Cosmology and Particle Physics

Problems for seminar

V.A. Rubakov

Problem 1a

Prove the Hubble law

$$z = H_0 r + O(r^2)$$

where z and r are redshift of a source and present distance to the source.

Problem 1b

Find the correction of order r^2 to the Hubble law (z and r are still redshift of a source and present distance to the source). Express this correction through the present value H_0 of the Hubble parameter and the present acceleration parameter

$$q_0 = \frac{1}{H_0^2}\left(\frac{\ddot{a}}{a}\right)_{t_0}$$

Hint: notice that, generally speaking, r is not equal to $(t_0 - t)$, where t is time of emission, and t_0 is present time.

Problem 2

Find lifetime of the Universe filled with non-relativistic matter without cosmological constant, when the Hubble parameter equals

$$H = H_0 = 67 \ \frac{\text{km}}{\text{s} \cdot \text{Mpc}}$$

Why is it shorter than the lifetime of our Universe with non-zero cosmological constant?

Problem 3a

Known neutrinos are in thermal equilibrium with cosmic plasma in the early Universe due to processes

$$\nu\bar{\nu} \longleftrightarrow e^+e^-$$

and crossing processes. Their cross sections in relativistic case (c.m. energy $E \gg m_e$) are

$$\sigma = C \cdot G_F^2 E^2 \,, \quad C = O(1) \,.$$

Estimate neutrino freeze-out temperature.

Problem 3b

Assuming that neutrinos are massless, find the present neutrino temperature.
Find the present number density of every type of neutrino (at this point the masslessness assumption is unnecessary).
Let neutrinos contribute less than 5% of dark matter mass density. Obtain the limit on neutrino mass.

Problem 4

Consider a model in which dark matter particle (WIMP) is a neutral scalar S which interacts only with the Standard Model Higgs doublet H:

$$L_{int} = \lambda_{SH} H^{\dagger} H S^2$$

Consider the case

$$m_b < m_S < m_H/2$$

(no fine tuning).

Estimate the value of the coupling constant λ_{SH}, such that S has correct relic mass density today. Estimate branching of invisible decay

$$h \to SS\,.$$

Is this model ruled out?