4 See-saw mechanism.

4.1 Dirac-Majorana mass term for one generation.

It is possible to consider mixed models in which both Majorana and Dirac mass terms are present. For simplicity sake we'll start with a **toy model** for one lepton generation.

Let us consider a theory containing two independent neutrino fields ν_L and ν_R:

$$\begin{cases} \nu_L \text{ would generally represent any active neutrino (e.g., } \nu_L = \nu_{eL}\text{),} \\ \nu_R \text{ can represents a right handed field unrelated to any of these or} \\ \quad \text{it can be charge conjugate of any of the active neutrinos (e.g., } \nu_R = (\nu_{\mu L})^c\text{).} \end{cases}$$

We can write the following generic mass term between ν_L and ν_R:

$$\mathcal{L}_m = - \underbrace{m_D \overline{\nu}_L \nu_R}_{\text{Dirac mass term}} - \underbrace{(1/2) \left[m_L \overline{\nu}_L \nu_L^c + m_R \overline{\nu}_R^c \nu_R \right]}_{\text{Majorana mass term}} + \text{H.c.} \tag{5}$$

- ⋆ As we know, the Dirac mass term respects L while the Majorana mass term violates it.
- ⋆ The parameter m_D in Eq. (5) is in general complex; to simplify matters, we'll assume it to be real but not necessarily positive.
- ⋆ The parameters m_L, and m_R in Eq. (5) can be chosen real and (by an appropriate rephasing the fields ν_L and ν_R) non-negative, but the latter is not assumed.
- ⋆ Obviously, neither ν_L nor ν_R is a mass eigenstate.

In order to obtain the *mass basis* we can apply the useful identity

$$\overline{\nu}_L \nu_R = (\overline{\nu}_R)^c (\nu_L)^c \tag{6}$$

> The identity (6) is a particular case of the more general relation
>
> $$\overline{\psi}_1 \Gamma \psi_2 = \overline{\psi}_2^c C \Gamma^T C^{-1} \psi_1^c,$$
>
> in which $\psi_{1,2}$ are Dirac spinors and Γ represents an arbitrary combination of the Dirac γ matrices.

Relation (6) allows us to rewrite Eq. (5) as follows

$$\mathcal{L}_m = -\frac{1}{2} \left(\overline{\nu}_L, (\overline{\nu}_R)^c \right) \begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix} \begin{pmatrix} (\nu_L)^c \\ \nu_R \end{pmatrix} + \text{H.c.} \equiv \; -\frac{1}{2} \overline{\boldsymbol{\nu}}_L \mathbf{M} \left(\boldsymbol{\nu}_L \right)^c + \text{H.c.}$$

If (again for simplicity) CP conservation is assumed the matrix $\mathbf{M}$ can be diagonalized by the orthogonal transformation that is rotation

$$\mathbf{V} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \quad \text{with} \quad \theta = \frac{1}{2} \arctan\left(\frac{2m_D}{m_R - m_L} \right).$$

and we have

$$\mathbf{V}^T \mathbf{M} \mathbf{V} = \text{diag}(m_1, m_2),$$

where $m_{1,2}$ are eigenvalues of $\mathbf{M}$ given by

$$m_{1,2} = \frac{1}{2} \left(m_L + m_R \pm \sqrt{(m_L - m_R)^2 + 4m_D^2} \right).$$

The eigenvalues are real if (as we assume) $m_{D,L,R}$ are real, but not necessarily positive. Let us define $\zeta_k = \text{sign}\, m_k$ and rewrite the mass term in the new basis:

$$\mathcal{L}_m = -\frac{1}{2}\left[\zeta_1 \left|m_1\right| \overline{\nu}_{1L} \left(\nu_{1L}\right)^c + \zeta_2 \left|m_2\right| \left(\overline{\nu}_{2R}\right)^c \nu_{2R}\right] + \text{H.c.}, \tag{7}$$

The new fields ν_{1L} and ν_{2R} represent chiral components of two different neutrino states with "masses" m_1 and m_2, respectively:

$$\begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix} = \mathbf{V} \begin{pmatrix} \nu_{1L} \\ \nu_{2R}^c \end{pmatrix} \quad \Longrightarrow \quad \begin{cases} \nu_{1L} = \cos\theta\, \nu_L - \sin\theta\, \nu_R^c, \\ \nu_{2R} = \sin\theta\, \nu_L^c + \cos\theta\, \nu_R. \end{cases}$$

Now we define two 4-component fields

$$\nu_1 = \nu_{1L} + \zeta_1 \left(\nu_{1L}\right)^c \quad \text{and} \quad \nu_2 = \nu_{2R} + \zeta_2 \left(\nu_{2R}\right)^c.$$

Certainly, these fields are self-conjugate with respect to the C transformation:

$$\nu_k^c = \zeta_k \nu_k \quad (k = 1, 2)$$

and therefore they describe Majorana neutrinos. In terms of these fields Eq. (7) reads

$$\mathcal{L}_m = -\frac{1}{2}\left(\left|m_1\right| \overline{\nu}_1 \nu_1 + \left|m_2\right| \overline{\nu}_2 \nu_2\right). \tag{8}$$

We can conclude therefore that $\nu_k(x)$ is the Majorana neutrino field with the definite (physical) mass $|m_k|$.

There are several special cases of the Dirac-Majorana mass matrix $\mathbf{M}$ which are of considerable phenomenological importance, in particular,

(A): $\mathbf{M} = \begin{pmatrix} 0 & m \\ m & 0 \end{pmatrix} \quad \Longrightarrow \quad |m_{1,2}| = m, \quad \theta = \frac{\pi}{4}$ (maximal mixing).

Two Majorana fields are equivalent to one Dirac field.

A generalization $|m_{L,R}| \ll |m_D|$, leads to the so-called

Pseudo-Dirac neutrinos.

(B): $\mathbf{M} = \begin{pmatrix} m_L & m \\ m & m_L \end{pmatrix} \quad \Longrightarrow \quad m_{1,2} = m_L \pm m_D, \quad \theta = \frac{\pi}{4}$ (maximal mixing);

(C): $\mathbf{M} = \begin{pmatrix} 0 & m \\ m & M \end{pmatrix}$ or, more generally, $|m_L| \ll |m_R|, \quad m_D > 0$.

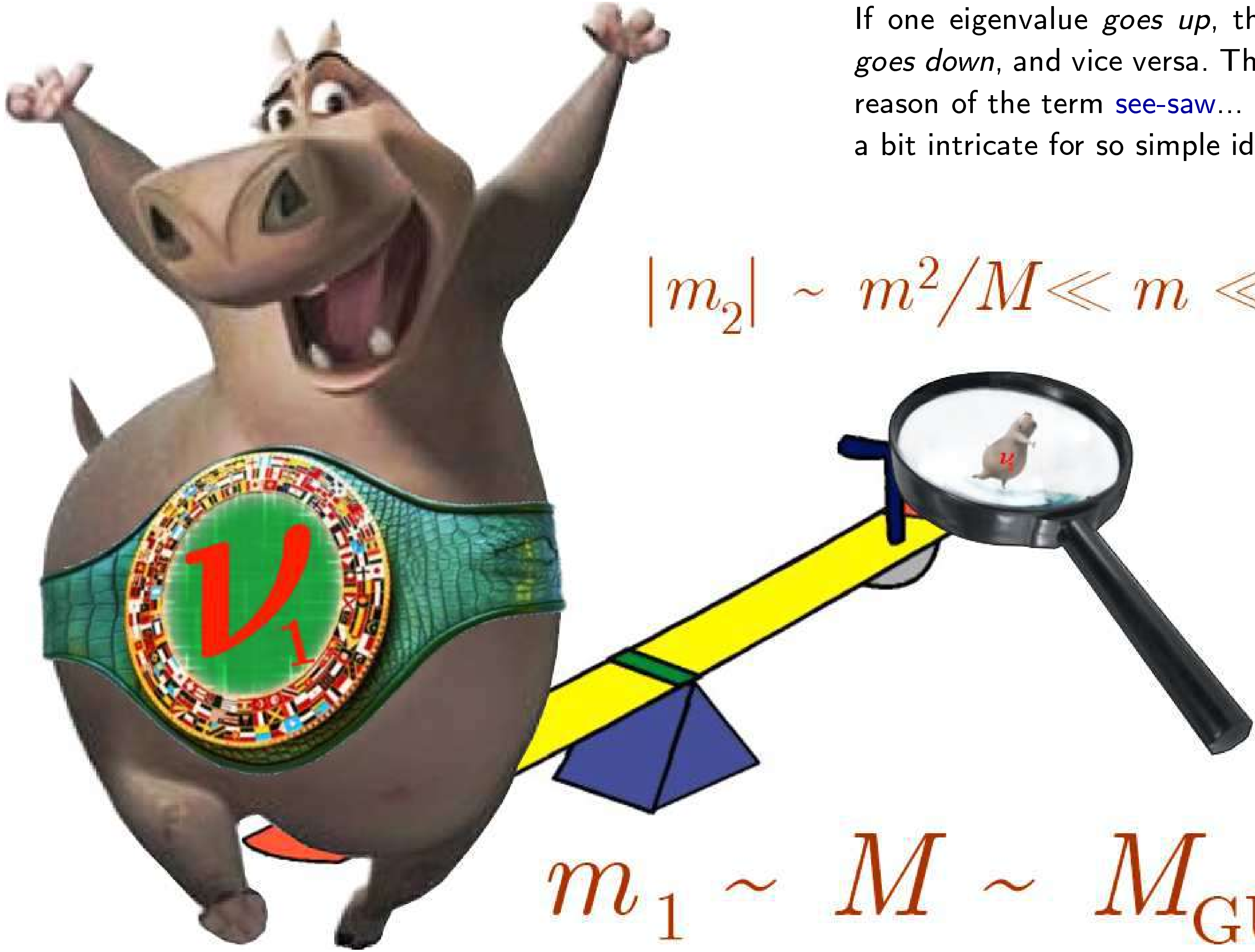
The see-saw

The case (C) with $m \ll M$ is the simplest example of the see-saw mechanism. It leads to two masses, one very large, $m_1 \approx M$, other very small, $m_2 \approx -m^2/M \ll m$, suppressed compared to the entries in $\mathbf{M}$. In particular, one can assume

$m \sim m_\ell$ or m_q (0.5 MeV to 200 GeV) and $M \sim M_{\text{GUT}} \sim 10^{15-16}$ GeV.

Then $|m_2|$ can ranges from $\sim 10^{-14}$ eV to ~ 0.04 eV. The mixing between the heavy and light neutrinos is extremely small: $\theta \approx m/M \sim 10^{-20} - 10^{-13} \lll 1$.

If one eigenvalue *goes up*, the other *goes down*, and vice versa. This is the reason of the term see-saw...
a bit intricate for so simple idea...



$$|m_2| \sim m^2/M \ll m \ll M$$

$$m_1 \sim M \sim M_{\mathrm{GUT}}$$

4.2 More neutral fermions.

A generalization of the above scheme to N generations is almost straightforward but technically rather cumbersome. Let's consider it schematically for the $N = 3$ case.

- If neutral fermions are added to the set of the SM fields, then the flavour neutrinos can acquire mass by mixing with them.
- The additional fermions can be[a]
  - Gauge chiral singlets per family $\mathcal{N}$ (e.g., right-handed neutrinos) [Type I seesaw], or
  - $SU(2) \times U(1)$ doublets (e.g., Higgsino in SUSY), or
  - $Y = 0$, $SU(2)_L$ triplets Σ (e.g., Wino in SUSY) [Type III seesaw].
- Addition of three right-handed neutrinos $\mathcal{N}_{iR}$ leads to the see-saw mechanism with the following mass terms:

$$\mathcal{L}_m = -\sum_{ij} \left[\overline{\nu}_{iL} M^D_{ij} \mathcal{N}_{jR} - \frac{1}{2} \left(\mathcal{N}_{iR}\right)^c M^R_{ij} \mathcal{N}_{jR} + \text{H.c.} \right].$$

- The above equation leads to the following 6×6 see-saw mass matrix:

$$\mathbf{M} = \begin{pmatrix} \mathbf{0} & \mathbf{m}_D^T \\ \mathbf{m}_D & \mathbf{M}_R \end{pmatrix}.$$

  Both $\mathbf{m}_D$ and $\mathbf{M}_R$ are 3×3 matrices in the generation space.

[a] Type II seesaw operates with additional $SU(2)_L$ scalar triplets Δ.

Similar to the one-generation case we assume that the eigenvalues of $\mathbf{M}_R$ are large in comparison with the eigenvalues of $\mathbf{m}_D$. Then $\mathbf{M}$ can be approximately block-diagonalized by an unitary transformation:

$$\mathbf{U}^\dagger \mathbf{M} \mathbf{U} = \mathrm{diag}\left(\mathbf{M}_1, \mathbf{M}_2\right) + \mathcal{O}\left(\mathbf{m}_D \mathbf{M}_R^{-1}\right),$$

where

$$\mathbf{U} = \begin{pmatrix} 1 + \frac{1}{2}\mathbf{m}_D^\dagger \left(\mathbf{M}_R \mathbf{M}_R^\dagger\right)^{-1} \mathbf{m}_D & \mathbf{m}_D^\dagger \left(\mathbf{M}_R^\dagger\right)^{-1} \\ -\mathbf{M}_R^{-1}\mathbf{m}_D & 1 + \frac{1}{2}\mathbf{M}_R^{-1}\mathbf{m}_D \mathbf{m}_D^\dagger \left(\mathbf{M}_R^\dagger\right)^{-1} \end{pmatrix}.$$

$$\mathbf{M}_1 \simeq \mathbf{M}_R \quad \text{and} \quad \mathbf{M}_2 \simeq -\mathbf{m}_D^T \mathbf{M}_R^{-1} \mathbf{m}_D$$

The mass eigenfields are surely Majorana neutrinos.

- Quadratic see-saw: If eigenvalues of $\mathbf{M}_R$ are of the order of a large scale parameter $M \sim M_{\mathsf{GUT}}$[a] [e.g., $\mathbf{M}_R = \mathbf{M}_1$] than the standard neutrino masses are suppressed:

  $$m_i \sim \frac{m_{Di}^2}{M} \lll m_{Di},$$

  Here $m_{Di} \sim Y_i \langle H \rangle$ are the eigenvalues of $\mathbf{m}_D$. As long as these eigenvalues (or Yukawa couplings Y_i) are hierarchical, the Majorana neutrino masses display quadratic hierarchy:

  $$m_1 : m_2 : m_3 \propto m_{D1}^2 : m_{D2}^2 : m_{D3}^2.$$

[a] Large M is natural in, e.g., $SO(10)$ inspired GUT models which therefore provide a nice framework to understand small neutrino masses.

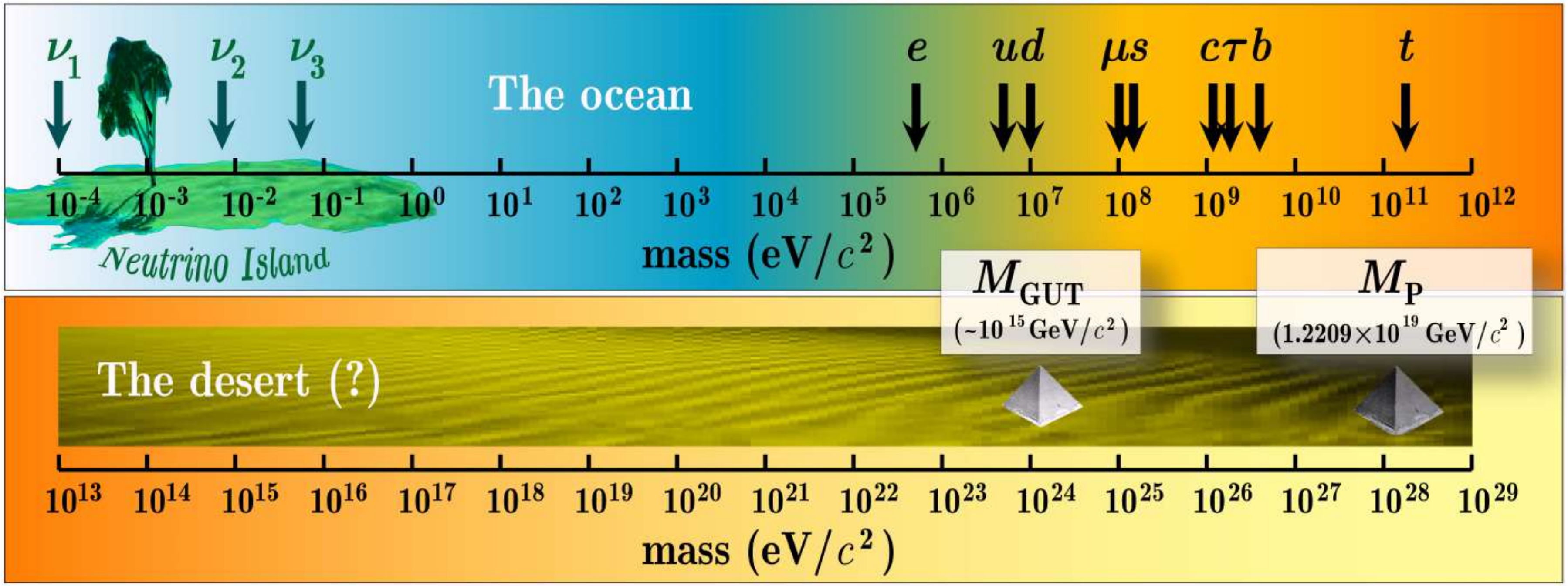
- Linear see-saw: In a more special case, $\mathbf{M}_R = (M/M_D)\mathbf{M}_D$, where M_D is the generic scale of the charged fermion masses than

$$m_i \sim \frac{M_D m_{Di}}{M} \lll m_{Di}$$

but the hierarchy is linear:

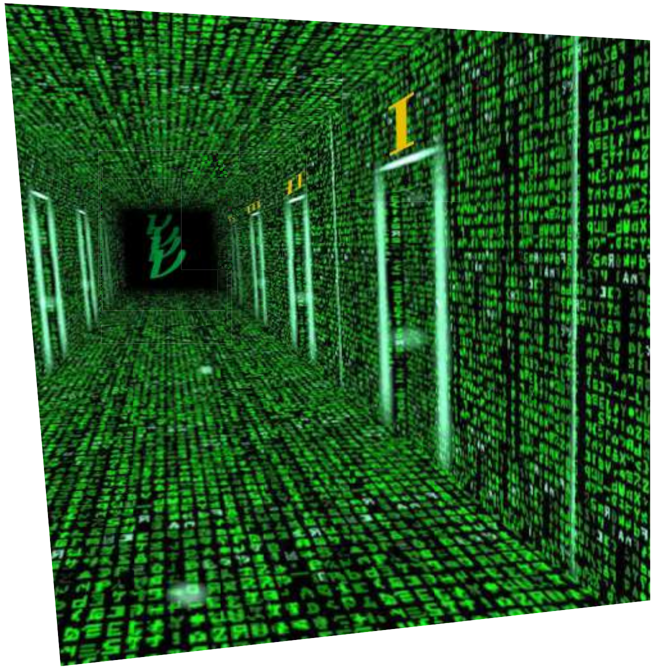
$$m_1 : m_2 : m_3 \propto m_{D1} : m_{D2} : m_{D3}.$$

The two mentioned possibilities are, in principle, experimentally distinguishable.



Beyond this section

- Double see-saw*
- Inverse see-saw*
- Radiative see-saw*
- SUSY & SUGRA see-saw
- TeV-scale gauged $B - L$ symmetry*
- TeV see-saw & large extra dimensions
- See-saw & Dark Matter
- See-saw & Leptogenesis
- See-saw & Baryogenesis
- Dirac see-saw
- Top (top-bottom) see-saw
- Cascade see-saw
- ...



* See Backup.

Conclusions (not really confirmed)

- The "mainstream" ν mass models, defined as see-saw models, are capable of describing the atmospheric–reactor–accelerator ν oscillation data, the LMA MSW solar neutrino solution, and cosmological limits. The SM and MSSM may naturally be extended to incorporate the see-saw mechanism.
- [A fly in the ointment] Wealth of the models ($\gg$ number of the authors of the models) greatly complicates the choice of the best one.

What do we know and don't know about neutrinos?

4.3 Here's what we know today (we're getting ahead of ourselves).

	νfit	Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 7.0$)	
		bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
With SK atmospheric data	$\sin^2\theta_{12}$	$0.304^{+0.012}_{-0.012}$	$0.269 \to 0.343$	$0.304^{+0.013}_{-0.012}$	$0.269 \to 0.343$
	$\theta_{12}/^\circ$	$33.45^{+0.77}_{-0.75}$	$31.27 \to 35.87$	$33.45^{+0.78}_{-0.75}$	$31.27 \to 35.87$
	$\sin^2\theta_{23}$	$0.450^{+0.019}_{-0.016}$	$0.408 \to 0.603$	$0.570^{+0.016}_{-0.022}$	$0.410 \to 0.613$
	$\theta_{23}/^\circ$	$42.1^{+1.1}_{-0.9}$	$39.7 \to 50.9$	$49.0^{+0.9}_{-1.3}$	$39.8 \to 51.6$
	$\sin^2\theta_{13}$	$0.02246^{+0.00062}_{-0.00062}$	$0.02060 \to 0.02435$	$0.02241^{+0.00074}_{-0.00062}$	$0.02055 \to 0.02457$
	$\theta_{13}/^\circ$	$8.62^{+0.12}_{-0.12}$	$8.25 \to 8.98$	$8.61^{+0.14}_{-0.12}$	$8.24 \to 9.02$
	$\delta_{\mathrm{CP}}/^\circ$	230^{+36}_{-25}	$144 \to 350$	278^{+22}_{-30}	$194 \to 345$
	$\dfrac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$
	$\dfrac{\Delta m^2_{3\ell}}{10^{-3}\ \mathrm{eV}^2}$	$+2.510^{+0.027}_{-0.027}$	$+2.430 \to +2.593$	$-2.490^{+0.026}_{-0.028}$	$-2.574 \to -2.410$

Three-flavor oscillation parameters from a recent fit to global data ("NuFIT 5.1") performed by the NuFIT team. Note that $\Delta m^2_{3\ell} \equiv \Delta m^2_{31} > 0$ for NO and $\Delta m^2_{3\ell} \equiv \Delta m^2_{32} < 0$ for IO.

[See I. Esteban *et al.* (The NuFIT team), "The fate of hints: updated global analysis of three-flavor neutrino oscillations," JHEP09(2020)178, arXiv:2007.14792 [hep-ph]. Present update (October 2021) is from $\langle$ http://www.nu-fit.org/ $\rangle$.]

List of data used in the NuFIT 5.1 analysis (October 2021)

Solar experiments:

Homestake chlorine total rate (1 dp), Gallex & GNO total rates (2 dp), SAGE total rate (1 dp), SK-I full energy and zenith spectrum (44 dp), SK-II full energy and day/night spectrum (33 dp), SK-III full energy and day/night spectrum (42 dp), SK-IV 2970-day day-night asymmetry and energy spectrum (24 dp), SNO combined analysis (7 dp), Borexino Phase-I 741-day low-energy data (33 dp), Borexino Phase-I 246-day high-energy data (6 dp), Borexino Phase-II 408-day low-energy data (42 dp).

Atmospheric experiments:

IceCube/DeepCore 3-year data (64 dp), SK-I–IV 364.8 kiloton years + χ^2 map.

Reactor experiments:

KamLAND separate DS1, DS2, DS3 spectra with Daya-Bay reactor $\overline{\nu}_e$ fluxes (69 dp), Double-Chooz FD/ND spectral ratio, with 1276-day (FD), 587-day (ND) exposures (26 dp), Daya-Bay 1958-day EH2/EH1 and EH3/EH1 spectral ratios (52 dp), RENO 2908-day FD/ND spectral ratio (45 dp).

Accelerator experiments:

MINOS 10.71 PoT_{20} ν_μ-disappearance data (39 dp), MINOS 3.36 PoT_{20} $\overline{\nu}_\mu$-disappearance data (14 dp), MINOS 10.60 PoT_{20} ν_e-appearance data (5 dp), MINOS 3.30 PoT_{20} $\overline{\nu}_e$-appearance (5 dp), T2K 19.7 PoT_{20} ν_μ-disappearance data (35 dp), T2K 19.7 PoT_{20} ν_e-appearance data (23 dp for the CCQE and 16 dp for CC1π samples), T2K 16.3 PoT_{20} $\overline{\nu}_\mu$-disappearance data (35 dp), T2K 16.3 PoT_{20} $\overline{\nu}_e$-appearance data (23 dp), NOvA 13.6 PoT_{20} ν_μ-disappearance data (76 dp), NOvA 13.6 PoT_{20} ν_e-appearance data (13 dp), NOvA 12.5 PoT_{20} $\overline{\nu}_\mu$-disappearance data (76 dp), NOvA 12.5 PoT_{20} $\overline{\nu}_e$-appearance data (13 dp).

Here dp = data point(s), $\text{PoT}_{20} = 10^{20}$ PoT (Protons on Target), and EH = Experiment Hall.

4.3.1 Neutrino oscillation parameter plot.

The regions of neutrino squared-mass splitting

$$\Delta m^2 = |\Delta m^2_{ij}| = |m_j^2 - m_i^2|$$

and $\tan^2\theta$ (where θ is one of the mixing angles θ_{ij} corresponding to a particular experiment) favored or excluded by various experiments. Contributed to RPP-2018[a] by Hitoshi Murayama (University of California, Berkeley).

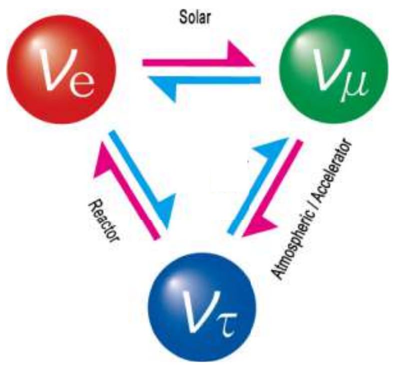
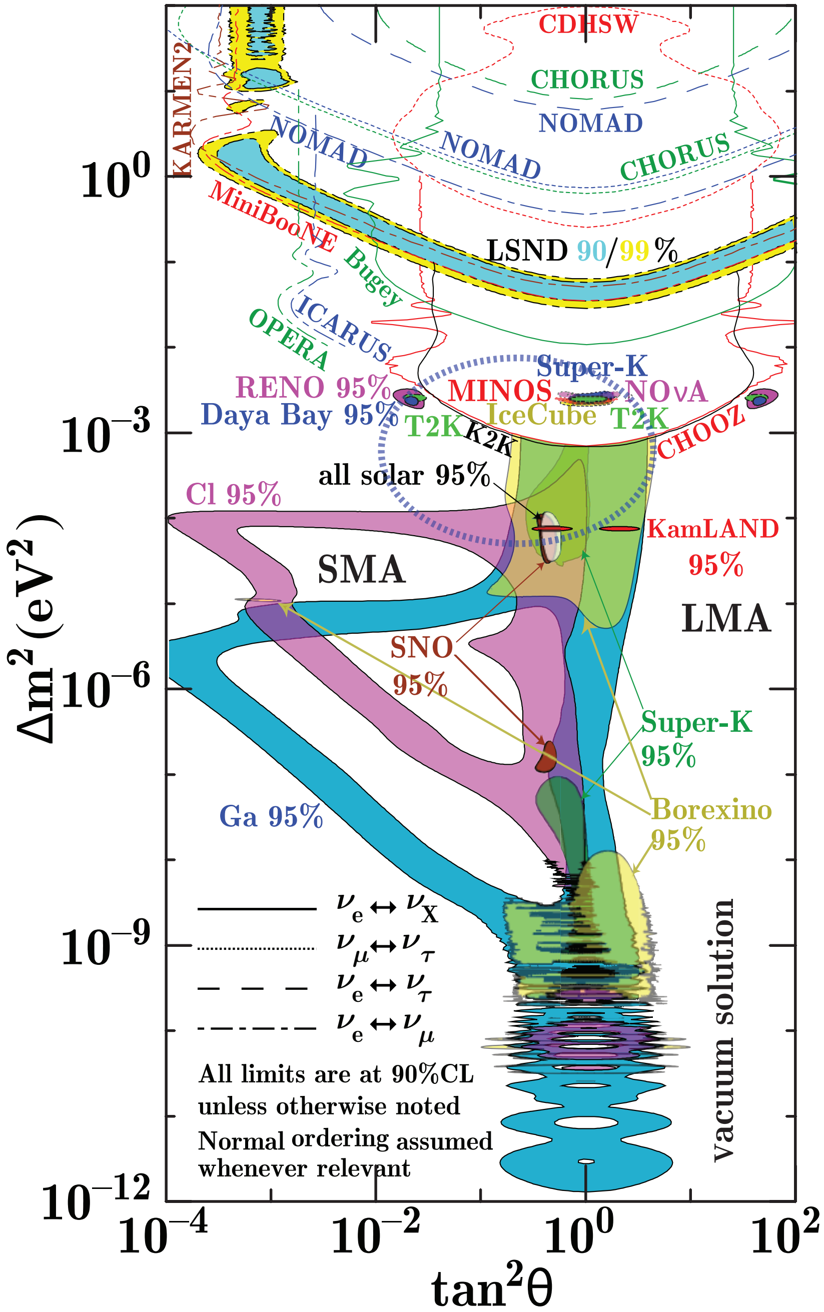


Figure includes the most rigorous results from before 2018, but data from many earlier experiments (e.g., BUST, NUSEX, Fréjus, IMB, Kamiokande, MACRO, SOUDAN 2) are ignored.

[a] M. Tanabashi *et al.* (Particle Data Group), "Review of Particle Physics", Phys. Rev. D **98** (2018) 030001.



In the absence of CP violation, the mixing angles may be represented as Euler angles relating the flavor eigenstates to the mass eigenstates. ▷

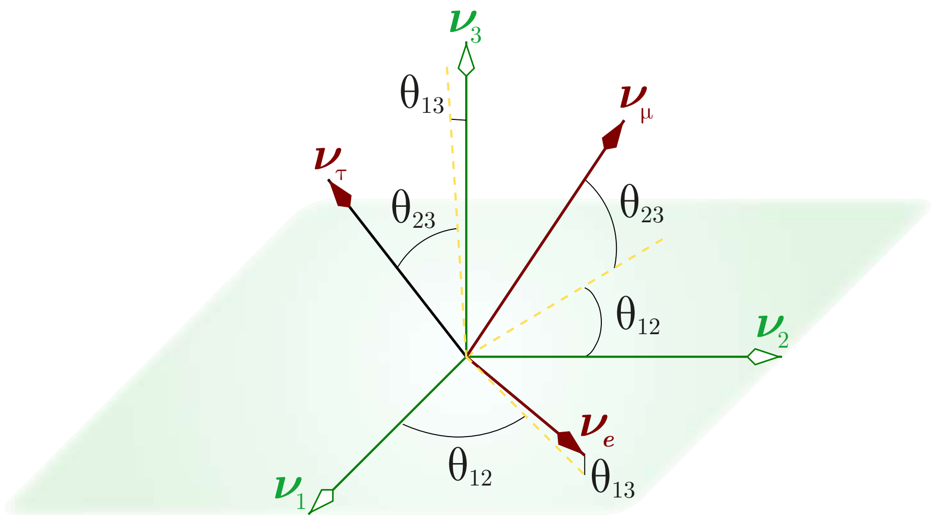
According to the NuFIT analysis (p. 45), the best-fit mixing angles and δ for the normal mass ordering (a bit preferred) are:

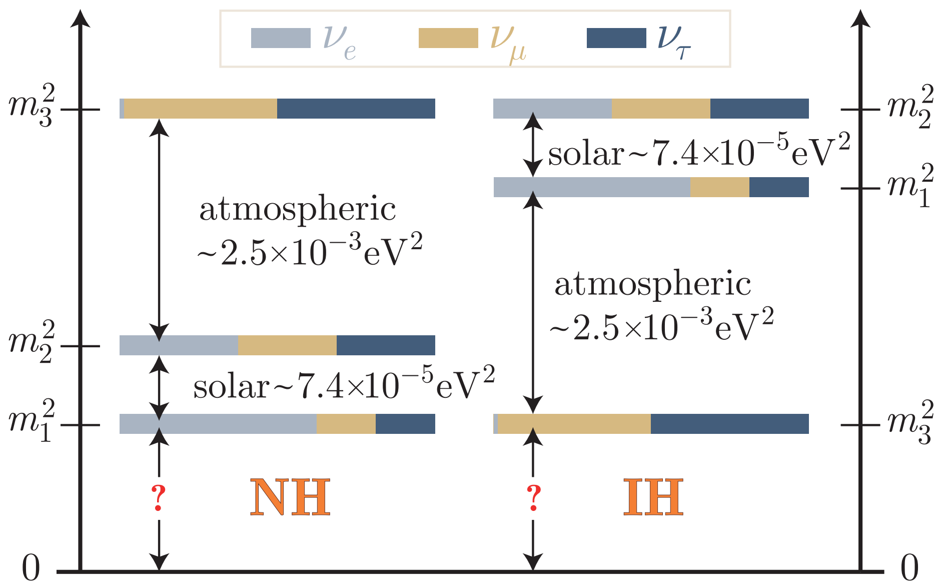
	PNMS	CKM
$\theta_{12}/^\circ$	$33.45^{+0.77}_{-0.75}$	13.04 ± 0.05
$\theta_{23}/^\circ$	$42.1^{+1.1}_{-0.9}$	2.38 ± 0.06
$\theta_{13}/^\circ$	$8.62^{+0.12}_{-0.12}$	0.201 ± 0.011
δ°	230^{+36}_{-25}	68.8 ± 4.5

The CKM angles and CP phase are also shown for comparison.

It should be stressed that the neutrino mass spectrum is still undetermined. ▷

[Figures (slightly modified and updated) are taken from S. F. King, "Neutrino mass and mixing in the seesaw playground," arXiv:1511.03831 [hep-ph].]

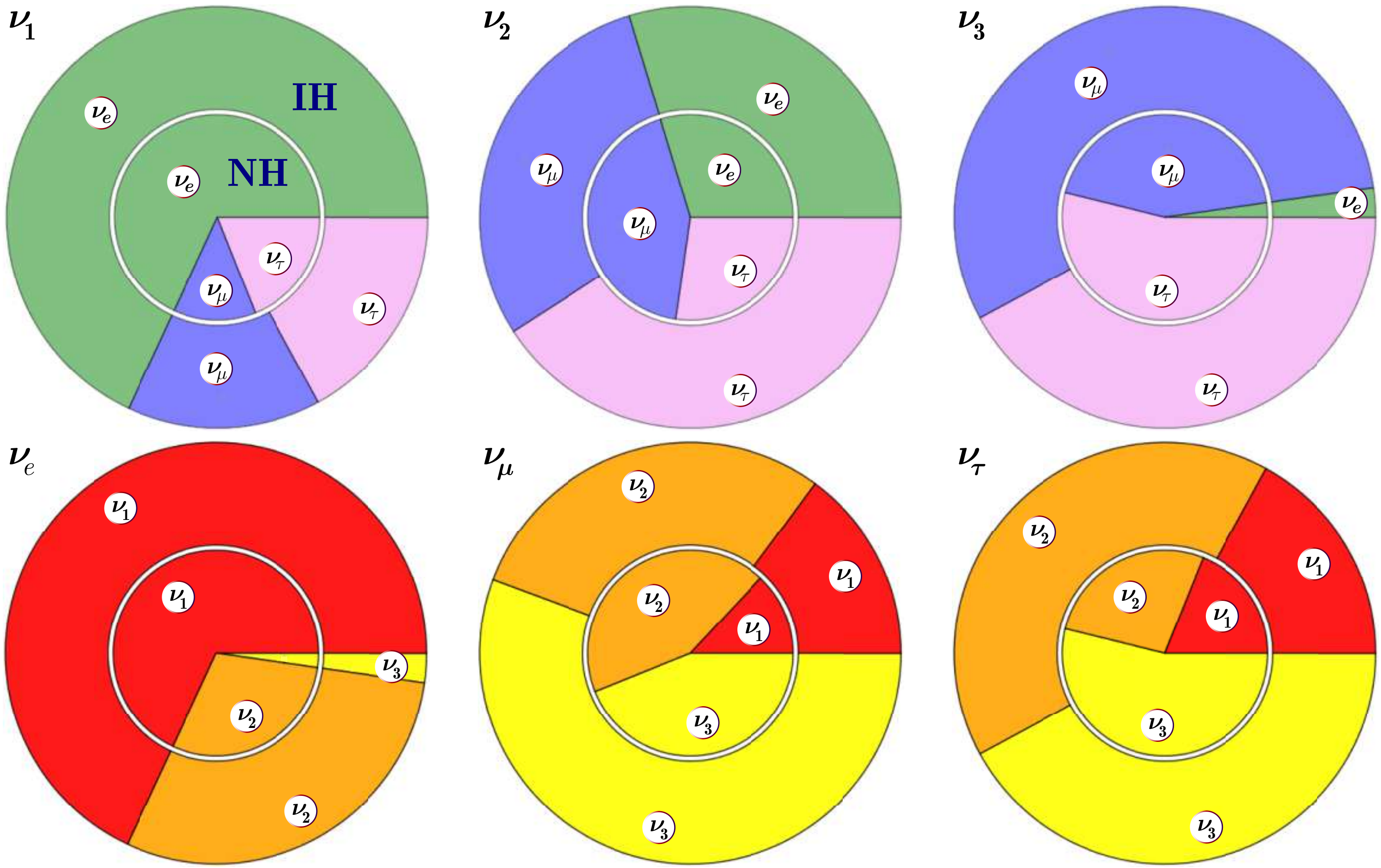




Flavor content of mass states and mass content of flavor states is the same for Dirac ν and $\overline{\nu}$ (CP phase δ only changes the sign for $\overline{\nu}$) and for Majorana left/right νs ($\left|V^{\mathsf{D}}_{\alpha i}\right| = \left|V^{\mathsf{M}}_{\alpha i}\right|$).

4.3.2 Flavor content of mass states and mass content of flavor states.

$$\left(|V_{\alpha i}|^2\right)_{\mathsf{NH}} = \begin{pmatrix} 0.681 & 0.297 & 0.0225 \\ 0.130 & 0.430 & 0.439 \\ 0.189 & 0.273 & 0.538 \end{pmatrix}, \quad \left(|V_{\alpha i}|^2\right)_{\mathsf{IH}} = \begin{pmatrix} 0.681 & 0.297 & 0.0224 \\ 0.149 & 0.294 & 0.557 \\ 0.170 & 0.409 & 0.421 \end{pmatrix}.$$



4.3.3 Current status of the neutrino masses from oscillation experiments.

So, NuFIT 5.1 provides the following constraints for the mass squared splittings:

$$
\begin{aligned}
m_2^2 - m_1^2 &= 7.42^{+0.21}_{-0.20} \times 10^{-5}\ \text{eV}^2 \quad (\text{“solar” for NH and IH})\\
m_3^2 - m_1^2 &= 2.51^{+0.027}_{-0.027} \times 10^{-3}\ \text{eV}^2 \quad (\text{“atmospheric” for NH})\\
m_2^2 - m_3^2 &= 2.49^{+0.026}_{-0.028} \times 10^{-3}\ \text{eV}^2 \quad (\text{“atmospheric” for IH})
\end{aligned}
$$

These result imply that at least two of the neutrino eigenfields have nonzero masses and thus there are (at least) two very different possible scenarios related to the mass ordering:

$$m_1 \ll m_2 < m_3 \quad (\text{for NH}) \quad \text{or} \quad m_3 \ll m_1 < m_2 \quad (\text{for IH}).$$

The data on Δm_{ij}^2 give the following estimates (henceforth $\sum m_\nu \equiv \sum_{i=1}^3 m_i$):

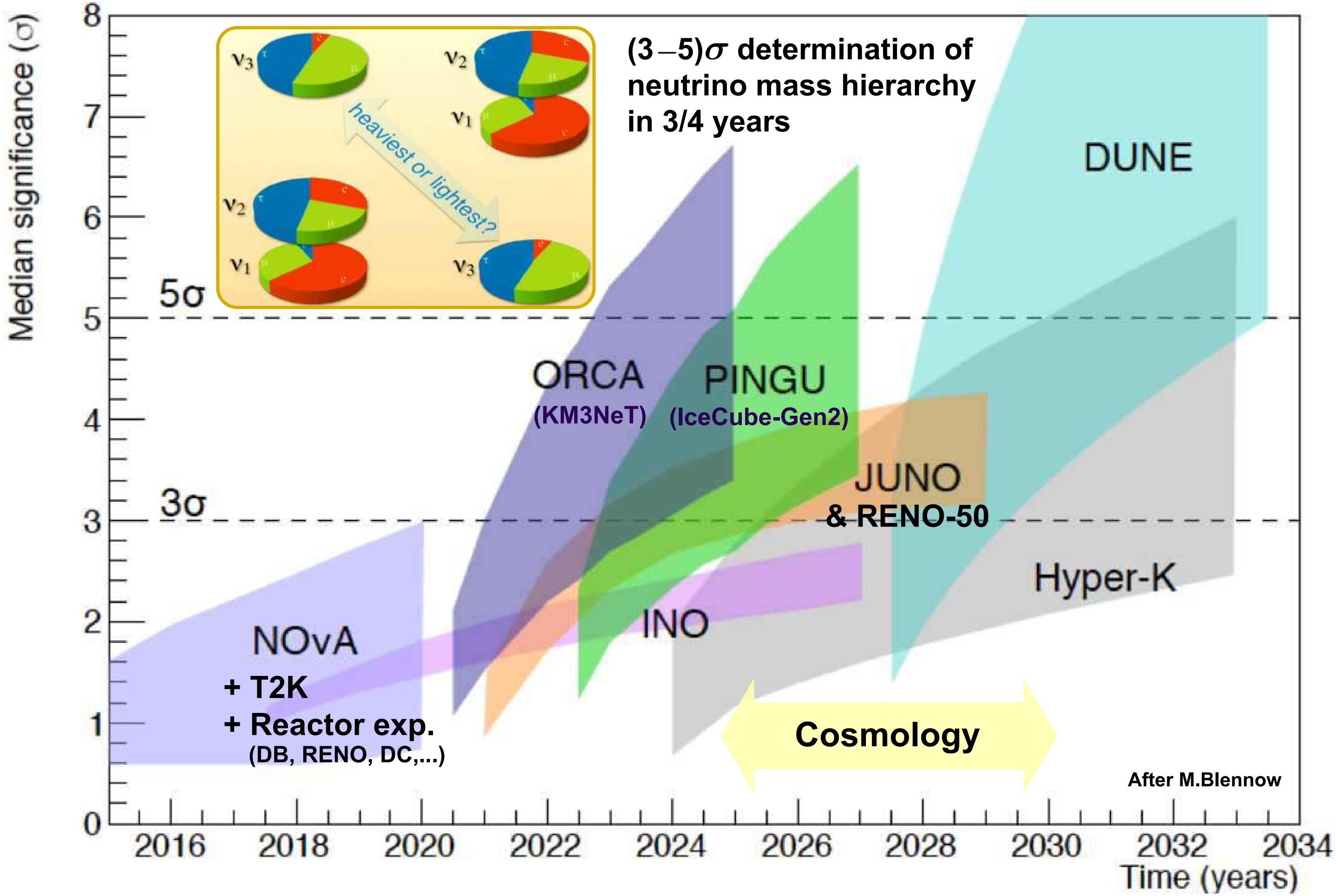
$$\begin{cases} m_2 = (8.61 \pm 0.122) \times 10^{-3}\ \text{eV}, \\ m_3 = (5.01 \pm 0.027) \times 10^{-2}\ \text{eV}, \end{cases} \implies \sum m_\nu \geq m_2 + m_3 = 0.0587 \pm 0.0003\ \text{eV} \ (\text{for NH}) \tag{9}$$

$$\begin{cases} m_2 = (4.99 \pm 0.028) \times 10^{-2}\ \text{eV}, \\ m_1 = (4.92 \pm 0.029) \times 10^{-2}\ \text{eV}, \end{cases} \implies \sum m_\nu \geq m_1 + m_1 = 0.0983 \pm 0.0006\ \text{eV} \ (\text{for IH}) \tag{10}$$

Therefore, the lower bounds on $\sum m_\nu$ at 1σ C.L. are:

$$\sum m_\nu^{\text{NH}} > 0.0584\ \text{eV} \quad \text{and} \quad \sum m_\nu^{\text{IH}} > 0.0977\ \text{eV}.$$

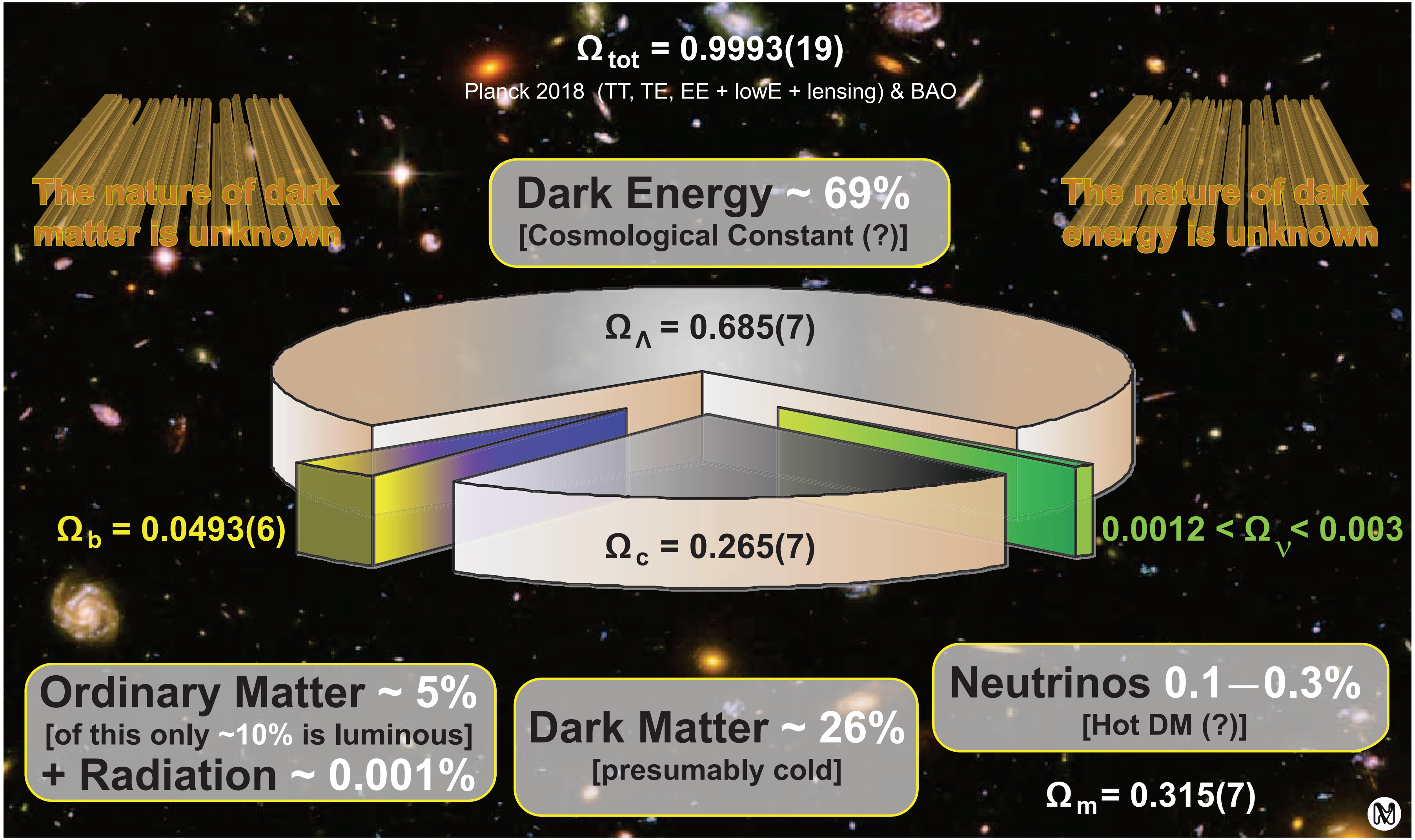
Note: Current accelerator and reactor data favor the NH scenario, but the question is not yet closed.



A summary of sensitivities to the neutrino mass hierarchy for various experimental approaches, with timescales, as claimed by the proponents in each case. Widths indicate main expected uncertainty.

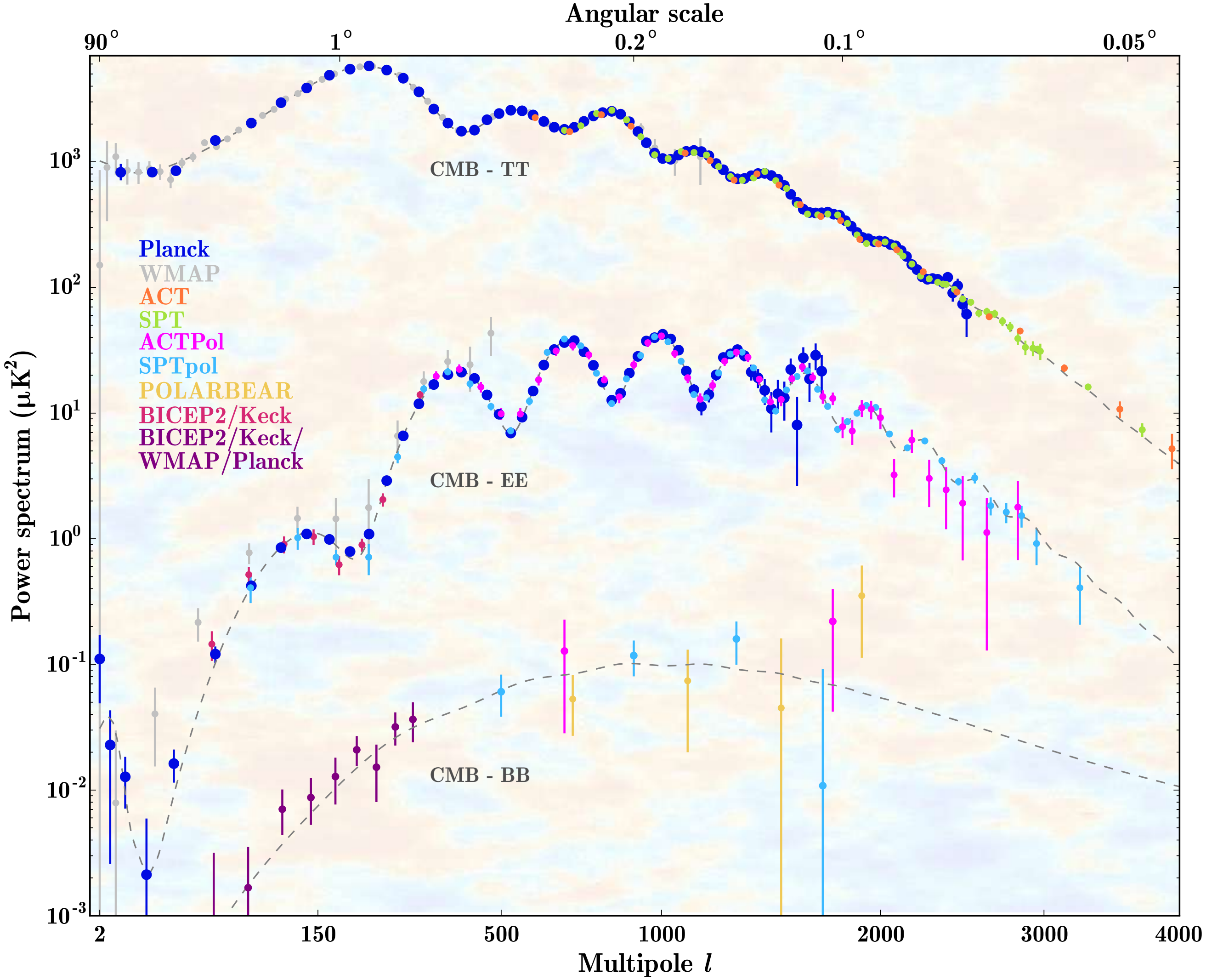
CνB.

Relict neutrinos (or Cosmic Neutrino Background, or CNB, or CνB) produce the largest neutrino flux on Earth, but compose only a very small fraction of invisible (non-luminous) matter in the Universe.



CMB as a probe of CνB.

It is not yet realistic to directly detect the νs created within the first second after the Big Bang, and which have too little energy now. However, for the first time, *Planck*, ESA's mission has unambiguously detected the effect CνB has on relic radiation maps. The quality of these maps is now such that the imprints left by dark matter and relic νs are clearly visible.[a]



[a] See N. Aghanim *et al.* (*Planck* Collaboration), "Planck 2018 results. I. Overview and the cosmological legacy of Planck", Astron. Astrophys. **641** (2020) A1, arXiv:1807.06205 [astro-ph.CO]; "Planck 2018 results. VI. Cosmological parameters", Astron. Astrophys. **641** (2020) A6, arXiv:1807.06209 [astro-ph.CO].

The relic photon spectrum almost exactly follows the blackbody spectrum with temperature

$$T_0 = 2.7255 \pm 0.0006 \text{ K}.$$

After many decades of experimental and theoretical efforts, the CMB is known to be almost isotropic but having small temperature fluctuations (called CMB anisotropy) with amplitude

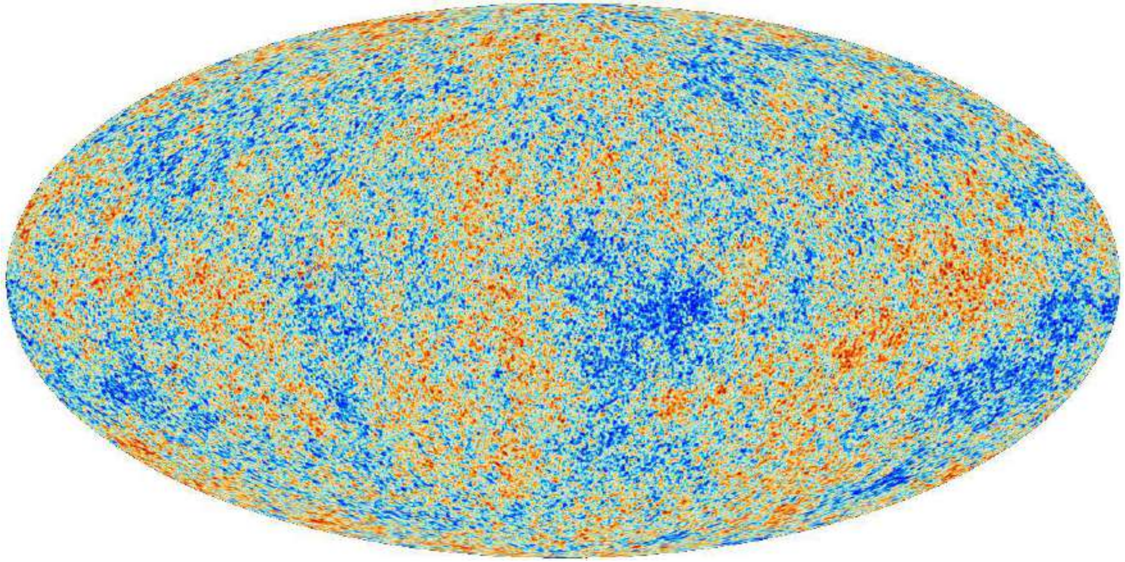
$$\delta T \sim \left(10^{-5} - 10^{-3}\right).$$

These fluctuations can be decomposed in a sum of spherical harmonics $Y_{lm}(\theta, \phi)$

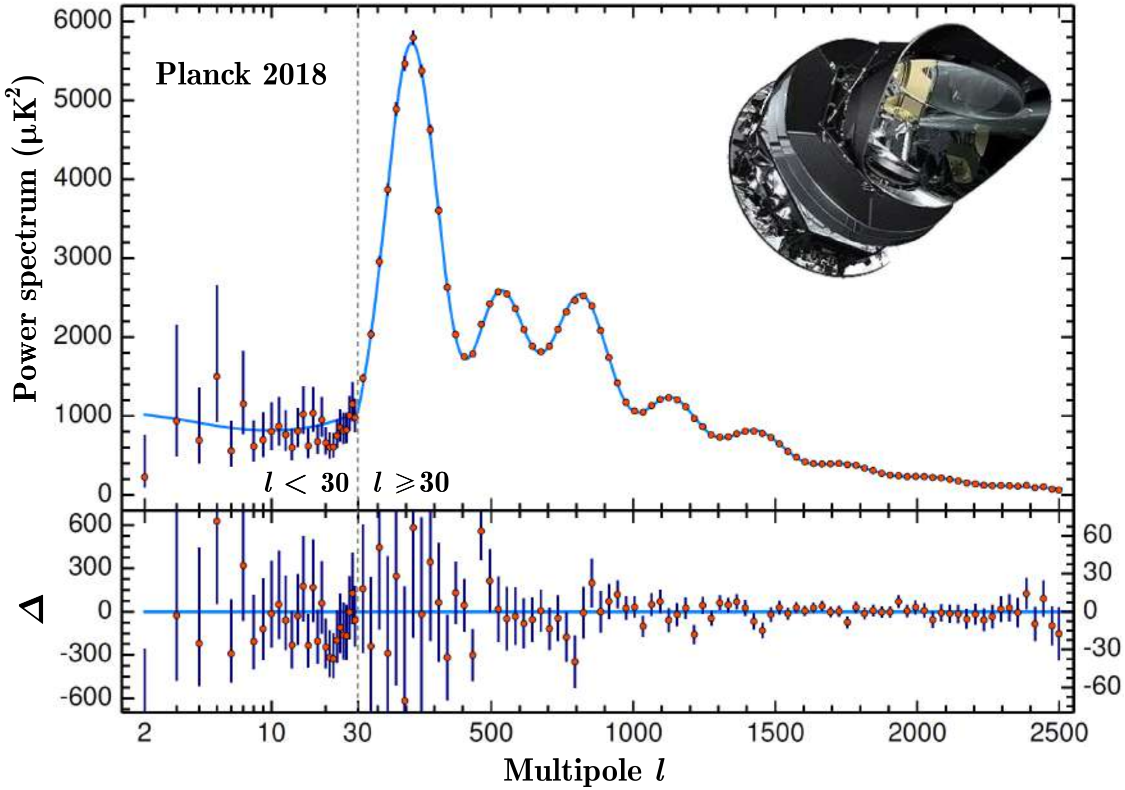
$$\delta T(\theta, \phi) = \sum_{l=1}^{\infty} \sum_{m=-l}^{l} a_{lm} Y_{lm}(\theta, \phi).$$

The averaged squared coefficients a_{lm} give the variance

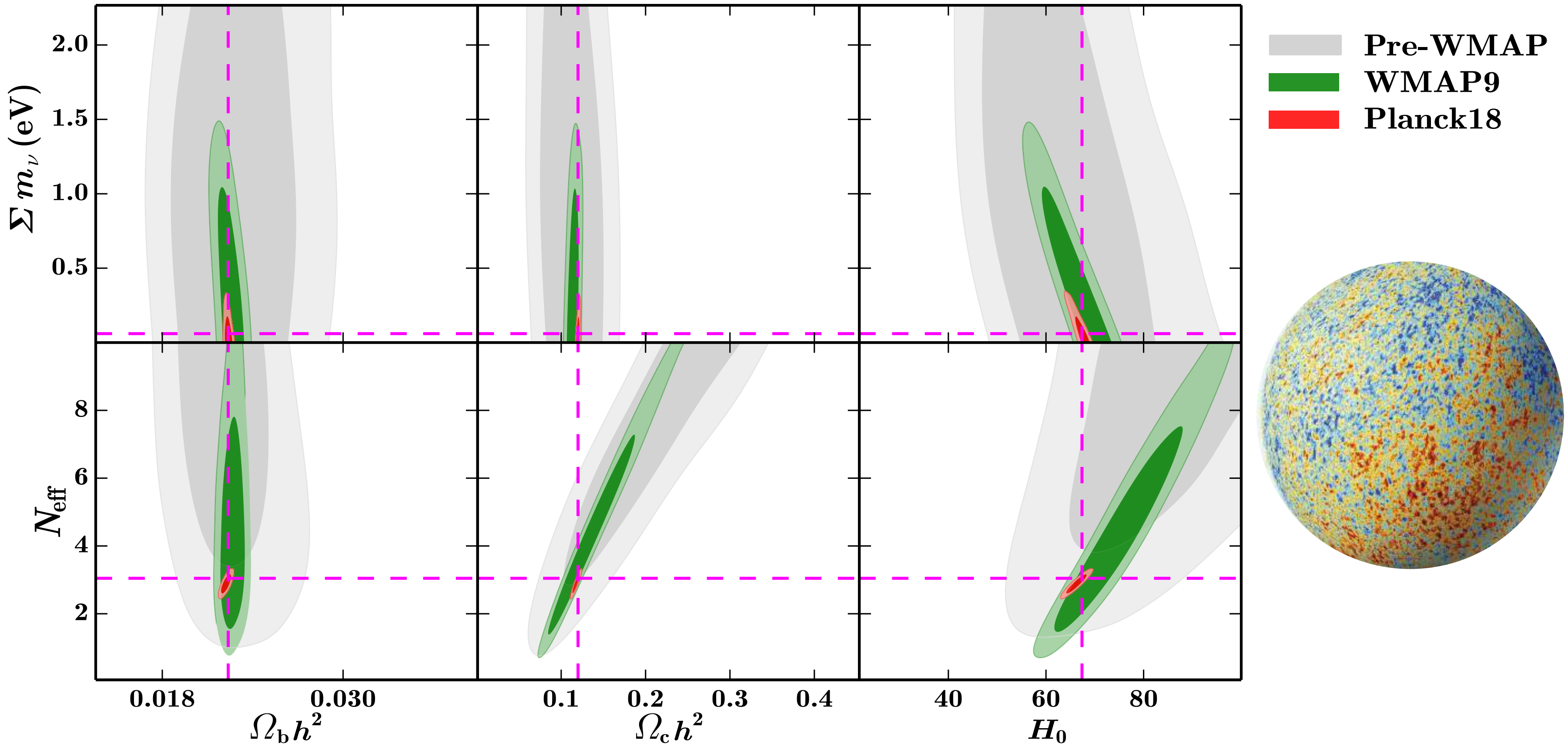
$$C_l = \langle |a_{lm}|^2 \rangle = \frac{1}{2l+1} \sum_{m=-l}^{l} |a_{lm}|^2.$$



CMB maps can be compressed into the power spectrum



Planck 2018: neutrino summary.



Successive reductions in the allowed parameter space for various one-parameter extensions to ΛCDM, from pre-WMAP (MAXIMA, DASI, BOOMERANG, VSA, CBI) to *Planck*. The contours display the 68 % and 95 % C.L. for the extra parameter vs. five other base-ΛCDM parameters. The dashed lines indicate the ΛCDM best-fit parameters or fixed default values of the extended parameters.

[Adopted from Aghanim *et al.* (*Planck* Collaboration), "Planck 2018 results. I. Overview and the cosmological legacy of Planck", Astron. Astrophys. **641** (2020) A1, arXiv:1807.06205 [astro-ph.CO];]

Finally *Planck* 2018 (+BAO) sets: $\boxed{\sum m_\nu < \mathbf{0.12}\ \text{eV}, \quad N_{\text{eff}} = \mathbf{2.99 \pm 0.17}, \quad \Delta N_{\text{eff}} < \mathbf{0.3}.}$

Here N_{eff} is the effective number or neutrino species; roughly speaking, $N_{\text{eff}} \simeq 3$ means that additional light neutrinos are not supported (although not excluded) by *Planck*.

But(!) this constraint implies degenerate mass hierarchy (DH), $m_i = \sum m_\nu/3$, and many other model assumptions. Results for other ν mass spectra have been obtained recently ($m_0 \equiv m_{\min}$):[a]

	Base			Base+SNe		
	DH	NH	IH	DH	NH	IH
$\Lambda\text{CDM} + \sum m_\nu$						
ω_c	0.1194 ± 0.0009	0.1192 ± 0.0009	0.1191 ± 0.0009	0.1193 ± 0.0009	0.1191 ± 0.0009	0.1189 ± 0.0009
ω_b	0.02242 ± 0.00013	$0.02242^{+0.00013}_{-0.00014}$	0.02243 ± 0.00013	0.02243 ± 0.00013	0.02244 ± 0.00013	0.02244 ± 0.00013
Θ_s	1.04100 ± 0.00029	1.04100 ± 0.00029	1.04100 ± 0.00029	1.04102 ± 0.00029	1.04103 ± 0.00029	1.04103 ± 0.00029
τ	$0.0554^{+0.0068}_{-0.0076}$	$0.0569^{+0.0066}_{-0.0076}$	$0.0585^{+0.0069}_{-0.0076}$	0.0556 ± 0.0071	$0.0573^{+0.0069}_{-0.0076}$	$0.0588^{+0.0068}_{-0.0077}$
n_s	0.9666 ± 0.0036	0.9668 ± 0.0037	0.9671 ± 0.0037	0.9669 ± 0.0036	0.9673 ± 0.0036	0.9675 ± 0.0037
$\ln[10^{10} A_s]$	$3.048^{+0.014}_{-0.015}$	$3.051^{+0.014}_{-0.015}$	3.053 ± 0.015	3.046 ± 0.014	3.049 ± 0.014	$3.052^{+0.014}_{-0.015}$
m_0 (eV)	< 0.040	< 0.040	< 0.042	< 0.038	< 0.038	< 0.039
$\sum m_\nu$ (eV)	< 0.12	< 0.15	< 0.17	< 0.11	< 0.14	< 0.16
H_0 (km/s/Mpc)	$67.81^{+0.54}_{-0.46}$	$67.50^{+0.49}_{-0.44}$	67.22 ± 0.45	$67.89^{+0.52}_{-0.45}$	67.59 ± 0.44	67.33 ± 0.43
σ_8	$0.814^{+0.010}_{-0.007}$	$0.806^{+0.009}_{-0.006}$	$0.799^{+0.008}_{-0.006}$	$0.815^{+0.010}_{-0.007}$	$0.806^{+0.008}_{-0.006}$	$0.799^{+0.008}_{-0.006}$
S_8	0.827 ± 0.011	0.823 ± 0.011	0.820 ± 0.011	0.826 ± 0.011	0.822 ± 0.011	0.818 ± 0.011
$\Delta\chi^2 = \chi^2 - \chi^2_{IH}$	-2.89	-0.95	0	-2.73	-1.27	0

Let's recall the latest oscillation lower limits: $\sum m_\nu^{\text{NH}} \gtrsim 0.058$ eV and $\sum m_\nu^{\text{IH}} \gtrsim 0.098$ eV.

[a] Sh. R. Choudhury & S. Hannestad, "Updated results on neutrino mass and mass hierarchy from cosmology with Planck 2018 likelihoods," JCAP07(2020)037, arXiv:1907.12598 [astro-ph.CO].

Afterward: Open problems in neutrino physics.

- Are neutrinos Dirac or Majorana fermions?
- What is the absolute mass scale of (known) neutrinos?
  Why neutrino masses are so small? [Does any version of see-saw work?]
  What is the neutrino mass spectrum? [sign$(\Delta m^2_{32}) \Longleftrightarrow$ NH or IH.]
  Can the lightest neutrinos be massless fermions? [Not quasiparticles in Weyl semimetals!]
- Why neutrino mixing is so different from quark mixing?
  ~~What physics is responsible for the octant degeneracy? [sign$(\theta_{23} - 45°)$.]~~
- What are the source and scale of CP/T violation in the neutrino sector?
  How many CP violating phases are there?
- Is CPT conserved in the neutrino sector?
- How many neutrino flavors are there?
- Whether the number of neutrinos with definite masses is equal to or greater than the number of flavor neutrinos? In other words, do sterile neutrinos exist? [a] If so,
  - What is their mass spectrum?
  - Do they mix with active neutrinos?
  - Do light (heavy) sterile neutrinos constitute hot (cold) dark matter?
- Are (all) neutrinos stable particles?

[a] Hints from LSND+MiniBooNE, Neutrino-4, SAGE+GALLEX+BEST are in tension with many other data.