиц,
$v \sim 1$ – относительная скорость фотона и центра рассеяния в плазме,
$\sigma = \alpha^2/T^2$ - сечение рассеяния

Система уравнений

Полученная система состоит из **двух независимых подсистем**: $\{\boldsymbol{f^x}, \boldsymbol{h_\times}\}$ и $\{\boldsymbol{f^y}, \boldsymbol{\Phi}, \boldsymbol{\Psi}, \boldsymbol{h_+}\}$

Φ и Ψ – скалярные степени свободы

Запишем и решим **первую подсистему** в терминах масштабного фактора a:

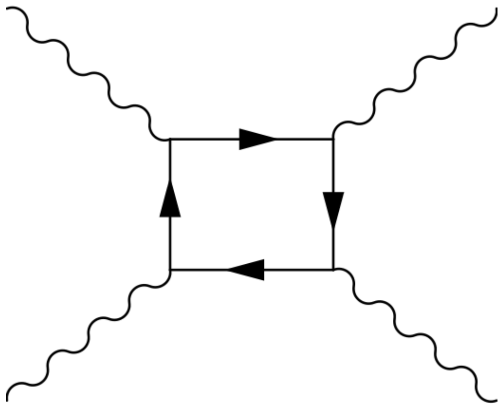
$$f^x\!: \; a^2H^2 f^{x''} + aH^2\left[1 + a\frac{H'}{H} + 8\frac{2B_0^2C_0 - a^4}{16B_0^2C_0 - a^4} + aH\Gamma\right] f^{x'}$$

$$+ \left[\frac{k^2}{a^2} + 2aHH' - 8H^2\frac{4B_0^2C_0 + a^4}{16B_0^2C_0 - a^4} + \omega_{pl}^2 + 2H\Gamma\right] f^x - \alpha\beta H^2\left(af^{x'} + 2f^x\right) = \frac{ikB_0}{a^4 m_{pl}} h_\times$$

$$h_x^y\!: \; a^2H^2h_\times'' + \left(4aH^2 + a^2HH'\right)h_\times' + \left[\frac{k^2}{a^2} - 16\pi G B_0^2 a\left(\frac{4B_0^2C_0}{a^4} - 1\right)\right] h_\times = 16\pi i k G B_0 a^3\left(1 - \frac{16B_0^2C_0}{a^4}\right) m_{pl} f^x$$

штрих означает производную по a.

Действие Гейзенберга-Эйлера



рассеяние фотона на фотоне

Для высоких температур:

$$C(T) = \sum_j \frac{\alpha^2(T) q_j^4}{90 m_j(T)^4}$$

где q_j – заряд частиц, дающих вклад в петлю (в единицах заряда электрона)

В искривленном пространстве-времени:

$$\mathcal{A}_{HE}^{(0)} = \int d^4x \sqrt{-g}\, C_0 \left[(F_{\mu\nu} F^{\mu\nu})^2 + \frac{7}{4} (\tilde{F}^{\mu\nu} F_{\mu\nu})^2 \right]$$

где дуальный тензор электромагнитного поля

$$\tilde{F}_{\alpha\beta} = \frac{\sqrt{-g}}{2} \epsilon_{\alpha\beta\mu\nu} F^{\mu\nu}, \quad \tilde{F}^{\alpha\beta} = \frac{1}{2\sqrt{-g}} \epsilon^{\alpha\beta\mu\nu} F_{\mu\nu}$$ и $$C_0 = \alpha^2/(90 m_e^4)$$

GW in flat spacetime

Einstein equation

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

G – gravitational constant
$R_{\mu\nu}$ – Ricci curvature tensor
$R = R^{\alpha}_{\alpha}$ – scalar curvature

Flat and empty spacetime

$$T_{\mu\nu} = 0$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

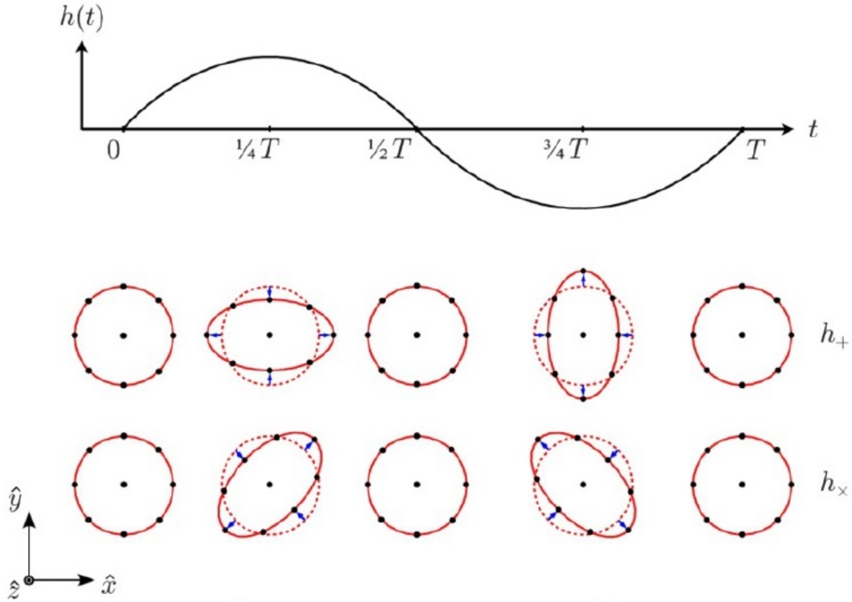
$$\downarrow$$

$$\Box h_{\mu\nu} = 0$$

Solution has

- $h_{\mu\nu} = h_{\nu\mu} \rightarrow$ 10 components
- calibration $\rightarrow$ 2 components

$$\boxed{\eta_{\mu\nu} = diag(1,-1,-1,-1)}$$



$$h_{\mu\nu} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & h_+ & h_\times & 0 \\ 0 & h_\times & -h_+ & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = h_+ e^+_{\mu\nu} + h_\times e^\times_{\mu\nu}$$

GW tensor properties

Gauge fixing

1) **Calibration**

$$\begin{aligned} h_{0\alpha} &= 0, \alpha = 0,1,2,3 \\ x'_\mu &= x_\mu + \xi_\mu \\ h'_{\mu\nu} &= h_{\mu\nu} - \frac{\partial \xi_\mu}{\partial x^\nu} - \frac{\partial \xi_\nu}{\partial x^\mu} \\ R_{\mu\nu} &= \frac{1}{2}\Box h_{\mu\nu} \end{aligned}$$

Fock harmonic gauge $\Box x^\mu = 0$ $<=>$ $\partial_\mu \left(h^\mu_\nu - \frac{1}{2}\delta^\mu_\nu h\right) = 0$

2) Einstein equation → Tracelessness → Transversality

$$h \equiv h^\mu_\mu = 0,\ \partial_\mu h^\mu_\nu = 0$$