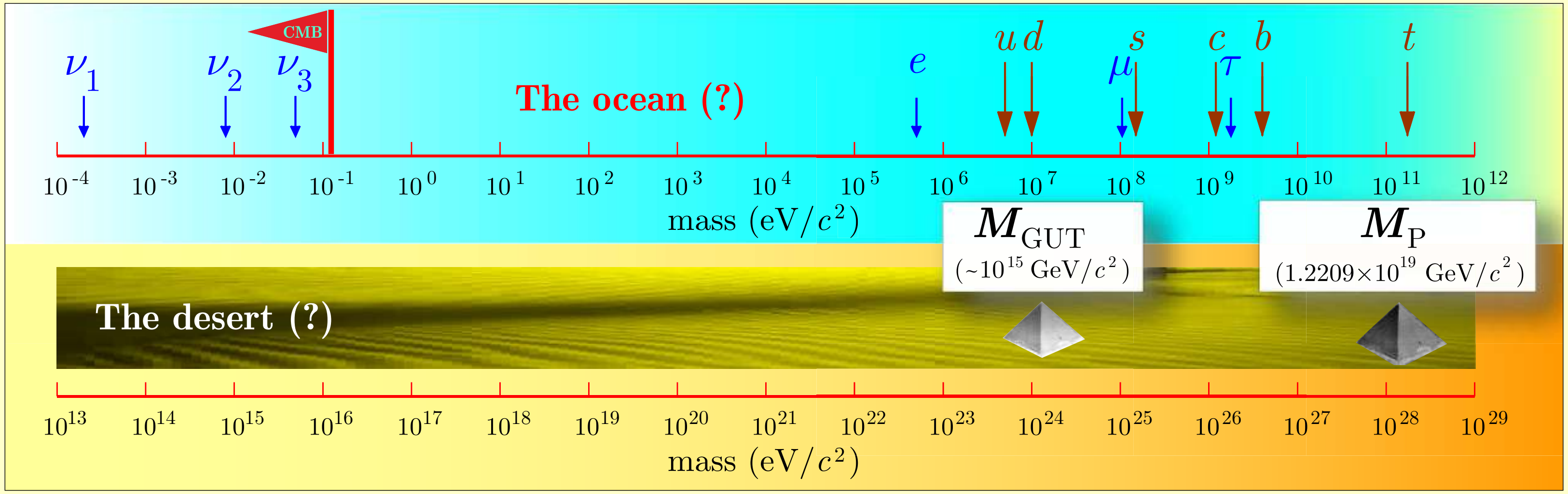
# Standard Model with Neutrino Masses



# 1 Interaction Lagrangian and weak currents.

In the Standard Model (SM), the charged and neutral current neutrino interactions with leptons are described by the following parts of the full Lagrangian:

$$\mathcal{L}_I^{\mathsf{CC}}(x) = -\frac{g}{2\sqrt{2}} j_\alpha^{\mathsf{CC}}(x) W^\alpha(x) + \text{H.c.} \quad \text{and} \quad \mathcal{L}_I^{\mathsf{NC}}(x) = -\frac{g}{2\cos\theta_{\mathsf{W}}} j_\alpha^{\mathsf{NC}}(x) Z^\alpha(x).$$

Here $g$ is the $SU(2)$ (electro-weak) gauge coupling constant

$$g^2 = 4\sqrt{2} m_W^2 G_F, \quad g\sin\theta_{\mathsf{W}} = |e|,$$

and $\theta_{\mathsf{W}}$ is the weak mixing (Weinberg) angle, ($\sin^2\theta_{\mathsf{W}}(M_Z) = 0.23120$).

The leptonic charged current and neutrino neutral current are given by the expressions:

$$j_\alpha^{\mathsf{CC}}(x) = 2 \sum_{\ell=e,\mu,\tau,\ldots} \overline{\nu}_{\ell,L}(x)\gamma_\alpha \ell_L(x) \quad \text{and} \quad j_\alpha^{\mathsf{NC}}(x) = \sum_{\ell=e,\mu,\tau,\ldots} \overline{\nu}_{\ell,L}(x)\gamma_\alpha \nu_{\ell,L}(x).$$

Phenomenologically, the charged and neutral currents may include (yet unknown) heavy neutrinos and corresponding heavy charged leptons. The left- and right-handed fermion fields are defined as usually:

$$\begin{cases} \nu_{\ell,L}(x) = P_L \nu_\ell(x), \;\; \ell_L(x) = P_L \ell(x), \;\; P_L \equiv \dfrac{1}{2}(1-\gamma_5), \\ \nu_{\ell,R}(x) = P_R \nu_\ell(x), \;\; \ell_L(x) = P_R \ell(x), \;\; P_R \equiv \dfrac{1}{2}(1+\gamma_5). \end{cases}$$

## Physical meaning of chiral projections for a massive Dirac fermion.

$$(\hat{p}-m)\psi = 0 \quad \Longrightarrow \quad \begin{pmatrix} p_0 - m & -\mathbf{p}\boldsymbol{\sigma} \\ \mathbf{p}\boldsymbol{\sigma} & -p_0 - m \end{pmatrix} \begin{pmatrix} \phi \\ \chi \end{pmatrix} = 0 \quad \Longrightarrow \quad \begin{cases} (\mathbf{p}\boldsymbol{\sigma})\chi = (p_0 - m)\phi, \\ (\mathbf{p}\boldsymbol{\sigma})\phi = (p_0 + m)\chi. \end{cases}$$

$$\Downarrow$$

$$\psi_L = P_L\psi = \frac{1}{2}\begin{pmatrix} \phi - \chi \\ \chi - \phi \end{pmatrix} = \begin{pmatrix} \phi_- \\ -\phi_- \end{pmatrix}$$

$$\psi_R = P_R\psi = \frac{1}{2}\begin{pmatrix} \phi + \chi \\ \phi + \chi \end{pmatrix} = \begin{pmatrix} \phi_+ \\ \phi_+ \end{pmatrix}$$

where $\quad \phi_\pm = \dfrac{1}{2}\left(1 \pm \dfrac{\mathbf{p}\boldsymbol{\sigma}}{p_0 + m}\right)\phi.$

Let $p_0 \gg m$ and thus $1 - |\mathbf{v}| \ll 1$, where $\mathbf{v} = \mathbf{p}/p_0$. Then, directing $\mathbf{v}$ along the $z$ axis we obtain

$$\phi_- \simeq \frac{1-\sigma_3}{2}\phi = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} \phi_\rightarrow \\ \phi_\leftarrow \end{pmatrix} = \begin{pmatrix} 0 \\ \phi_\leftarrow \end{pmatrix}, \quad \phi_+ \simeq \frac{1+\sigma_3}{2}\phi = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} \phi_\rightarrow \\ \phi_\leftarrow \end{pmatrix} = \begin{pmatrix} \phi_\rightarrow \\ 0 \end{pmatrix}.$$

**Reminder: Pauli & Dirac matrices**

$$\sigma_0 \equiv \mathbf{1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

$$\gamma^0 = \gamma_0 = \begin{pmatrix} \sigma_0 & 0 \\ 0 & -\sigma_0 \end{pmatrix}, \quad \gamma^k = -\gamma_k = \begin{pmatrix} 0 & \sigma_k \\ -\sigma_k & 0 \end{pmatrix}, \quad k = 1,2,3, \quad \gamma^5 = \gamma_5 = \begin{pmatrix} 0 & \sigma_0 \\ \sigma_0 & 0 \end{pmatrix}.$$

Note that the kinetic term of the Lagrangian includes both L and R handed neutrinos and moreover, it can include other sterile neutrinos:

$$\mathcal{L}_0 = \frac{i}{2}\left[\overline{\boldsymbol{\nu}}(x)\gamma^\alpha\partial_\alpha\boldsymbol{\nu}(x) - \partial_\alpha\overline{\boldsymbol{\nu}}(x)\gamma^\alpha\boldsymbol{\nu}(x)\right] \equiv \frac{i}{2}\overline{\boldsymbol{\nu}}(x)\overleftrightarrow{\boldsymbol{\partial}}\boldsymbol{\nu}(x) = \frac{i}{2}\left[\overline{\boldsymbol{\nu}}_L(x)\overleftrightarrow{\boldsymbol{\partial}}\boldsymbol{\nu}_L(x) + \overline{\boldsymbol{\nu}}_R(x)\overleftrightarrow{\boldsymbol{\partial}}\boldsymbol{\nu}_R(x)\right],$$

$$\boldsymbol{\nu}(x) = \boldsymbol{\nu}_L(x) + \boldsymbol{\nu}_R(x) = \begin{pmatrix} \nu_e(x) \\ \nu_\mu(x) \\ \nu_\tau(x) \\ . \\ . \\ . \end{pmatrix}, \quad \boldsymbol{\nu}_{L/R}(x) = \begin{pmatrix} \nu_{e,L/R}(x) \\ \nu_{\mu,L/R}(x) \\ \nu_{\tau,L/R}(x) \\ . \\ . \\ . \end{pmatrix} = \frac{1 \mp \gamma_5}{2}\begin{pmatrix} \nu_e(x) \\ \nu_\mu(x) \\ \nu_\tau(x) \\ . \\ . \\ . \end{pmatrix}.$$

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Neutrino chirality: $\gamma_5\nu_L = -\nu_L$ and $\gamma_5\nu_R = +\nu_R$.

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The Lagrangian of the theory with massless neutrinos is invariant with respect to the global gauge transformations

$$\nu_\ell(x) \to e^{i\Lambda_\ell}\nu_\ell(x), \quad \ell(x) \to e^{i\Lambda_\ell}\ell(x) \quad \text{with} \quad \Lambda_\ell = \text{const}.$$

By Noether's theorem this leads to conservation of the individual lepton flavor numbers (more rarely called lepton flavor charges) $L_\ell$. It is agreed that

$$\boxed{L_\ell(\ell^-, \nu_\ell) = +1, \quad L_\ell(\ell^+, \overline{\nu}_\ell) = -1, \quad \ell^\pm = e^\pm,\ \mu^\pm,\ \tau^\pm, \text{ etc.}}$$

**Lepton flavor conservation is not the case for massive neutrinos.**

There are two fundamentally different kinds of neutrino mass terms: Dirac and Majorana.

# 2 Dirac neutrinos

The conventional Dirac mass term for a single spinor field $\psi(x)$ is well known:

$$-m\overline{\psi}(x)\psi(x) = -m\left[\overline{\psi}_R\psi_L + \overline{\psi}_L\psi_R\right] = -m\overline{\psi}_R(x)\psi_L(x) + \text{H.c.}$$

(the identities $\overline{\psi}_L\psi_L = \overline{\psi}_R\psi_R = 0$ and $(\overline{\psi}_R\psi_L)^\dagger = \overline{\psi}_L\psi_R$ are used here).

The most general extension to the $N$-generation Dirac neutrino case reads:

$$\mathcal{L}_\text{D}(x) = -\overline{\boldsymbol{\nu}}_R(x)\mathbf{M}_\text{D}\boldsymbol{\nu}_L(x) + \text{H.c.},$$

where $\mathbf{M}_\text{D}$ is a nonsingular [to exclude massless case] complex $N \times N$ matrix.

> In general, $N \geq 3$ since the column $\boldsymbol{\nu}_L$ may include both *active* and *sterile* neutrino fields which do not enter into the standard charged and neutral currents.

Any nonsingular complex matrix can be diagonalized by means of an appropriate bi-unitary transformation

$$\mathbf{M}_\text{D} = \widetilde{\mathbf{V}}\mathbf{m}\mathbf{V}^\dagger, \quad \mathbf{m} = ||m_k\delta_{kl}|| = \text{diag}\,(m_1, m_2, \ldots, m_N),$$

where $\mathbf{V}$ and $\widetilde{\mathbf{V}}$ are unitary matrices and $m_k \geq 0$.

$$\Longrightarrow \qquad \mathcal{L}_\text{D}(x) = -\overline{\boldsymbol{\nu}'}_R(x)\mathbf{m}\boldsymbol{\nu}'_L(x) + \text{H.c.} = -\overline{\boldsymbol{\nu}'}(x)\mathbf{m}\boldsymbol{\nu}'(x) = -\sum_{k=1}^{N} m_k\overline{\nu}_k(x)\nu_k(x),$$

where the new fields $\nu_k$ are defined by

$$\boldsymbol{\nu}'_L(x) = \mathbf{V}^\dagger\boldsymbol{\nu}_L(x), \quad \boldsymbol{\nu}'_R(x) = \widetilde{\mathbf{V}}^\dagger\boldsymbol{\nu}_R(x), \quad \boldsymbol{\nu}'(x) = (\nu_1, \nu_2, \ldots, \nu_N)^T.$$

The fields $\boldsymbol{\nu}'_R(x)$ do not enter into $\mathcal{L}_I \Longrightarrow$ the matrix $\widetilde{\mathbf{V}}$ remains out of play...

Since $\mathbf{V}\mathbf{V}^\dagger = \mathbf{V}^\dagger\mathbf{V} = \mathbf{1}$ and $\widetilde{\mathbf{V}}^\dagger\widetilde{\mathbf{V}} = \widetilde{\mathbf{V}}\widetilde{\mathbf{V}}^\dagger = \mathbf{1}$, the neutrino kinetic term in the Lagrangian is transformed to

$$\mathcal{L}_0 = \frac{i}{2}\left[\overline{\boldsymbol{\nu}}'_L(x)\overleftrightarrow{\boldsymbol{\partial}}\boldsymbol{\nu}'_L(x) + \overline{\boldsymbol{\nu}}'_R(x)\overleftrightarrow{\boldsymbol{\partial}}\boldsymbol{\nu}'_R(x)\right] = \frac{i}{2}\overline{\boldsymbol{\nu}'}(x)\overleftrightarrow{\boldsymbol{\partial}}\boldsymbol{\nu}'(x) = \frac{i}{2}\sum_k \overline{\nu}_k(x)\overleftrightarrow{\boldsymbol{\partial}}\nu_k(x).$$

$$\Downarrow$$

$\nu_k(x)$ is the field of a Dirac neutrino with the mass $m_k$ and the flavor LH neutrino fields $\nu_{\ell,L}(x)$ involved into the SM weak lepton currents are linear combinations of the LH components of the fields of the neutrinos with definite masses:

$$\boxed{\boldsymbol{\nu}_L = \mathbf{V}\boldsymbol{\nu}'_L \quad \text{or} \quad \nu_{\ell,L} = \sum_k V_{\ell k}\nu_{k,L}.}$$

The matrix $\mathbf{V}$ is referred to as the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino mixing matrix while the matrix $\widetilde{\mathbf{V}}$ is not honored with a personal name.

**Quark-lepton complementarity (QLC):** Of course the PMNS matrix it is not the same as the CKM (Cabibbo-Kobayashi-Maskawa) quark mixing matrix. However the PMNS and CKM matrices may be, in a sense, *complementary* to each other.

The QLC means that *in the same (PDG) parametrizations* the sums of (small) quark and (large) lepton mixing angles are almost (i.e., within errors) equal to $\pi/4$ for $(ij) = (12)$ and $(23)$:

$$\theta_{12}^{\text{CKM}} + \theta_{12}^{\text{PMNS}} = (46.49 \pm 0.77)^\circ, \quad \theta_{23}^{\text{CKM}} + \theta_{23}^{\text{PMNS}} = (44.48 \pm 1.10)^\circ, \quad \text{sum} = (90.97 \pm 1.34)^\circ.$$

The origin of the data (but not QLC) will be explained below.

## 2.1 Parametrization of mixing matrix for Dirac neutrinos.

It is well known that a complex $n \times n$ unitary matrix depends on $n^2$ *real* parameters.

The classical result by Francis Murnaghan [F. D. Murnaghan, "The unitary and rotation groups (Lectures on Applied Mathematics, Volume 3)," Spartan Books, Washington, D.C. (1962)] states that any $n \times n$ matrix from the unitary group $U(n)$ can be presented as product of the diagonal phase matrix

$$\boldsymbol{\Gamma} = \text{diag}\left(e^{i\alpha_1}, e^{i\alpha_2}, \ldots, e^{i\alpha_n}\right),$$

containing $n$ phases $\alpha_k$, and $n(n-1)/2$ matrices $\mathbf{U}$ whose main building blocks have the form

$$\begin{pmatrix} \cos\theta & \sin\theta\, e^{-i\phi} \\ -\sin\theta\, e^{+i\phi} & \cos\theta \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & e^{+i\phi} \end{pmatrix} \underbrace{\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}}_{\text{Euler rotation}} \begin{pmatrix} 1 & 0 \\ 0 & e^{-i\phi} \end{pmatrix}.$$

Therefore any $n \times n$ unitary matrix can be parametrized in terms of

$n(n-1)/2$ "angles" (taking values within $[0, \pi/2]$)

and

$n(n+1)/2$ "phases" (taking values within $[0, 2\pi)$).

The usual parametrization of both the CKM and PMNS matrices is of this type.

**IMPORTANT:** Murnaghan's factorization method does not specify the sequence of the building blocks $\boldsymbol{\Gamma}$ and $\mathbf{U}$.

One can reduce the number of the phases further by taking into account that the Lagrangian with the Dirac mass term is invariant with respect to the transformation

$$\ell \mapsto e^{ia_\ell}\ell, \quad \nu_k \mapsto e^{ib_k}\nu_k, \quad V_{\ell k} \mapsto e^{i(b_k - a_\ell)}V_{\ell k},$$

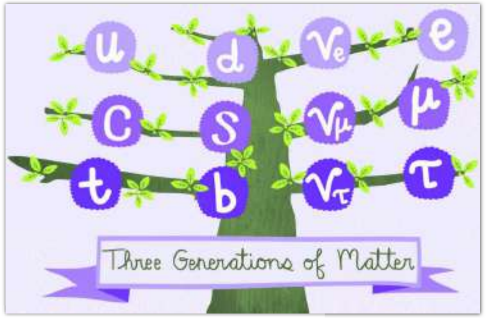
and to the global gauge transformation

$$\ell \mapsto e^{i\Lambda}\ell, \quad \nu_k \mapsto e^{i\Lambda}\nu_k, \quad \text{with} \quad \Lambda = \text{const}. \tag{1}$$

Therefore $2N - 1$ phases are unphysical and the number of physical (Dirac) phases is

$$n_{\text{D}} = \frac{N(N+1)}{2} - (2N-1) = \frac{N^2 - 3N + 2}{2} = \frac{(N-1)(N-2)}{2} \qquad (N \geq 2);$$

$$n_{\text{D}}(2) = 0, \quad n_{\text{D}}(3) = 1, \quad n_{\text{D}}(4) = 3, \ldots$$




- The global symmetry (1) leads to conservation of the lepton charge

$$L = \sum_{\ell = e, \mu, \tau, \ldots} L_\ell$$

common to all charged leptons and all neutrinos $\nu_k$. However

$$\boxed{\text{The individual lepton flavor numbers } L_\ell \text{ are no longer conserved.}}$$

- The nonzero physical phases lead to the $CP$ (and $T$) violation in the neutrino sector.[a] This could have important implications for particle physics and cosmology (leptogenesis, baryogenesis,...).

[a]The proof can be found, e.g., in Sec. 4.6 of C. Giunti and C. W. Kim, "Fundamentals of neutrino physics and astrophysics" (Oxford University Press Inc., New York, 2007) or in Sec. 6.3 of S. M. Bilenky, "Introduction to the physics of massive and mixed neutrinos" (2nd ed.), Lect. Notes Phys. **947** (2018) 1–276. Note the differences in notation and in representation for the matrix $C$.

### 2.1.1 Three-neutrino case.

In the most interesting (today!) case of three lepton generations one defines the orthogonal rotation matrices in the $ij$-planes which depend upon the mixing angles $\theta_{ij}$:

$$\mathbf{O}_{12} = \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{Solar matrix}}, \quad \mathbf{O}_{13} = \underbrace{\begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{pmatrix}}_{\text{Reactor matrix}}, \quad \mathbf{O}_{23} = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{\text{Atmospheric matrix}},$$

(where $c_{ij} \equiv \cos\theta_{ij}$, $s_{ij} \equiv \sin\theta_{ij}$) and the diagonal matrix with the Dirac phase factor:

$$\boldsymbol{\Gamma}_{\mathsf{D}} = \mathsf{diag}\left(1, 1, e^{i\delta}\right).$$

The parameter $\delta$ is commonly referred to as the Dirac $CP$-violation/violating phase.

Finally, by applying Murnaghan's factorization, the PMNS matrix for the Dirac neutrinos can be parametrized as

$$\mathbf{V}_{(\mathsf{D})} = \mathbf{O}_{23}\boldsymbol{\Gamma}_{\mathsf{D}}\mathbf{O}_{13}\boldsymbol{\Gamma}_{\mathsf{D}}^{\dagger}\mathbf{O}_{12} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}.$$

- ⋆ This is the *Chau–Keung presentation* advocated by the PDG for both CKM and PMNS matrices.
- ⋆ Remember that the positioning of the factors in $\mathbf{V}_{(\mathsf{D})}$ is not fixed by the Murnaghan (or any other) algorithm and is just a subject-matter of agreement.
- ⋆ Today we believe we know a lot about the entries of this matrix.

### 2.1.2 Lepton numbers are not conserved, so what of it?.

Since the Dirac mass term violates conservation of the individual lepton numbers, $L_e, L_\mu, L_\tau$, it allows many lepton family number violating processes, like

$$\mu^\pm \to e^\pm + \gamma, \quad \mu^\pm \to e^\pm + e^+ + e^-,$$
$$K^+ \to \pi^+ + \mu^\pm + e^\mp, \quad K^- \to \pi^- + \mu^\pm + e^\mp,$$
$$\mu^- + (A,Z) \to e^- + (A,Z), \quad \tau^- + (A,Z) \to \mu^- + (A,Z), \ldots$$

However the $(\beta\beta)_{0\nu}$ decay or the kaon semileptonic decays like

$$K^+ \to \pi^- + \mu^+ + e^+, \quad K^- \to \pi^+ + \mu^- + e^-,$$

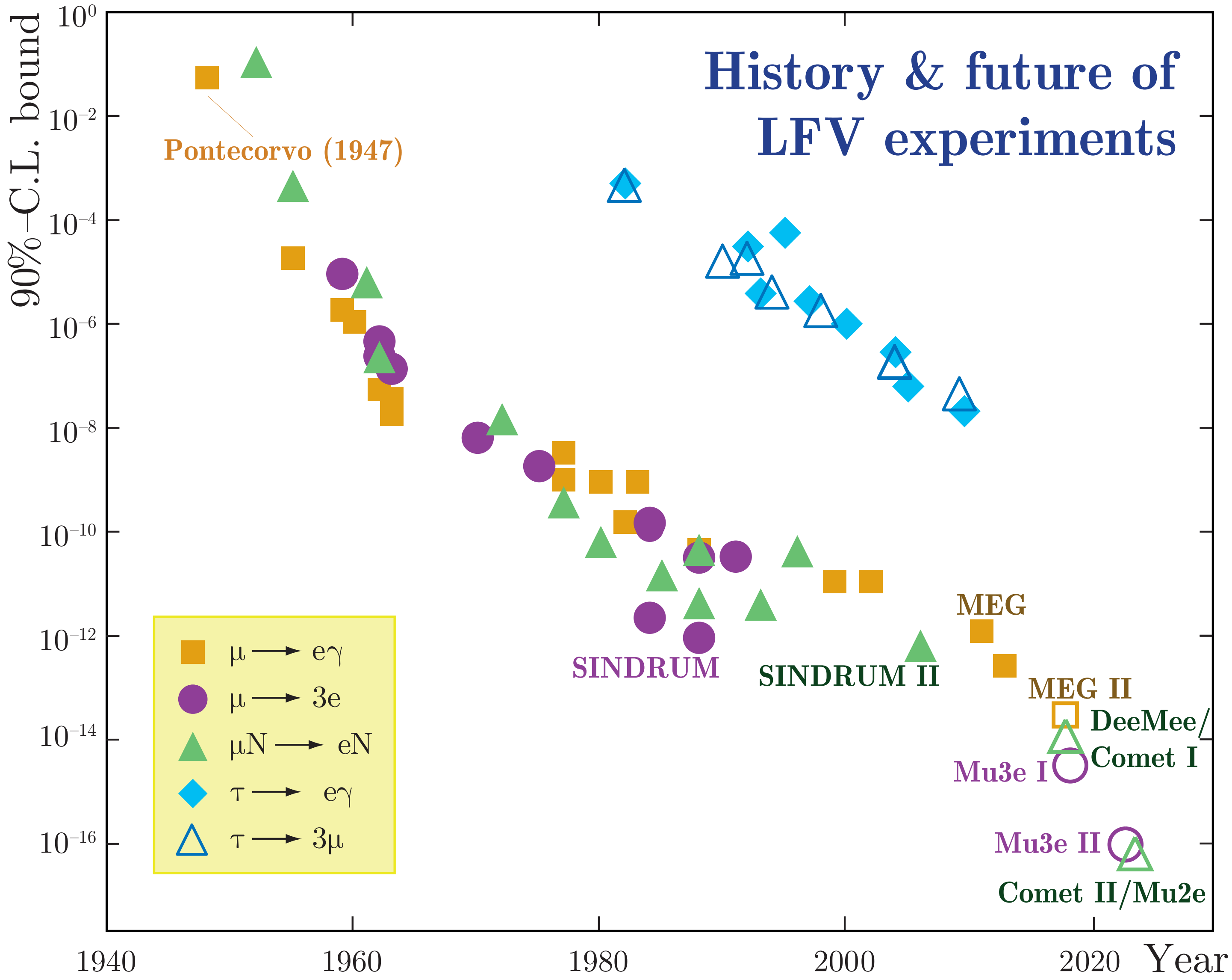
etc. are still *forbidden* as a consequence of the total lepton charge conservation.

Current limits on the simplest lepton family number violating $\mu$ and $\tau$ decays (2020). [a]

| Decay Modes | Fraction | C.L. | Decay Modes | Fraction | C.L. |
|---|---|---|---|---|---|
| $\mu^- \to e^- \nu_e \overline{\nu}_\mu$ | $< 1.2\%$ | 90% | $\tau^- \to e^- \gamma$ | $< 3.3 \times 10^{-8}$ | 90% |
| $\mu^- \to e^- \gamma$ | $< 4.2 \times 10^{-13}$ | 90% | $\tau^- \to \mu^- \gamma$ | $< 4.4 \times 10^{-8}$ | 90% |
| $\mu^- \to e^- e^+ e^-$ | $< 1.0 \times 10^{-12}$ | 90% | $\tau^- \to e^- \pi^0$ | $< 8.0 \times 10^{-8}$ | 90% |
| $\mu^- \to e^- 2\gamma$ | $< 7.2 \times 10^{-11}$ | 90% | $\tau^- \to \mu^- \pi^0$ | $< 1.1 \times 10^{-7}$ | 90% |

These limits are not quite as impressive as might appear at first glance.

[a] P. A. Zyla *et al.* (Particle Data Group), "Review of Particle Physics", PTEP **2020** (2020) 083C01.




[From N. Berger, "Charged lepton flavour violation experiments," talk at the Zürich Phenomenology Workshop, January 2015. For details, see W. J. Marciano, T. Mori, and J. M. Roney, "Charged lepton flavor violation experiments," Ann. Rev. Nucl. Part. Sci. **58** (2008) 315–341. Is not yet updated!]

### 2.1.3 Neutrinoless muon decay in SM.

The $L_\mu$ and $L_e$ violating muon decay $\mu^- \to e^- \gamma$ is allowed if $V_{\mu k}^* V_{ek} \neq 0$ for $k = 1, 2$ or 3. The corresponding Feynman diagrams include $W$ loops and thus the decay width is strongly suppressed by the neutrino to $W$ boson mass ratios:

$$R = \frac{\Gamma\left(\mu^- \to e^- \gamma\right)}{\Gamma\left(\mu^- \to e^- \nu_\mu \overline{\nu}_e\right)} = \frac{3\alpha}{32\pi} \left| \sum_k V_{\mu k}^* V_{ek} \frac{m_k^2}{m_W^2} \right|^2 .$$

Since $m_k/m_W \approx 1.244 \times 10^{-12}\,(m_k/0.1\text{ eV})$, the ratio can be estimated as

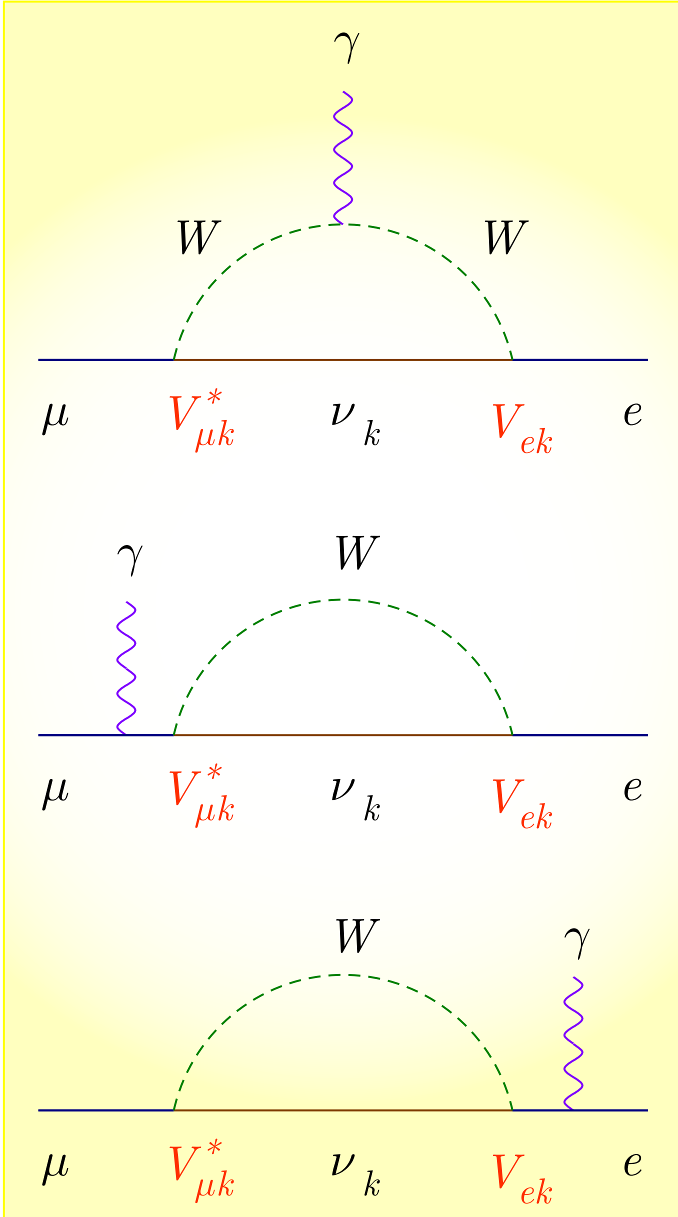
$$R \approx 5.22 \times 10^{-52} \left| \sum_k V_{\mu k}^* V_{ek} \left( \frac{m_k}{0.1\text{ eV}} \right)^2 \right|^2 \lesssim 8 \times 10^{-54},$$

while the current experimental upper limit is (at least!) 40 orders of magnitude larger (see Table in p. 16):

$$R_{(\text{exp})} < 4.2 \times 10^{-13} \text{ at } 90\% \text{ C.L.} \quad (\textbf{NO GO!})$$

Some nonstandard models are much more optimistic.

We must deeply appreciate the oscillation phenomenon which makes the miserable $\nu$ mass effect measurable.



## 2.2 Nuclear beta decay.

The method of measurement of the (anti)neutrino mass through the investigation of the high-energy part of the $\beta$-spectrum was proposed by Perrin (1933) and Fermi (1934).
The first experiments on the measurement of the neutrino mass with this method have been done by Curran, Angus and Cockcroft (1948) and Hanna and Pontecorvo (1949).

The energy spectrum of electrons in the decay $(A, Z) \to (A, Z+1) + e^- + \overline{\nu}_e$ is[a]

$$\frac{d\Gamma}{dT} = \sum_k |V_{ek}|^2 \frac{d\Gamma_k}{dT}, \tag{2}$$

$$\frac{d\Gamma_k}{dT} = \frac{(G_F \cos\theta_C)^2}{2\pi^3} p p_k \,(T + m_e)\,(Q - T)\,|\mathcal{M}|^2 \,F(T, Z)\theta\,(Q - T - m_k). \tag{3}$$

Here $G_F$ is the Fermi constant, $\theta_C$ is the Cabibbo angle, $m_e$, $p$ and $T$ are the mass, magnitude of the momentum and kinetic energy of the electron, respectively,

$$p_k = \sqrt{E_k^2 - m_k^2} = \sqrt{(Q-T)^2 - m_k^2} \quad \text{and} \quad Q = E_k + T = E_{A,Z} - E_{A,Z+1} - m_e$$

are, respectively, the magnitude of the neutrino momentum and energy released in the decay (the endpoint of the $\beta$ spectrum in case $m_k = 0$), $\mathcal{M}$ is the nuclear matrix element, and $F(T, Z)$ is the Fermi function, which describes the Coulomb interaction of the final-state nucleus and electron.
The step function in Eq. (3) ensures that a neutrino state $\nu_k$ is only produced if its total energy is larger than its mass: $E_k = Q - T \geq m_k$.

[a] The recoil of the final nucleus and radiative corrections (luckily small) are neglected.

As it is seen from Eq. (2), the largest distortion of the $\beta$-spectrum due to neutrino masses can be observed in the region

$$Q - T \sim m_k. \tag{4}$$

However, for $\max(m_k) \simeq 0.1$ eV only a very small part (about $10^{-(13-14)}$) of the decays give contribution to the region (4). This is the reason why in the analysis of the results of the measurement of the $\beta$-spectrum a relatively large part of the spectrum is used.[a]

Taking this into account and applying unitarity of the mixing matrix, we can write

$$\begin{aligned}
\sum_k |V_{ek}|^2 \, p_k &\approx \sum_k |V_{ek}|^2 \,(Q-T) \left[1 - \frac{m_k^2}{2(Q-T)^2}\right] && \Longleftarrow \quad 4E_k^2 \gg m_k^2 \\
&= (Q-T) \left[1 - \frac{1}{2(Q-T)^2} \sum_k |V_{ek}|^2 \, m_k^2\right] && \Longleftarrow \quad \sum_k |V_{ek}|^2 = 1 \\
&\approx \sqrt{(Q-T)^2 - m_\beta^2},
\end{aligned}$$

where the effective neutrino mass $m_\beta$ is defined by

$$m_\beta^2 = \sum_k |V_{ek}|^2 \, m_k^2$$

and it was assumed that

$$\max_k \left(m_k^2\right) \ll 4(Q-T)^2.$$

[a]For example, in the Mainz tritium experiment (see below) the last 70 eV of the spectrum is used.

Finally, the $\beta$-spectrum that is used for fitting the data can be presented as

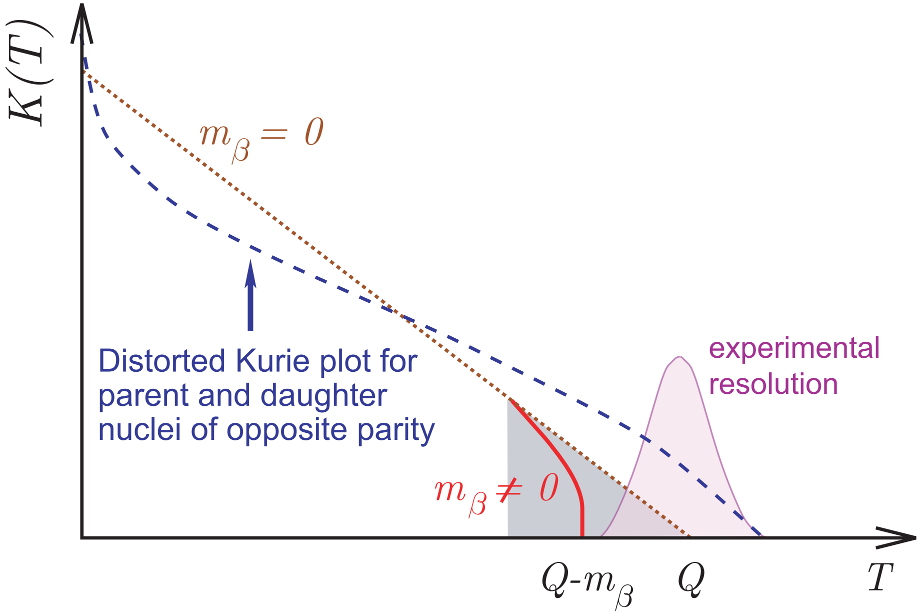
$$\frac{d\Gamma}{dT} \propto p\,(T+m_e)\,|\mathcal{M}|^2\,F(T)K^2(T),$$

where we have defined the *Kurie function* (sometimes called Fermi-Kurie function)

$$K(T) \propto \sqrt{\frac{d\Gamma/dT}{p\,(T+m_e)\,|\mathcal{M}|^2\,F(T)}}$$
$$\approx (Q-T)\left[1-\frac{m_\beta^2}{(Q-T)^2}\right]^{1/4}$$

developed by Franz Newell Devereux Kurie.

Unfortunately, the real-life situation is much more complicated.




Kurie plot for allowed processes is a sensitive test of $m_\beta$, while the first order forbidden processes should have a distorted Kurie plot.

In an actual experiment, the measurable quantity is a sum of $\beta$ spectra, leading each with probability $P_n = P_n(E_0 - V_n - E)$ to a final state $n$ of excitation energy $V_n$:

$$\frac{d\Gamma(T,Q)}{dT} \longmapsto \sum_n P_n\,(E_0 - V_n - E)\,\frac{d\Gamma\,(T, E_0 - V_n)}{dT}.$$

Here $E_0 = Q - \mathcal{E}$ the ground-state energy and $\mathcal{E}$ is the recoil energy of the daughter nucleus.

### 2.2.1 Tritium beta decay.

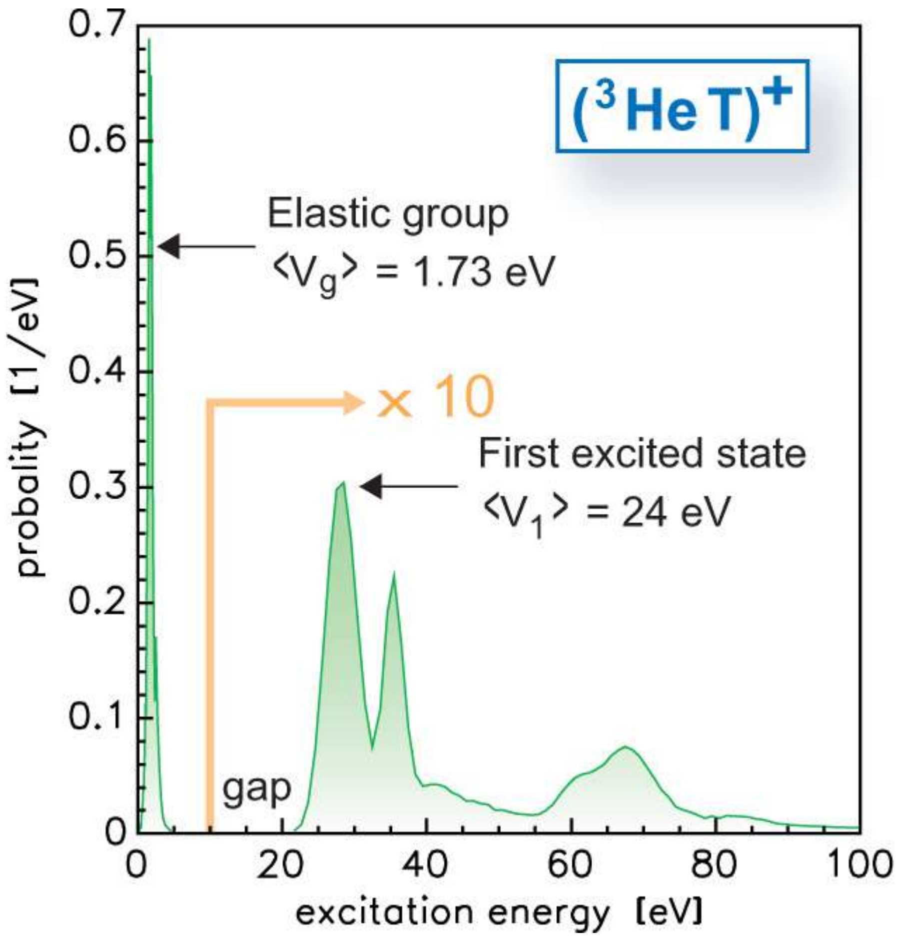
An important issue is the decay of molecular tritium $\mathsf{T}_2 \to \left({}^3\mathsf{HeT}\right)^+ + e^- + \overline{\nu}_e$. Considering the most precise direct determination of the mass difference

$$m(\mathsf{T}) - m\left({}^3\mathsf{He}\right) = (18590.1 \pm 1.7)\ \mathsf{eV}/c^2$$

and taking into account the recoil and apparative effects (these are taken for the Mainz experiment) one derives an endpoint energy of the molecular ion $\left({}^3\mathsf{HeT}\right)^+$ ground state:

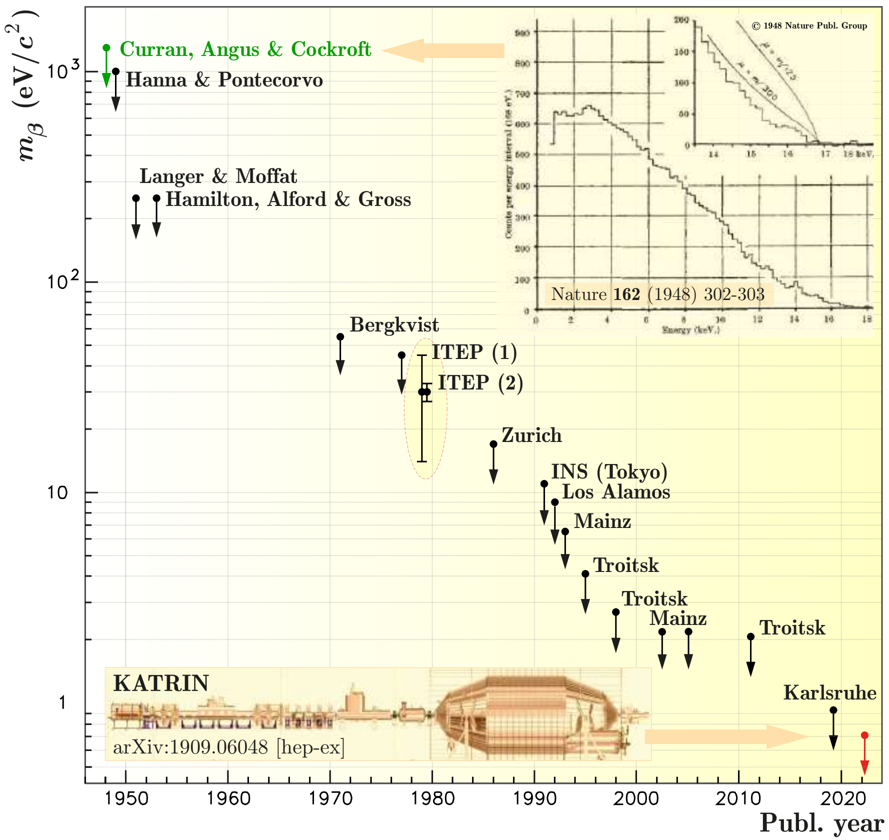
$$E_0 = (18574.3 \pm 1.7)\ \mathsf{eV}.$$

The excitation spectrum is shown in the figure. The first group concerns rotational and vibrational excitation of the molecule in its electronic ground state; it comprises a fraction of $P_g = 57.4\%$ of the total rate.




Excitation spectrum of the daughter molecular ion $\left({}^3\mathsf{HeT}\right)^+$ in $\beta$ decay of molecular tritium.

For more details, see C. Kraus *et al.*, "Final results from phase II of the Mainz neutrino mass search in tritium $\beta$ decay," Eur. Phys. J. C **40** (2005) 447–468, hep-ex/0412056.




Progress of the neutrino mass measurements in tritium $\beta$ decay, including the final Mainz phase II, Troitsk, and KATRIN upper limits (see below).

[The compilation is taken from V. M. Lobashev, "Direct search for mass of neutrino," in Proceedings of the 18th International Conference on Physics in Collision ("PIC 98"), Frascati, June 17–19, 1998, pp. 179–194 and supplemented with the recent data.]

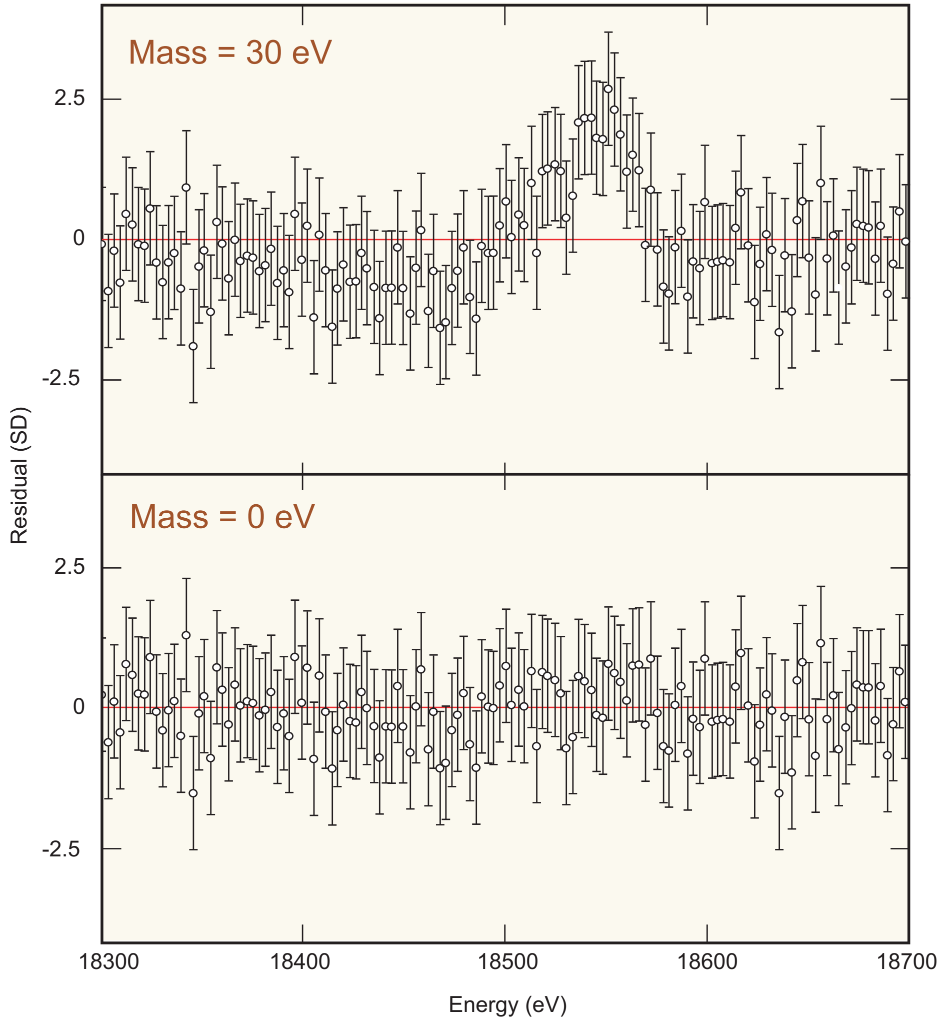
◁ The history of the search for the neutrino mass in the tritium $\beta$ decay counts more than 60 years. In 1980, the steady improvement of the upper limit was suddenly speeded up by a report of the ITEP group (Moscow) on the observation of the nonzero neutrino mass effect in the $\beta$-spectrum in the valine molecule ($C_5H_9T_2NO_2$). The reported result was[a]

$$14 \leq m_\beta \leq 46 \text{ eV}/c^2 \quad (99\% \text{ C.L.})$$

This research stimulated more than 20 experimental proposals with an intention to check this clime. Alas!... in several years the experimental groups from Zürich, Tokyo, Los Alamos, and then Livermore refuted the ITEP result.

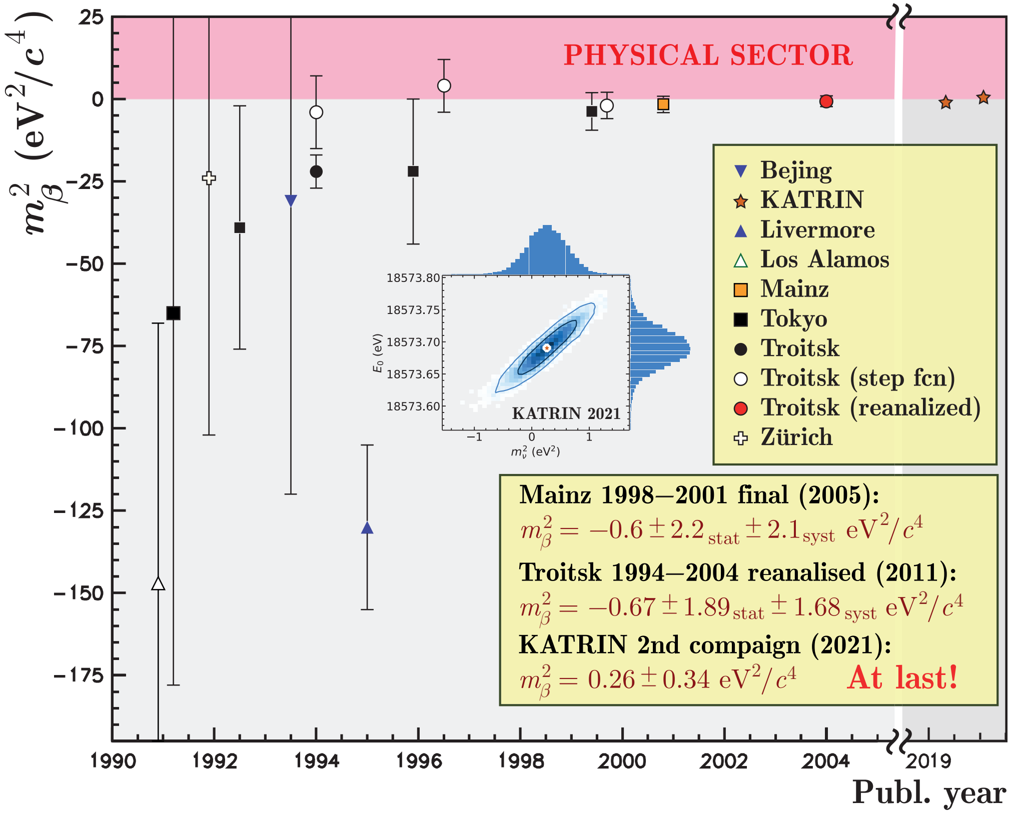
[a] V. A. Lyubimov, E. G. Novikov, V. Z. Nozik, E. F. Tretyakov, and V. S. Kosik, "An estimate of the $\nu_e$ mass from the $\beta$-spectrum of tritium in the valine molecule," Phys. Lett. B **94** (1980) 266–268 ($\sim$ 500 citations in InSPIRE! by the end of 2021).

The top figure shows the data points from the tail of the $\beta$-spectrum measured in the Los Alamos tritium experiment compared with the expected values (the straight line) for $m_\beta = 30$ eV. The data wander from the line, ruling out the possibility of a 30-eV neutrino.
The bottom figure shows the same data points compared with the expectation for $m_\beta = 0$. While the data clearly favor a neutrino mass of zero, the best fit is actually for a slightly negative $m_\beta$. (Note that in the bottom plot, the data points lie, on average, slightly above the line, so this is not a perfect fit.)
Both plots display "residuals," which indicate how many standard deviations each data point is from a particular hypothesis.




Did the neutrino weigh 30 electron volts?

[Borrowed from T. J. Bowles and R. G. H. Robertson, "Tritium beta decay and the search for neutrino mass," Los Alamos Sci. **25** (1997) 6–11.]




◁ The figure shows the results on the $m_\beta^2$ measurements in the tritium $\beta$ decay experiments reported after 1990.

The already finished experiments at Los Alamos, Zürich, Tokyo, Beijing and Livermore used magnetic spectrometers, while the experiments at Troitsk ($\nu$ mass), Mainz, and Karlsruhe (KATRIN) are using high-resolution electrostatic filters with magnetic adiabatic collimation.

The progress in the observable $m_\beta$ of the latest Mainz, Troitsk, and KATRIN results as compared to the most sensitive earlier experiments approaches two orders of magnitude.

[The figure in this slide includes the data from C. Kraus *et al.*, Eur. Phys. J. C **40** (2005) 447–468, hep-ex/0412056; V. N. Aseev *et al.*, Phys. Rev. D **84** (2011) 112003, arXiv:1108.5034 [hep-ex]; M. Aker *et al.*, Phys. Rev. Lett. **123** (2019) 221802, arXiv:1909.06048 [hep-ex] M. Aker *et al.*, arXiv:2105.08533 [hep-ex]. ]

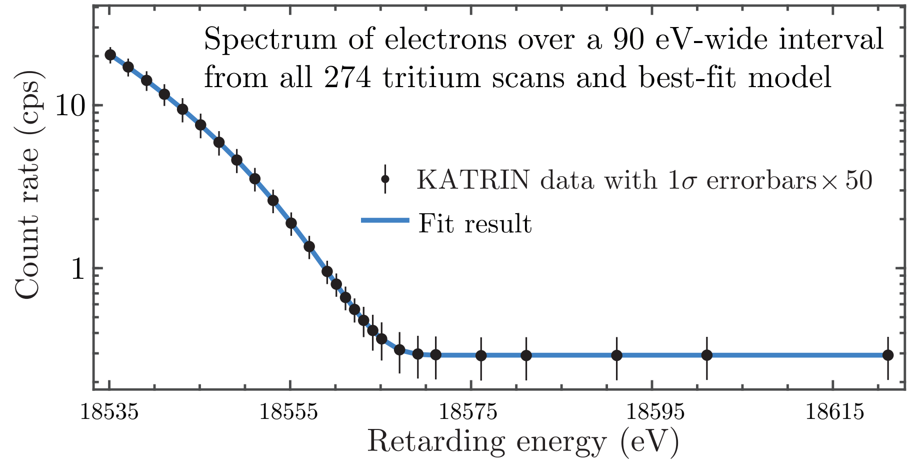
The negative $m_\beta^2$ most probably *was* "instrumental". After KATRIN (2021), only a very small space remains for fans of heterodox models with *tachyonic* neutrino states (more generally – superpositions of *bradyon-luxon-tachyon* states), *pseudotachyonic* ($m_\nu^2 < 0$, $v = E/p$), or perhaps *superbradyonic* ($m_\nu > 0$, $v > 1$) neutrinos.

### 2.2.2 Summary of the KATRIN result from the first science run (KNM1).

The best fit value of the effective neutrino mass square was found to be[a]

$$m_\beta^2 = \left(-1.0^{+0.9}_{-1.1}\right)\ \mathrm{eV}^2.$$

This result corresponds to a $1\sigma$ statistical fluctuation to negative values of $m_\beta^2$ possessing a $p$-value of 0.16. The total uncertainty budget of $m_\beta^2$ is largely dominated by $\sigma_{\text{stat}}$ (0.97 eV$^2$) as compared to $\sigma_{\text{syst}}$ (0.32 eV$^2$). These uncertainties are smaller by a factor of 2 and 6, respectively, compared to the final results of Troitsk and Mainz.




The methods of Lokhov and Tkachov (LT) and of Feldman and Cousins (FC) are then used to calculate the upper limit on the absolute mass scale of neutrino:

$$\boxed{m_\beta < 1.1\ \text{eV at } 90\%\ \text{C.L.}\ \text{(LT)},\quad m_\beta < 0.8\ (0.9)\ \text{eV at } 90\ (95)\%\ \text{C.L.}\ \text{(FC)}.}$$

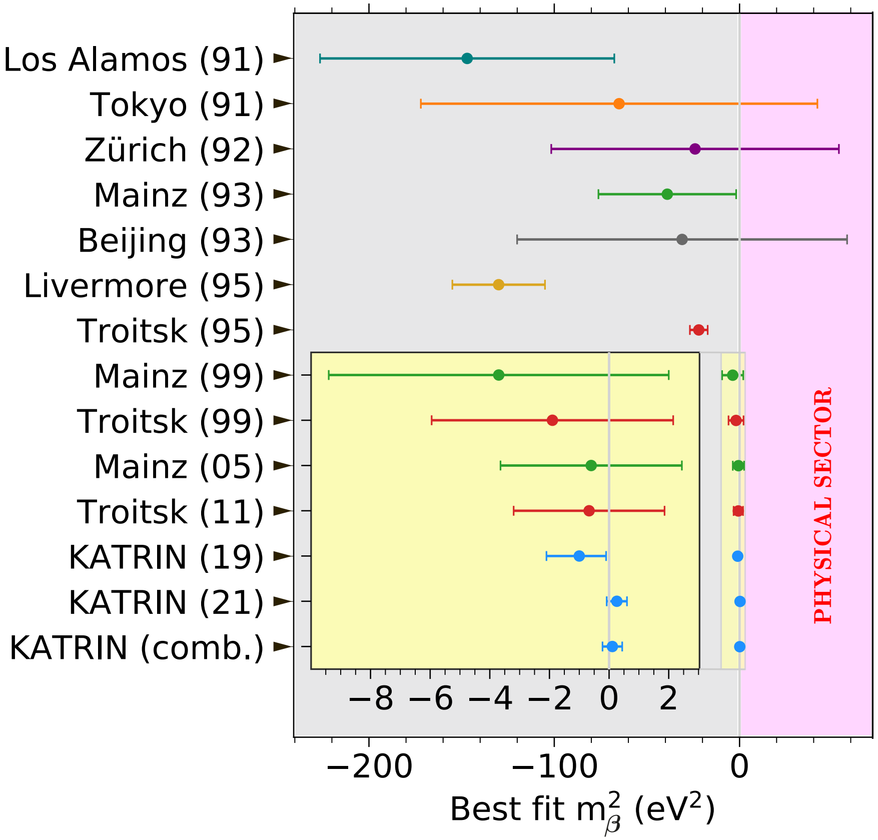
The LT value (the central result of the experiment) coincides with the KATRIN sensitivity. It is based on a purely kinematic method and improves upon previous works by almost a factor of two after a measuring period of only four weeks while operating at reduced column density.

> After 1000 days of data taking at nominal column density and further reductions of systematics the Karlsruhe Tritium Neutrino experiment KATRIN will reach a sensitivity of 0.2 eV (90% C.L.) on $m_\beta$.

[a] M. Aker *et al.*, "An improved upper limit on the neutrino mass from a direct kinematic method by KATRIN," Phys. Rev. Lett. **123** (2019) 221802, arXiv:1909.06048 [hep-ex].

### 2.2.3 Summary of the KATRIN result from the second science run (KNM2).

In the 2nd physics run, the source activity was increased by a factor of 3.8 and the background was reduced by 25 % with respect to the 1st campaign.[a] A sensitivity on $m_\beta$ of 0.7 eV at 90 % C.L. was reached. This is the first sub-eV sensitivity from a direct neutrino-mass experiment.




The best fit to the spectral data yields $m_\beta = 0.26 \pm 0.34$ eV, resulting in an upper limit of $m_\beta < 0.9$ eV (90 % C.L.), using the Lokhov-Tkachov method. The Feldman-Cousins technique yields the same limit. The resulting Bayesian limit at 90 % C.L. is $m_\beta < 0.85$ eV.

A simultaneous fit of both KNM1 and KNM2 data sets yields $m_\beta = 0.1 \pm 0.3$ eV, resulting an improved limit of $m_\beta < 0.8$ eV (90 % C.L.).

As both data sets are statistics-dominated, correlated systematic uncertainties between both campaigns are negligible.

◁ The figure displays the evolution of best-fit $m_\beta$ results from historical $\nu$-mass measurements (c.f. p. 25).

$m_\beta < 0.9$ eV at 90 % C.L. (KNM2), $m_\beta < 0.8$ eV at 90 % C.L. (KNM1+KNM2).

[a] M. Aker *et al.*, "First direct neutrino-mass measurement with sub-eV sensitivity", Nature Phys. **18** (2022) 160–166, arXiv:2105.08533 [hep-ex]; see also arXiv:2203.08059 [nucl-ex], submitted to Nature Physics.

# 3 Majorana neutrinos.


The charge conjugated bispinor field $\psi^c$ is defined by the transformation

$$\psi \longmapsto \psi^c = C\overline{\psi}^T, \quad \overline{\psi} \longmapsto \overline{\psi^c} = -\psi^T C,$$

where $C$ is the charge-conjugation matrix which satisfies the conditions

$$C\gamma_\alpha^T C^\dagger = -\gamma_\alpha, \quad C\gamma_5^T C^\dagger = \gamma_5, \quad C^\dagger = C^{-1} = C, \quad C^T = -C,$$

and thus coincides (up to a phase factor) with the inversion of the axes $x_0$ and $x_2$: $C = \gamma_0\gamma_2$.

Clearly the charged fermion field $\psi$ is different from the charge-conjugated field $\psi^c$ but a neutral fermion field $\nu$ can coincide with the charge-conjugated one $\nu^c$. In other words: for a neutral fermion (neutrino, neutralino) field $\nu(x)$ the following condition is not forbidden: [a]

$\nu^c(x) = \nu(x)$ (Majorana condition) $\Longleftrightarrow$ Majorana neutrino and antineutrino coincide!

**A few more details:** In the chiral representation

$$\nu = \begin{pmatrix} \phi \\ \chi \end{pmatrix}, \quad \nu^c = C\overline{\nu}^T = \begin{pmatrix} -\sigma_2\chi^* \\ +\sigma_2\phi^* \end{pmatrix}. \quad \Longrightarrow \quad \begin{cases} \phi = -\sigma_2\chi^*, \\ \chi = +\sigma_2\phi^* \end{cases} \quad \Longrightarrow \quad \phi + \chi = \sigma_2 (\phi - \chi)^*.$$

The Majorana neutrino is two-component, i.e., it is defined by only one chiral projection. Then (c.f. p. 9)

$$\nu_L = P_L\nu = \begin{pmatrix} \phi - \chi \\ \chi - \phi \end{pmatrix} \quad \text{and} \quad \nu_R = P_R\nu = \begin{pmatrix} \phi + \chi \\ \phi + \chi \end{pmatrix} = \nu_L^c. \quad \Longrightarrow \quad \boxed{\nu = \nu_L + \nu_R = \nu_L + \nu_L^c.}$$

[a] The simplest generalization of the Majorana condition, $\nu^c(x) = e^{i\varphi}\nu(x)$ ($\varphi = \text{const}$), is not very interesting.

The Majorana mass term in the general $N$-neutrino case is [Gribov & Pontecorvo (1969)]:

$$\mathcal{L}_{\mathsf{M}}(x) = -\frac{1}{2}\overline{\boldsymbol{\nu}}^{c}_{L}(x)\mathbf{M}_{\mathsf{M}}\boldsymbol{\nu}_{L}(x) + \text{H.c.},$$

Here $\mathbf{M}_{\mathsf{M}}$ is a $N \times N$ complex *nondiagonal* matrix and, in general, $N \geq 3$.

It can be proved that the $\mathbf{M}_{\mathsf{M}}$ should be symmetric, $\mathbf{M}_{\mathsf{M}}^{T} = \mathbf{M}_{\mathsf{M}}$. Assuming for simplicity that its spectrum is non-degenerated, the mass matrix can be diagonalized by means of the following transformation [Bilenky & Petcov (1987)]

$$\mathbf{M}_{\mathsf{M}} = \mathbf{V}^{*}\mathbf{m}\mathbf{V}^{\dagger}, \quad \mathbf{m} = ||m_k\delta_{kl}|| = \text{diag}\,(m_1, m_2, \ldots, m_N),$$

where $\mathbf{V}$ is a unitary matrix and $m_k \geq 0$. Therefore

$$\mathcal{L}_{\mathsf{M}}(x) = -\frac{1}{2}\left[(\overline{\boldsymbol{\nu}'}_{L})^{c}\mathbf{m}\boldsymbol{\nu}'_{L} + \overline{\boldsymbol{\nu}'}_{L}\mathbf{m}(\boldsymbol{\nu}'_{L})^{c}\right] = -\frac{1}{2}\,\overline{\boldsymbol{\nu}'}\mathbf{m}\boldsymbol{\nu}' = -\frac{1}{2}\sum_{k=1}^{N} m_k\overline{\nu}_k\nu_k,$$

$$\boldsymbol{\nu}'_{L} = \mathbf{V}^{\dagger}\boldsymbol{\nu}_{L}, \quad (\boldsymbol{\nu}'_{L})^{c} = C\left(\overline{\boldsymbol{\nu}'_{L}}\right)^{T}, \quad \boldsymbol{\nu}' = \boldsymbol{\nu}'_{L} + (\boldsymbol{\nu}'_{L})^{c}.$$

The last equality means that the fields $\nu_k(x)$ are Majorana neutrino fields. Considering that the kinetic term in the neutrino Lagrangian is transformed to[a]

$$\mathcal{L}_0 = \frac{i}{2}\,\overline{\boldsymbol{\nu}'}(x)\overleftrightarrow{\boldsymbol{\partial}}\,\boldsymbol{\nu}'(x) = \frac{i}{2}\sum_{k}\overline{\nu}_k(x)\overleftrightarrow{\boldsymbol{\partial}}\,\nu_k(x),$$

one can conclude that $\nu_k(x)$ is the field with the definite mass $m_k$.

[a]This also explains the origin of the factor $1/2$ in the Majorana mass term.

The flavor LH neutrino fields $\nu_{\ell,L}(x)$ present in the standard weak lepton currents are linear combinations of the LH components of the fields of neutrinos with definite masses:

$$\boxed{\boldsymbol{\nu}_L = \mathbf{V}\boldsymbol{\nu}'_L \quad \text{or} \quad \nu_{\ell,L} = \sum_k V_{\ell k}\nu_{k,L}.}$$

Of course neutrino mixing matrix $\mathbf{V}$ is not the same as in the case of Dirac neutrinos.

There is no global gauge transformations under which the Majorana mass term (in its most general form) could be invariant. This implies that there are no conserved lepton charges that could allow us to distinguish Majorana $\nu$s and $\overline{\nu}$s. In other words,

Majorana neutrinos are truly neutral fermions.

## 3.1 Parametrization of mixing matrix for Majorana neutrinos.

Since the Majorana neutrinos are not rephasable, there may be a lot of extra phase factors in the mixing matrix. The Lagrangian with the Majorana mass term is invariant with respect to the transformation

$$\ell \mapsto e^{ia_\ell}\ell, \quad V_{\ell k} \mapsto e^{-ia_\ell}V_{\ell k}$$

Therefore $N$ phases are unphysical and the number of the physical phases now is

$$\frac{N(N+1)}{2} - N = \frac{N(N-1)}{2} = \underbrace{\frac{(N-1)(N-2)}{2}}_{\text{Dirac phases}} + \underbrace{(N-1)}_{\text{Majorana phases}} = n_{\mathsf{D}} + n_{\mathsf{M}};$$

$$n_{\mathsf{M}}(2) = 1, \quad n_{\mathsf{M}}(3) = 2, \quad n_{\mathsf{M}}(4) = 3, \ldots$$

- In fact all phases are Majorana and the above notation is provisional and unorthodox.

In the case of three lepton generations one defines the diagonal matrix with the extra phase factors: $\boldsymbol{\Gamma}_{\mathsf{M}} = \mathrm{diag}\left(e^{i\alpha_1/2}, e^{i\alpha_2/2}, 1\right)$, where $\alpha_{1,2}$ are commonly referred to as the Majorana $CP$-violation phases. Then the PMNS matrix can be parametrized as

$$\mathbf{V}_{(\mathsf{M})} = \mathbf{O}_{23}\boldsymbol{\Gamma}_{\mathsf{D}}\mathbf{O}_{13}\boldsymbol{\Gamma}_{\mathsf{D}}^{\dagger}\mathbf{O}_{12}\boldsymbol{\Gamma}_{\mathsf{M}} = \mathbf{V}_{(\mathsf{D})}\boldsymbol{\Gamma}_{\mathsf{M}}$$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

---

Neither $L_\ell$ nor $L = \sum_\ell L_\ell$ is now conserved allowing a lot of new processes, for example, $\tau^- \to e^+(\mu^+)\pi^-\pi^-$, $\tau^- \to e^+(\mu^+)\pi^-K^-$, $\pi^- \to \mu^+\overline{\nu}_e$, $K^+ \to \pi^-\mu^+e^+$, $K^+ \to \pi^0e^+\overline{\nu}_e$, $D^+ \to K^-\mu^+\mu^+$, $B^+ \to K^-e^+\mu^+$, $\Xi^- \to p\mu^-\mu^-$, $\Lambda_c^+ \to \Sigma^-\mu^+\mu^+$, etc.

---

Needless to say that no one was discovered yet [see RPP] but (may be!?) the $(\beta\beta)_{0\nu}$ decay. The following section will discuss this issue with some detail.

## 3.2 Neutrinoless double beta decay.

The theory with Majorana neutrinos allows the decay

$$(A, Z) \to (A, Z+2) + 2e^- \quad [0\nu\beta\beta \equiv (\beta\beta)_{0\nu}]$$

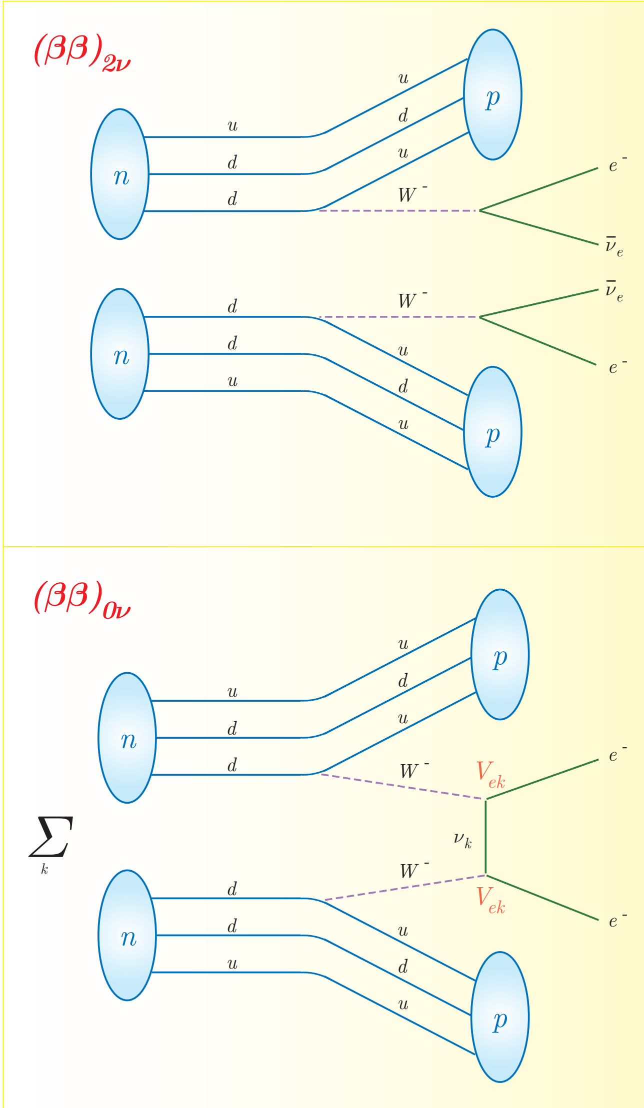
with $\Delta L = 2$. The decay rate for this process is expressed as follows:

$$\left[T_{1/2}^{0\nu}\right]^{-1} = G_Z^{0\nu} \left|m_{\beta\beta}\right|^2 \left|\mathcal{M}_{\mathsf{F}}^{0\nu} - (g_A/g_V)^2 \mathcal{M}_{\mathsf{GT}}^{0\nu}\right|^2,$$

where $G_Z^{0\nu}$ is the two-body phase-space factor including coupling constant, $\mathcal{M}_{\mathsf{F/GT}}^{0\nu}$ are the Fermi/Gamow-Teller nuclear matrix elements. The constants $g_V$ and $g_A$ are the vector and axial-vector relative weak coupling constants, respectively. The complex parameter $m_{\beta\beta}$ is the *effective Majorana electron neutrino mass* given by

$$m_{\beta\beta} = \sum_k V_{ek}^2 m_k = \sum_k |V_{ek}|^2 e^{i\phi_k} m_k$$
$$= |V_{e1}|^2 m_1 + |V_{e2}|^2 m_2 e^{i\phi_2} + |V_{e3}|^2 m_3 e^{i\phi_3}.$$

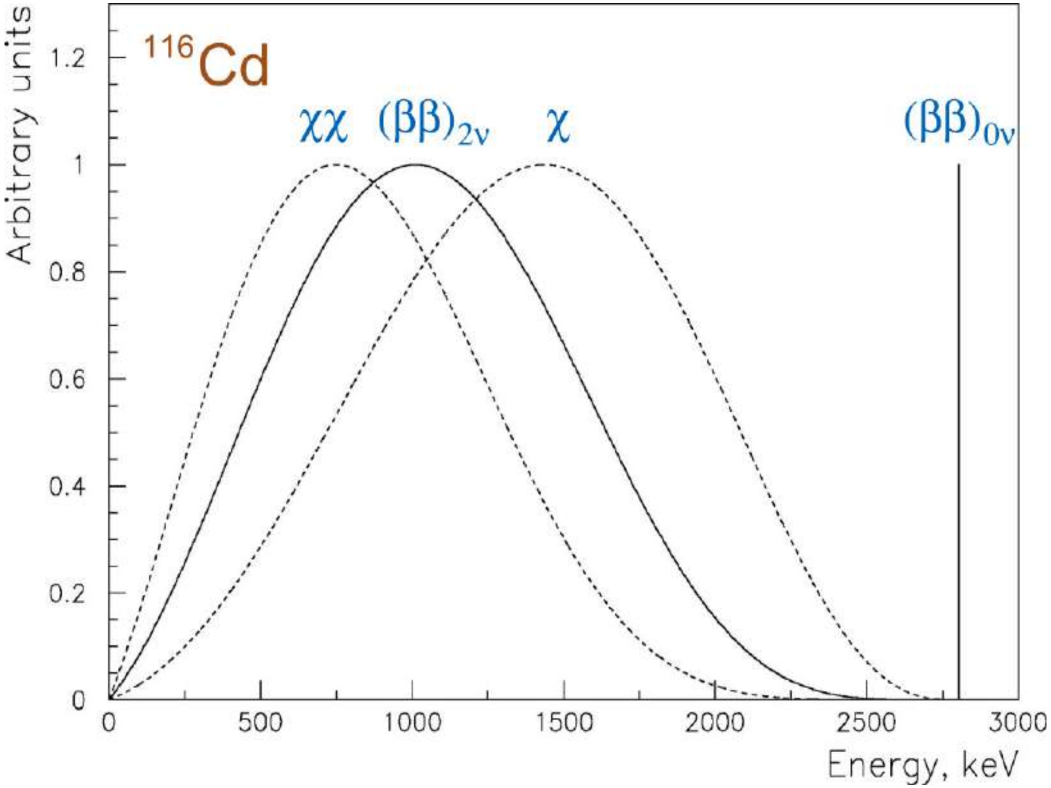
Here $\phi_1 = 0$, $\phi_2 = \alpha_2 - \alpha_1$ (pure Majorana phase) and $\phi_3 = -(\alpha_2 + 2\delta)$ (mixture of Dirac and Majorana $CP$-violation phases).



The electron sum energy spectrum of the $(\beta\beta)_{2\nu}$ mode as well as of the exotic modes with one or two majorons in final state,

$$(A, Z) \to (A, Z+2) + 2e^- + \chi,$$
$$(A, Z) \to (A, Z+2) + 2e^- + 2\chi,$$

is *continuous* because the available energy release ($Q_{\beta\beta}$) is shared between the electrons and other final state particles. In contrast, the two electrons from the $(\beta\beta)_{0\nu}$ decay carry the full available energy, and hence the electron sum energy spectrum has a sharp peak at the $Q_{\beta\beta}$ value. This feature allows one to distinguish the $(\beta\beta)_{0\nu}$ decay signal from the background.

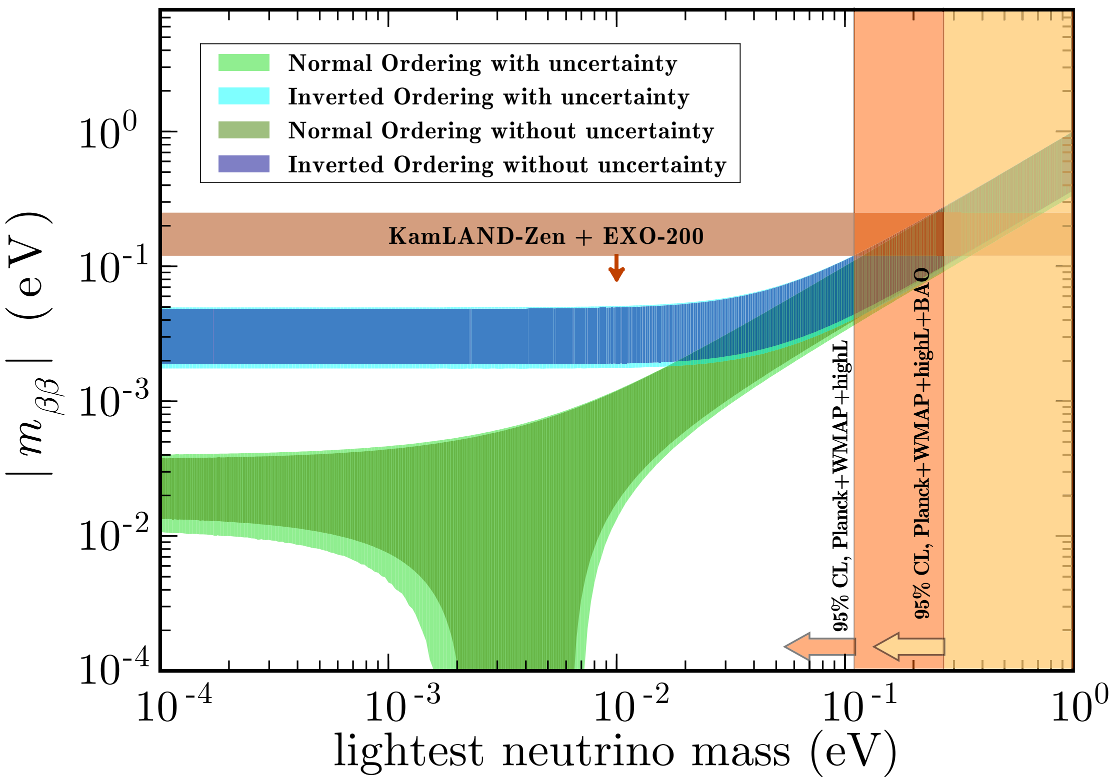



The electron sum energy spectra calculated for the different $\beta$ decay modes of cadmium-116.

[From Y. Zdesenko, "Colloquium: The future of double beta decay research," Rev. Mod. Phys. **74** (2003) 663–684.]

**Majoron** is a Nambu-Goldstone boson, – a hypothetical neutral pseudoscalar zero-mass particle which couples to Majorana neutrinos and may be emitted in the neutrinoless $\beta$ decay. It is a consequence of the spontaneous breaking of the global $B-L$ symmetry.

The currently allowed ranges of $m_{\beta\beta}$ observables of $0\nu\beta\beta$ decay is shown as a function of the lightest neutrino mass $m_0$. In the case of normal (inverted) mass ordering the ranges are shown by green (blue) color. The light (dark) colored regions are computed by taking into account (without taking account) the current $1\sigma$ uncertainties of the relevant mixing parameters.
Also shown are the limits on $m_{\beta\beta}$ coming from KamLAND-Zen and EXO-200 (by the light brown band and arrow) and the bounds on $m_0$ obtained by *Planck*.




Note that the "KamLAND-Zen+EXO 200" bound spans a broad band (rather than a line) because of the nuclear matrix element uncertainty.

It is remarkable that the effect of the $1\sigma$ uncertainties of the mixing parameters is quite small. In contrast, variation over the Majorana phases gives much larger impact on allowed region of $m_{\beta\beta}$, not only producing sizeable width but also creating a down-going branch at $10^{-3}$ eV $\lesssim m_0 \lesssim 10^{-2}$ eV for the case of the normal mass ordering due to the strong cancellation of the three mass terms.

[From H. Minakata, H. Nunokawa, and A. A. Quiroga, "Constraining Majorana CP phase in the precision era of cosmology and the double beta decay experiment," PTEP **2015** (2015) 033B03, arXiv:1402.6014 [hep-ph].]