Cosmology and Particle Physics

V.A. Rubakov

Institute for Nuclear Research
of the Russian Academy of Sciences

Department of Particle Physics and Cosmology
Physics Faculty
M.V. Lomonosov Moscow State University

Outline of Lecture 1

- Expanding Universe
- Dark matter: evidence
- WIMPs

Expanding Universe

- The Universe at large is homogeneous, isotropic and expanding.

  3d space is Euclidean (observational fact!)

  Sum of angles of a triangle = 180^o, even for triangles as large as the size of the visible Universe.

  All this is encoded in space-time metric (Friedmann–Lemâitre–Robertson–Walker)

$$ds^2 = dt^2 - a^2(t)\mathbf{dx}^2$$

  $\mathbf{x}$: comoving coordinates, label distant galaxies.

  $a(t)dx$: physical distances.

  $a(t)$: scale factor, grows in time; a_0: present value (matter of convention)

Space-time metric

$$ds^2 = dt^2 - a^2(t)\mathbf{dx}^2$$

$a(t)dx$: physical distances.

$a(t)$: scale factor, grows in time; a_0: present value

$$z(t) = \frac{a_0}{a(t)} - 1 : \qquad \text{redshift}$$

Light of wavelength λ emitted at time t has now wavelength
$\lambda_0 = \frac{a_0}{a(t)}\lambda = (1+z)\lambda$; $z(t)$: redshift

Momenta of all free particles scale as $p \propto a^{-1} \propto (1+z)$. Example: neutrinos were relativistic early on, non-relativistic now.

$$H(t) = \frac{\dot{a}}{a} : \qquad \text{Hubble parameter, expansion rate}$$

$H^{-1}(t)$: time scale at a given epoch.

- Present value (Planck; under some debate)

$$H_0 = (67.4 \pm 0.5)\ \frac{\text{km/s}}{\text{Mpc}} = (14 \cdot 10^9\ \text{yrs})^{-1}$$

$1\ \text{Mpc} = 3 \cdot 10^6\ \text{light yrs} = 3 \cdot 10^{24}\ \text{cm}$

NB: length scales today:

  - visible part of a galaxy $\sim$ 10 kpc
  - dark halo of a galaxy $\sim$ 100 kpc =0.1 Mpc
  - cluster of galaxies $\sim$ 1 – 3 Mpc
  - visible Universe = 14 Gpc

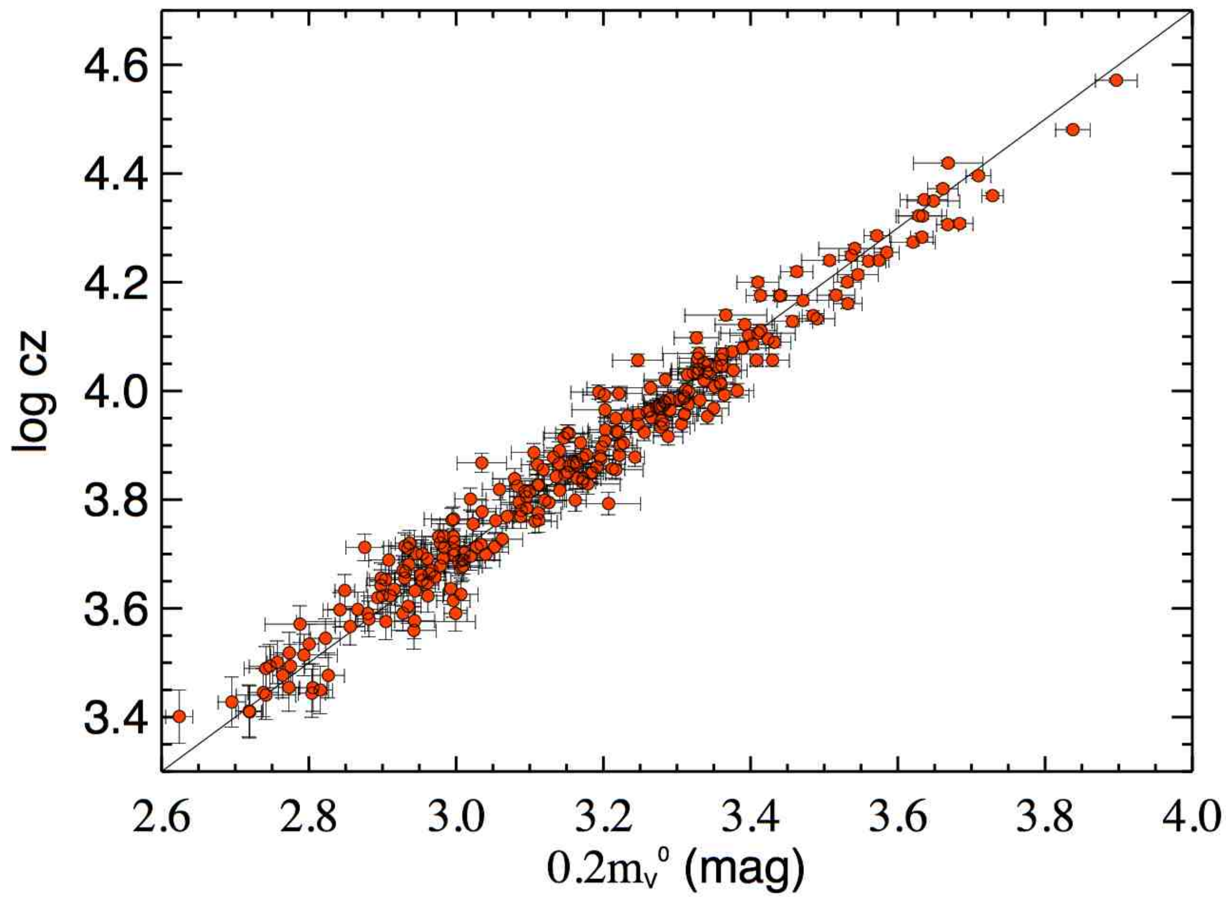
- Hubble law (valid at $z \ll 1$)

$$z = H_0 r$$

Fig

Problem: prove the Hubble law

Hubble diagram, log–log plot



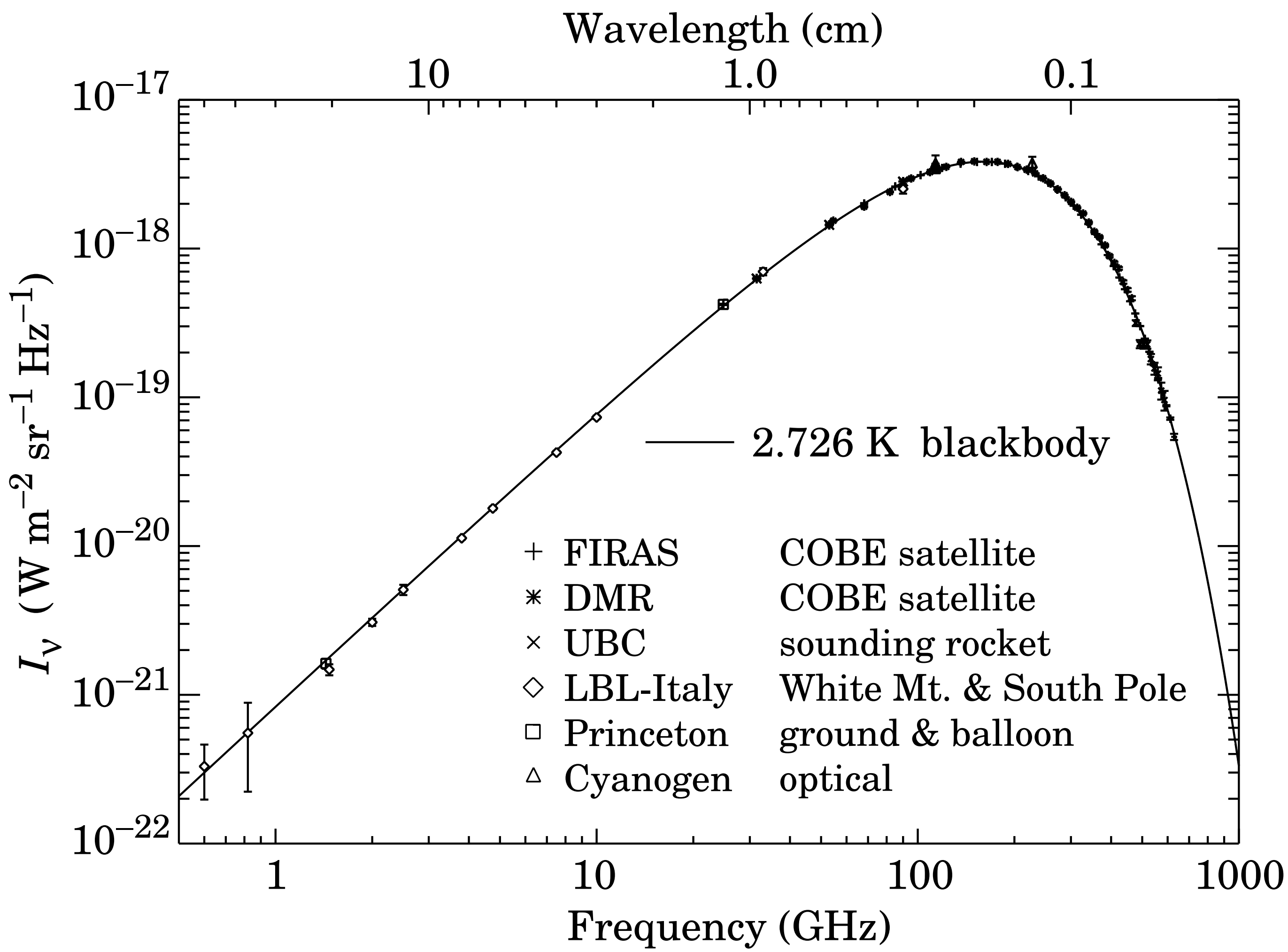
- The Universe is warm: CMB temperature today

$$T_0 = 2.7255 \pm 0.0006\ \mathrm{K}$$

Fig.

It was denser and warmer at early times.

CMB spectrum



$T = 2.726$ K

- Present number density of photons

$$n_\gamma = \# T^3 = 410 \frac{1}{\mathrm{cm}^3}$$

- Present entropy density

$$s = 2 \cdot \frac{2\pi^2}{45} T_0^3 + \text{neutrino contribution} = 3000 \frac{1}{\mathrm{cm}^3}$$

In early Universe (Bose–Einstein, Fermi–Dirac)

$$s = \frac{2\pi^2}{45} g_* T^3$$

g_*: number of relativistic degrees of freedom with $m \lesssim T$; fermions contribute with factor $7/8$.

Slow expansion $\Longrightarrow$ entropy conservation $\Longrightarrow$
Entropy density scales exactly as a^{-3}

Temperature scales approximately as a^{-1}.

Dynamics of expansion

- Friedmann equation: expansion rate of the Universe vs total energy density ρ ($M_{Pl} = G^{-1/2} = 10^{19}$ GeV):

$$\left(\frac{\dot{a}}{a}\right)^2 \equiv H^2 = \frac{8\pi}{3M_{Pl}^2}\rho$$

Einstein equations of General Relativity specified to homogeneous isotropic space-time with zero spatial curvature

- Present energy density

$$\rho_0 = \rho_c = \frac{3M_{Pl}^2}{8\pi}H_0^2 = 5 \cdot 10^{-6} \ \frac{\text{GeV}}{\text{cm}^3}$$
$$= 5\frac{m_p}{\text{m}^3}$$

$\hbar = c = k_B = 1$ in what follows

Present composition of the Universe

$$\Omega_i = \frac{\rho_{i,0}}{\rho_c}$$

present fractional energy density of i-th type of matter.

$$\sum_i \Omega_i = 1$$

- Dark energy: $\Omega_\Lambda = 0.69$
  ρ_Λ stays (almost?) constant in time [defining property]
- Non-relativistic matter: $\Omega_M = 0.31$
  $\rho_M = mn(t)$ scales as $\left(\frac{a_0}{a(t)}\right)^3$
  - Dark matter: $\Omega_{DM} = 0.26$
  - Usual matter (baryons): $\Omega_B = 0.049$
- Relativistic matter (radiation): $\Omega_{rad} = 8.6 \cdot 10^{-5}$ (for massless neutrinos)
  $\rho_{rad} = \omega(t)n(t)$ scales as $\left(\frac{a_0}{a(t)}\right)^4$

Friedmann equation

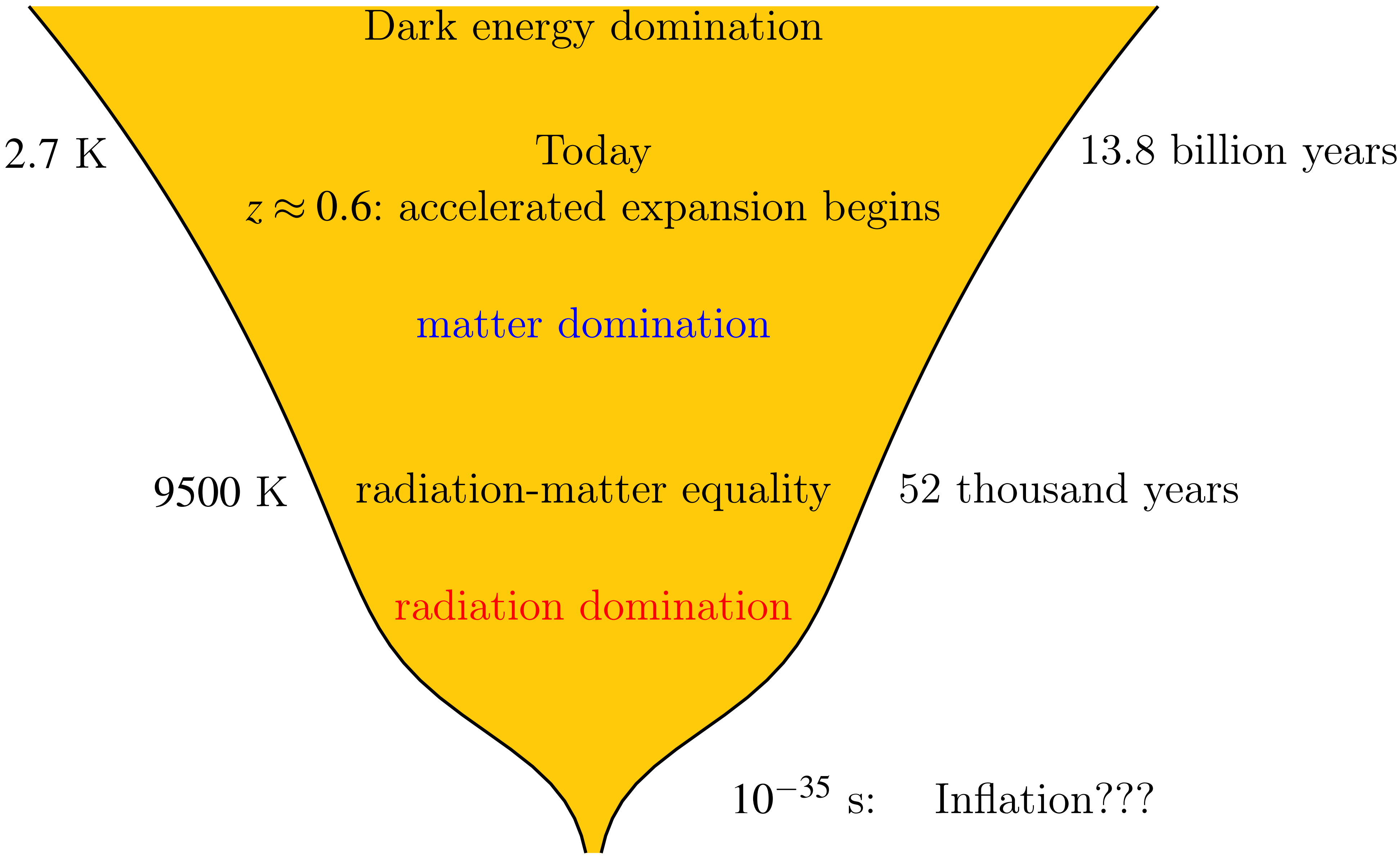
$$H^2(t) = \frac{8\pi}{3M_{Pl}^2}\left[\rho_\Lambda + \rho_M(t) + \rho_{rad}(t)\right] = H_0^2\left[\Omega_\Lambda + \Omega_M\left(\frac{a_0}{a(t)}\right)^3 + \Omega_{rad}\left(\frac{a_0}{a(t)}\right)^4\right]$$

$\ldots \Longrightarrow$ Radiation domination $\Longrightarrow$ Matter domination $\Longrightarrow$ Λ–domination

$z_{eq} = 3500$ — now

$T_{eq} = 9500$ K $= 0.8$ eV

$t_{eq} = 52 \cdot 10^3$ yrs


Dark energy domination
2.7 K
Today
13.8 billion years
$z \approx 0.6$: accelerated expansion begins
matter domination
9500 K
radiation-matter equality
52 thousand years
radiation domination
10^{-35} s: Inflation???

Expansion at radiation domination

- Expansion law:

$$H^2 = \frac{8\pi}{3M_{Pl}^2}\rho \implies \frac{\dot{a}^2}{a^2} = \frac{\text{const}}{a^4}$$

Solution:

$$a(t) = \text{const} \cdot \sqrt{t}$$

- $t = 0$: Big Bang singularity

$$H = \frac{\dot{a}}{a} = \frac{1}{2t}\,, \qquad \rho \propto \frac{1}{t^2}$$

- Decelerated expansion: $\ddot{a} < 0$.

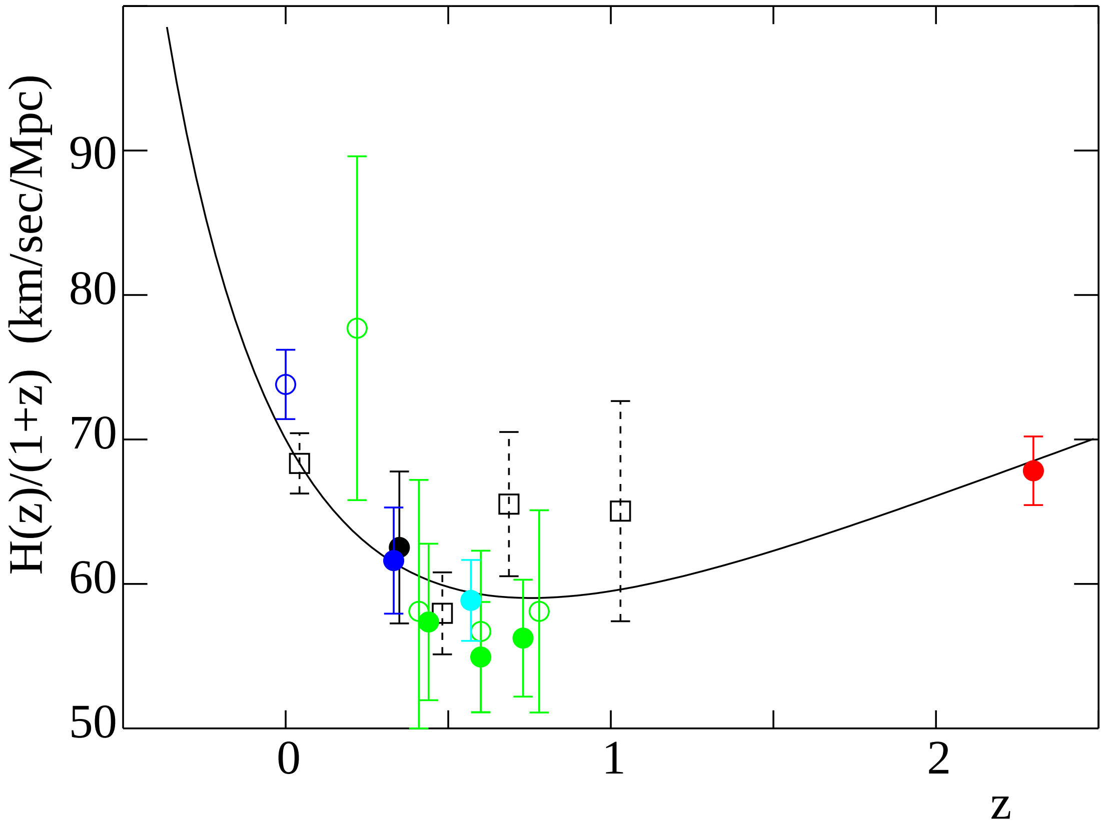
NB: Matter domination: $a(t) = \text{const} \cdot t^{2/3}$;
decelerated expansion: $\ddot{a} < 0$

- NB: Λ-domination

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi}{3M_{Pl}^2}\rho_\Lambda = \text{const} \Longrightarrow a(t) = \mathrm{e}^{H_\Lambda t}$$

accelerated expansion Fig.

Deceleration to acceleration



$$\frac{H}{1+z} = \frac{\dot{a}}{a} \cdot \frac{a}{a_0} = \frac{\dot{a}}{a_0} = \dot{a}(z)\,;$$ large $z \leftrightarrow$ early time

Cosmological (particle) horizon

Light travels along $ds^2 = dt^2 - a^2(t)d\mathbf{x}^2 = 0 \Longrightarrow dx = dt/a(t)$.
If emitted at $t = 0$, travels finite coordinate distance

$$\eta = \int_0^t \frac{dt'}{a(t')} \propto \sqrt{t} \quad \text{at radiation domination}$$

$\eta \propto \sqrt{t} \Longrightarrow$ visible Universe increases in time

Fig.

Physical size of causally connected region at time t (horizon size)

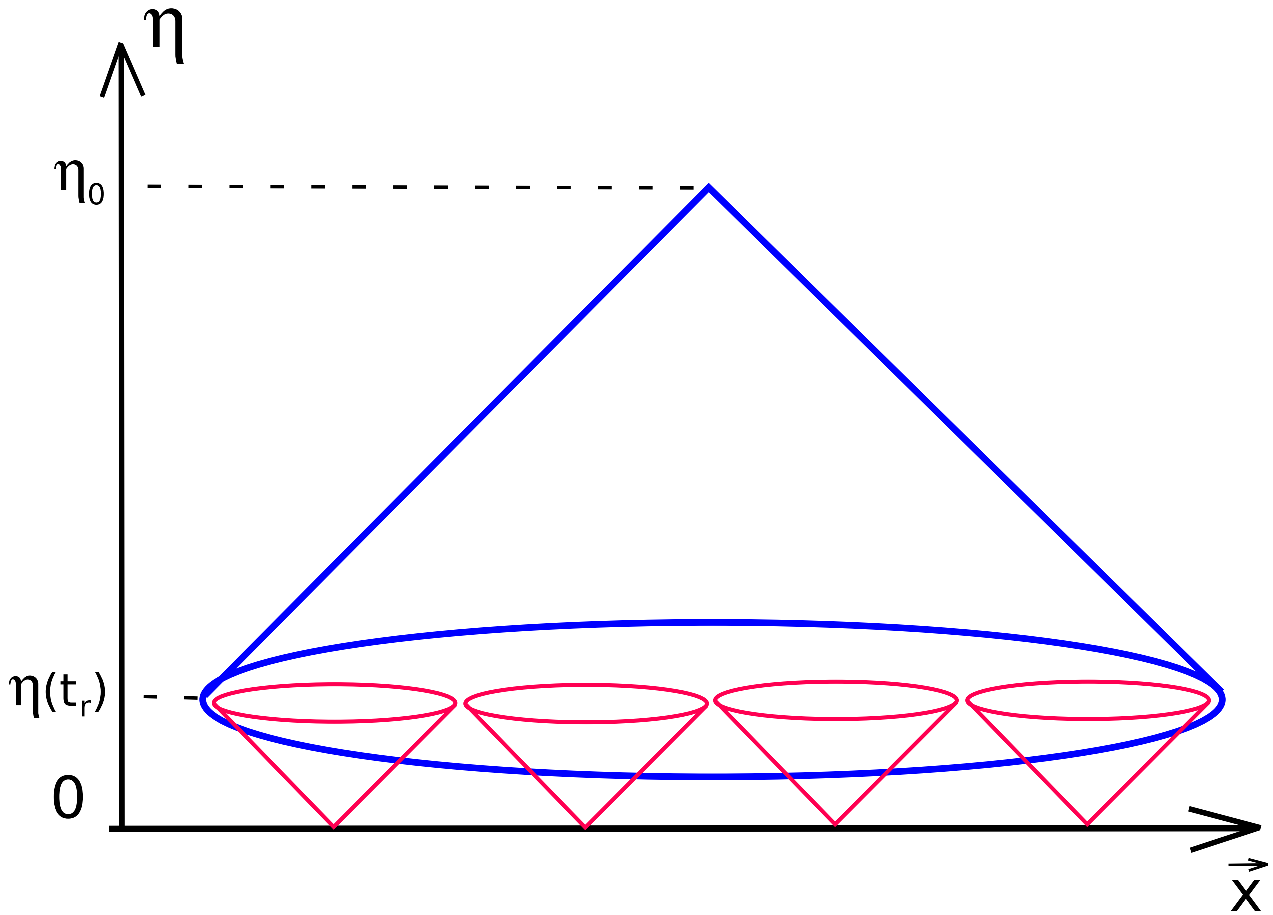
$$l_{H,t} = a(t)\int_0^t \frac{dt'}{a(t')} = 2t \quad \text{at radiation domination}$$

In hot Big Bang theory at both radiation and matter domination

$$l_{H,t} \sim t \sim H^{-1}(t)$$

Today $l_{H,t_0} \approx 15\ \text{Gpc} = 4.5 \cdot 10^{28}\ \text{cm}$

Causal structure of space-time in hot Big Bang theory



Regions of Hubble size have not talked to each other (are causally disconnected).

We see many such regions. Why are they all the same?

Cornerstones of thermal history

- Neutrino decoupling: $T = 2 - 3$ MeV $\sim 3 \cdot 10^{10}$K, $\quad t \sim 0.1 - 1$s
- Big Bang Nucleosynthesis, epoch of thermonuclear reactions

$$\begin{aligned} p + n &\rightarrow {}^2H \\ {}^2H + p &\rightarrow {}^3He \\ {}^3He + n &\rightarrow {}^4He \\ \text{up to} &\quad {}^7Li \end{aligned}$$

Abundances of light elements: measurements vs theory

$$T = 10^{10} \rightarrow 10^9 \text{ K}, \quad t = 1 \rightarrow 300 \text{ s}$$

Earliest time in thermal history probed quantitatively so far

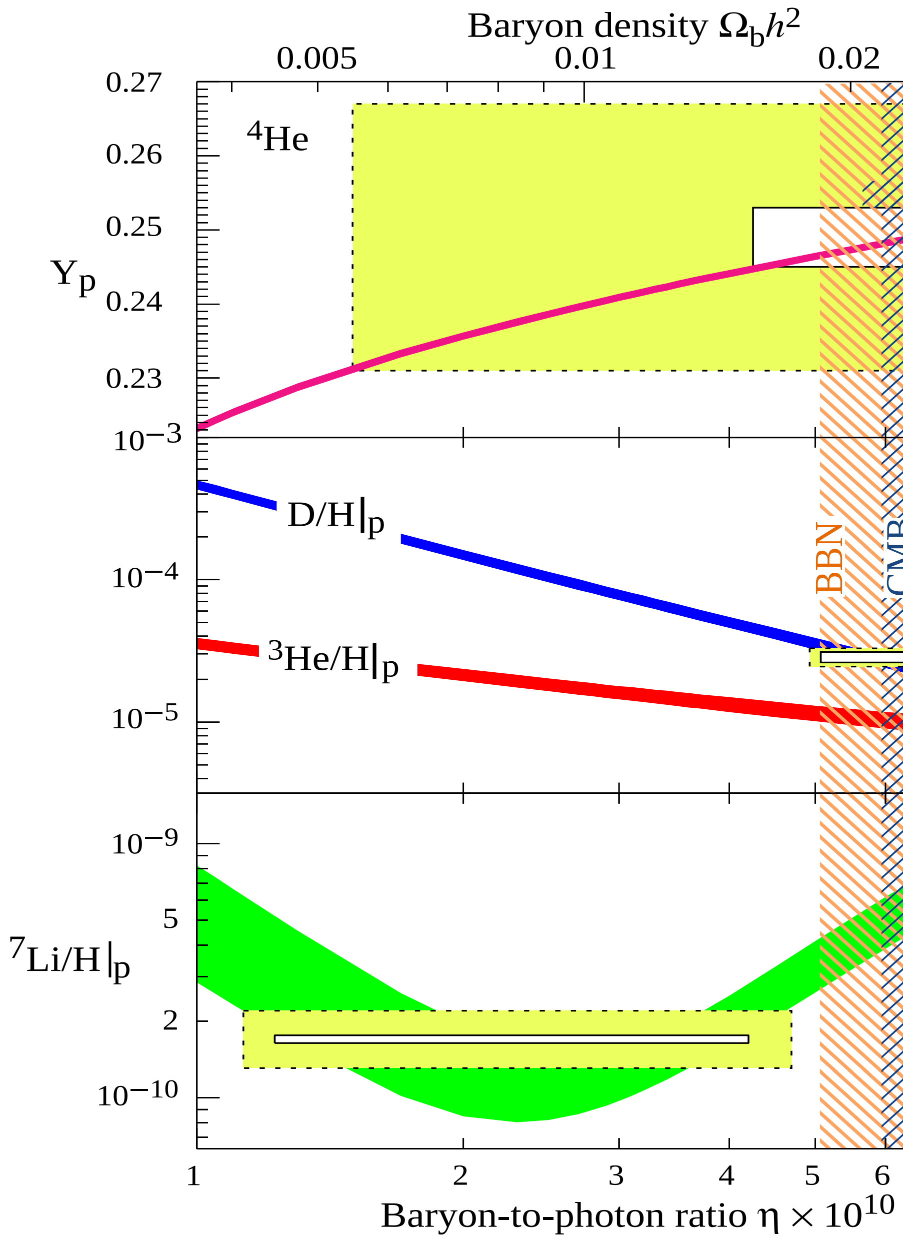
Fig.

- Recombination, transition from plasma to gas.

$$z = 1090, \ T = 3000 \text{ K}, \quad t = 380\,000 \text{ years}$$

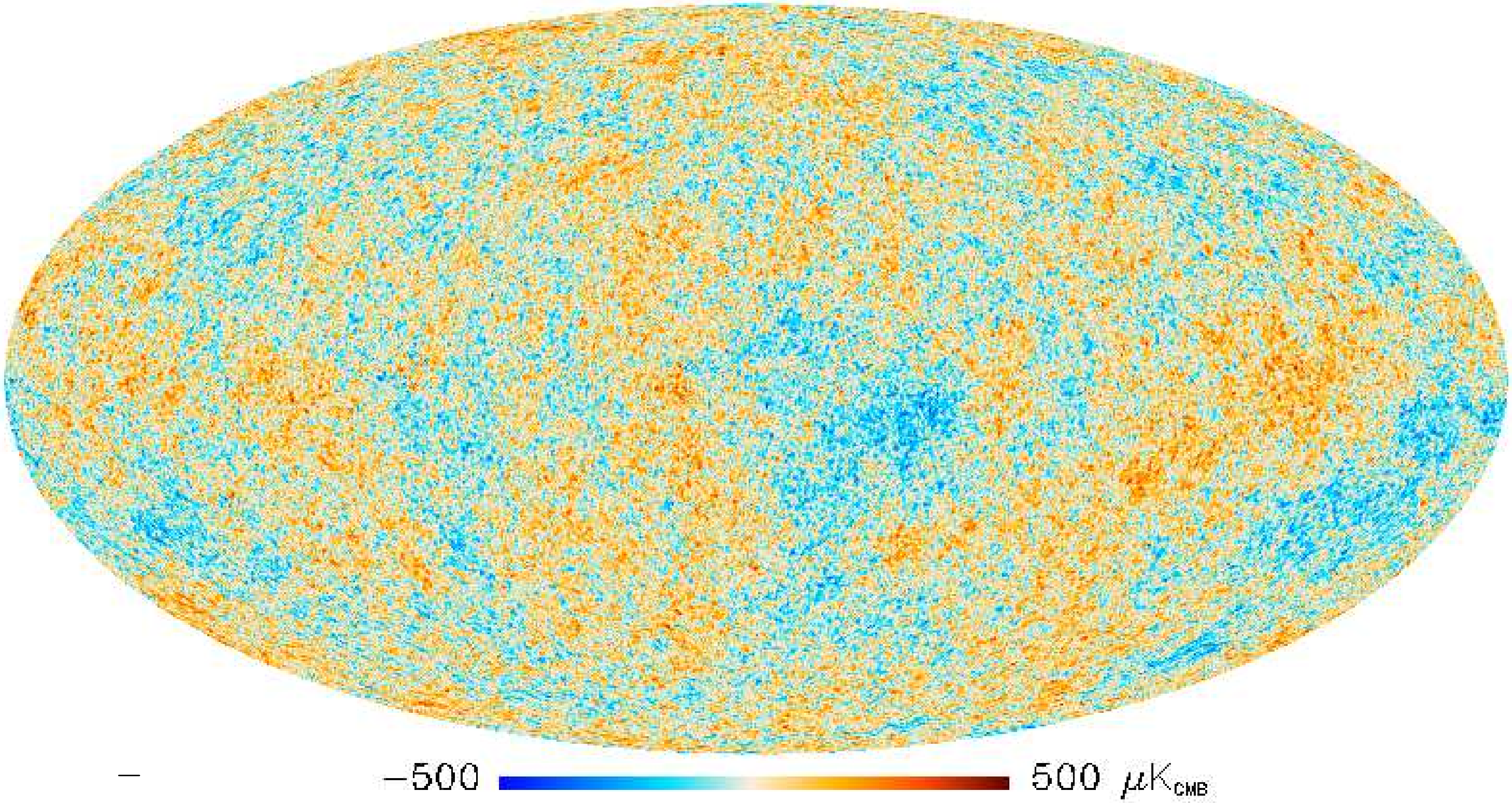
Last scattering of CMB photons

Photographic picture of young Universe Fig.



$\eta_{10} = \eta \cdot 10^{-10}$ = baryon-to-photon ratio. Consistent with CMB determination of η

$$T = 2.726^\circ K, \quad \frac{\delta T}{T} \sim 10^{-4} - 10^{-5}$$



Planck

Fourier decomposition of temperatue fluctuations:

$$\frac{\delta T}{T}(\theta,\varphi) = \sum_{l,m} a_{lm} Y_{lm}(\theta,\varphi)$$

a_{lm}: independent Gaussian random variables, $\langle a_{lm} a^*_{l'm'} \rangle \propto \delta_{ll'} \delta_{mm'}$

$\langle a^*_{lm} a_{lm} \rangle = C_l$ are measured; usually shown $D_l = \frac{l(l+1)}{2\pi} C_l$

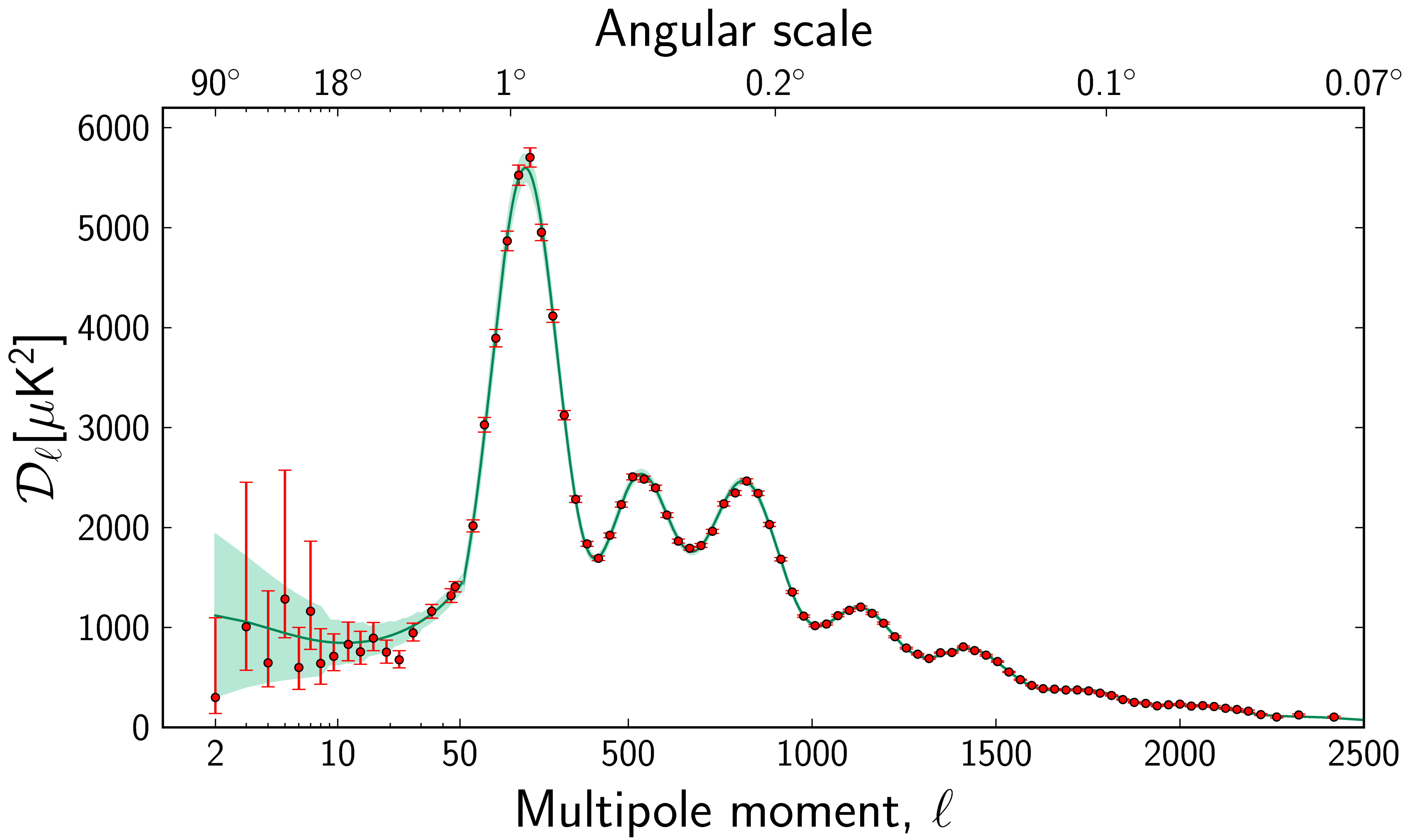
larger $l \Longleftrightarrow$ smaller angular scales, shorter wavelengths

NB: One Universe, one realization of an ensemble $\Longrightarrow$ cosmic variance $\Delta C_l / C_l \simeq 1/\sqrt{2l}$

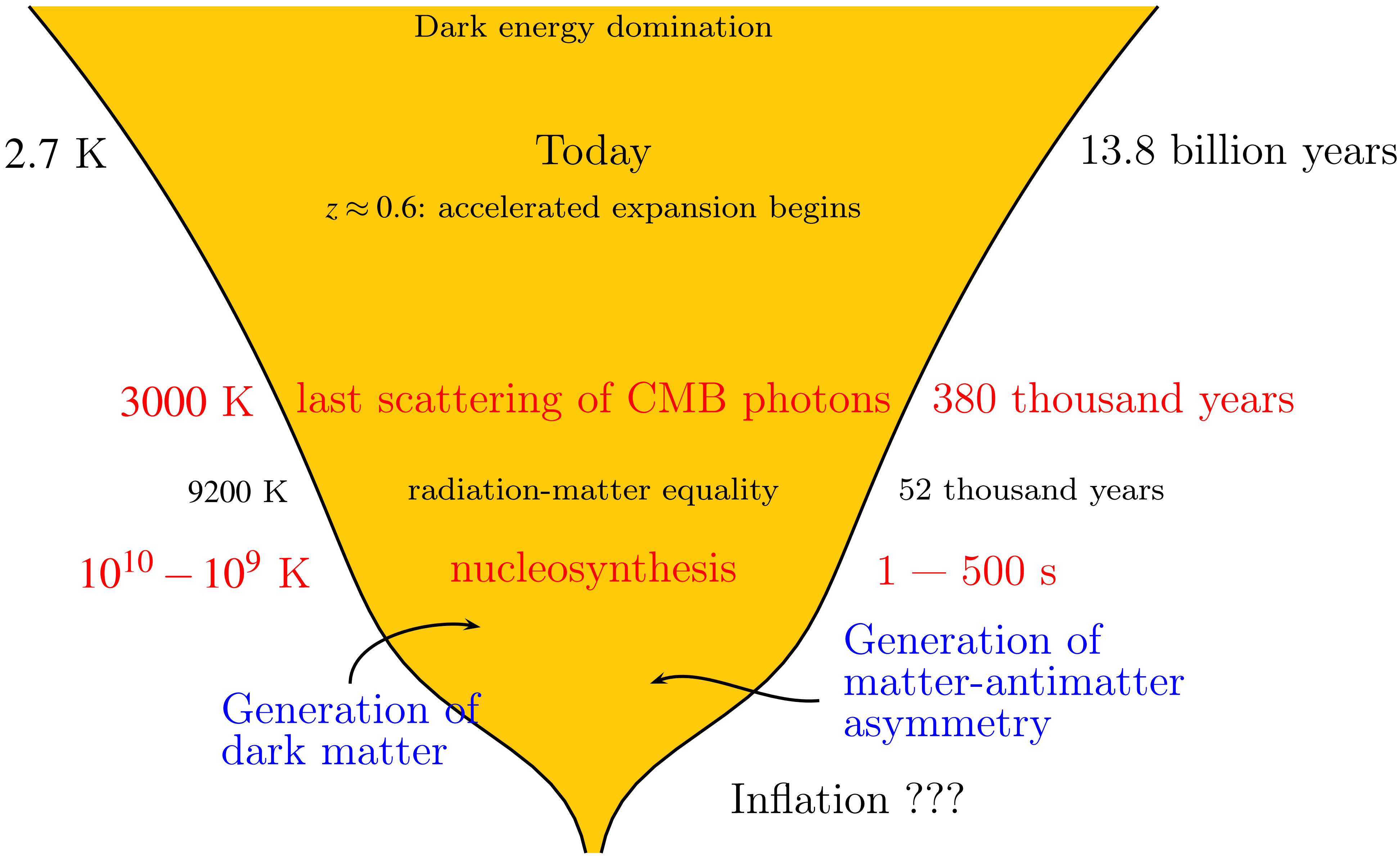
- Physics:
  - Primordial perturbations
  - Development of sound waves in cosmic plasma from early hot stage to recombination;
    gravitational potentials due to dark matter at recombination $\Longrightarrow$ composition of cosmic plasma
  - Propagation of photons after recombination
    $\Longrightarrow$ expansion history of the Universe

CMB angular spectrum

Planck

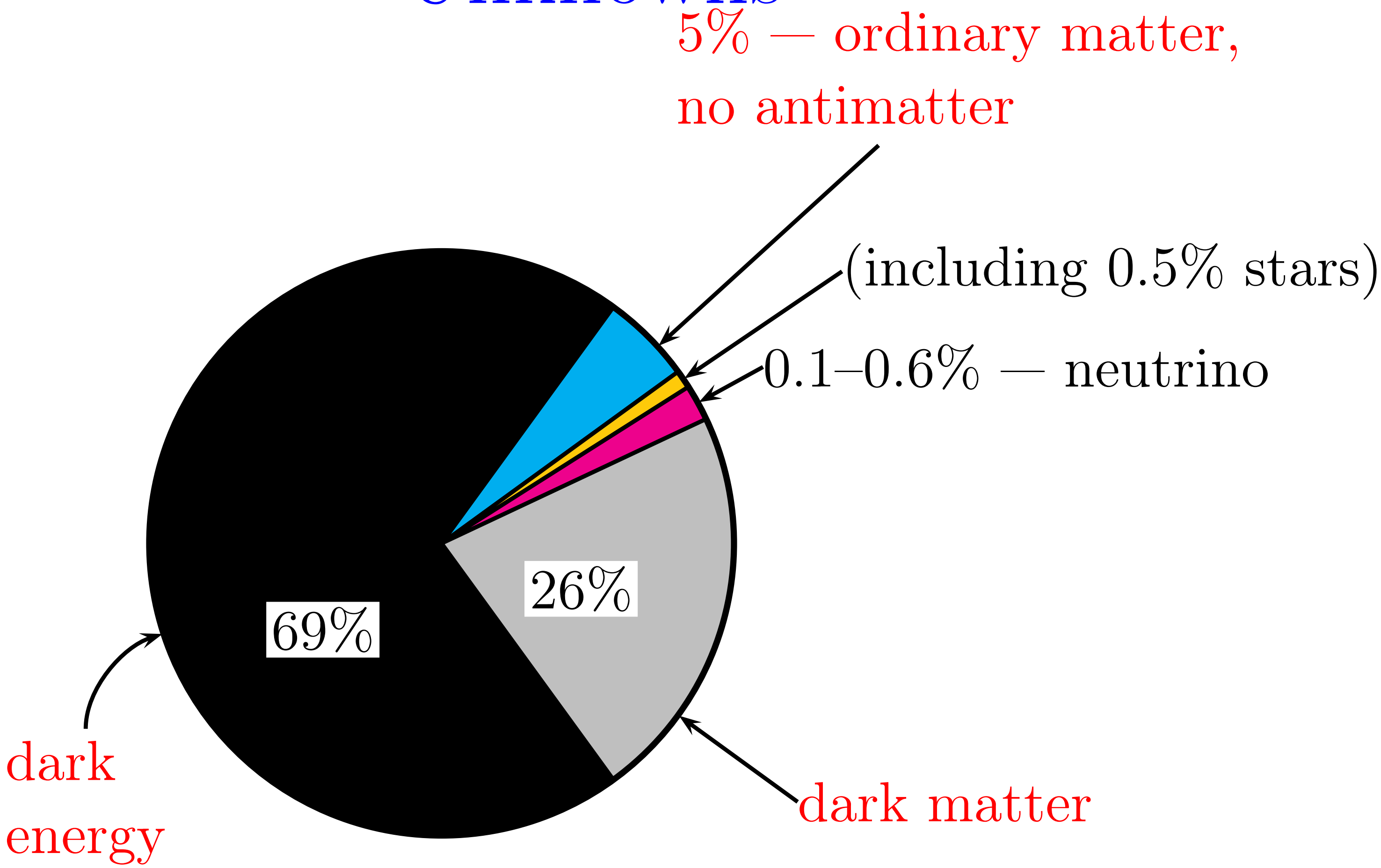


Angular scale of acoustic peaks known with precision 0.03% (!)



Dark energy domination
2.7 K
Today
13.8 billion years
$z \approx 0.6$: accelerated expansion begins
3000 K
last scattering of CMB photons
380 thousand years
9200 K
radiation-matter equality
52 thousand years
$10^{10} - 10^9$ K
nucleosynthesis
1 — 500 s
Generation of dark matter
Generation of matter-antimatter asymmetry
Inflation ???

Unknowns


5% — ordinary matter,
no antimatter
(including 0.5% stars)
0.1–0.6% — neutrino
26%
69%
dark
energy
dark matter