Modification of AdS/CFT correspondence in top-down approach to near-throat D3-black brane geometry

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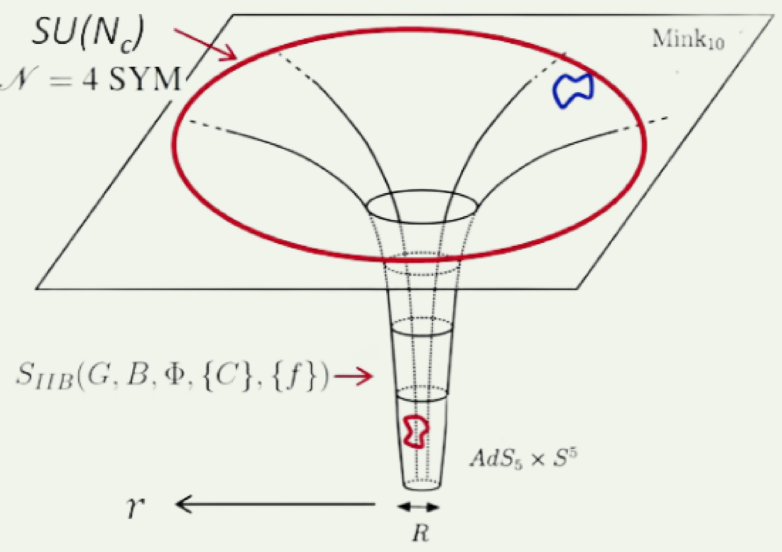
D3-black brane geometry and AdS/CFT ansatz



We can define the metric around the stack of D3-branes from equation of motion for type IIB SUGRA with a source:

$$S = S_{IIB} + T_3 \int d^4x + q \int d^4x \epsilon_{0\ldots3} C_{0\ldots3} + O(\alpha', G_N)$$

And solution for this equations looks like a funnel with infinite throat [1]:

$$ds^2 = f(r) dx^\beta dx_\beta + \frac{1}{f(r)}(dr^2 + r^2 d\Omega_5^2)$$

$$f(r) = \left(1 + R^4/r^4\right)^{-\frac{1}{2}}, \; R^4 = 4\pi g_s N \alpha'^2$$

When R denotes the AdS radius, N is a number of branes in stack, g_s and α' are string coupling constant and string scale resp., G_N is 10 dimensional Newton constant.

BPS saturation condition: This solution, also known as the black D3-brane, can be considered as a background for type IIB superstrings, when BPS saturation is fulfilled$\Rightarrow$ $T_3 = \frac{qN}{\sqrt{16\pi G_N}} = \frac{N}{(2\pi)^3 g_s \alpha'^2}$

Near-throat limit: in first order of r/R the metric is:

$$ds^2 = \frac{r^2}{R^2}(-dt^2 + d\bar{x}^2) + \frac{R^2}{r^2} dr^2 + R^2 d\Omega_5^2 = \underbrace{\frac{R^2}{z^2}(dz^2 + dx^\mu dx_\mu)}_{AdS_5} + \underbrace{R^2 d\Omega_5^2}_{S_5}; \quad r = \frac{R^2}{z}$$

Of interest for us here are the following constraints:

(I) SUGRA description

- $\rightarrow$ static D-branes: $r \gg \sqrt{\alpha'} \Leftrightarrow \sqrt{\alpha'} \rightarrow 0$;
- $\rightarrow$ semi-classical SUGRA (tree level): $g_s \rightarrow 0$;
- $\rightarrow$ exclusion of string winding around the throat:$\frac{R}{l_s} = (4\pi g_s N)^{1/4} \gg 1$; $2l_s^2 = \alpha'$ - characteristic string length.

(II) $\mathcal{N} = 4$ SYM description

- $\rightarrow$ coupling constant: $g_{ym}^2 = 4\pi g_s$;
- $\rightarrow$ t'Hooft constant: $\lambda = g_{ym}^2 N = 4\pi g_s N$;
- $\rightarrow$ planar limit: $N \rightarrow \infty$;
- $\rightarrow$ from gravity side: $g_s N \gg 1 \Leftrightarrow \lambda \rightarrow \infty$;

(III) Holographic identification of energy scale on the boundary and radial bulk coordinate: $E \propto 1/z$

- $\rightarrow$ rescaling of radial coordinate: $z \rightarrow z\frac{\alpha'}{R^2} \Leftrightarrow E = \frac{1}{z}$; $\partial AdS_5 : z = \frac{\alpha'}{R} = \epsilon \rightarrow 0$
- $\rightarrow$ Low energy limit $E\sqrt{\alpha'} \rightarrow 0$ [2]: string dynamics becomes more diverse when striving for a throat in the system of an infinitely distant observer

AdS/CFT ansatz: Formally, the relation $\mathcal{N}$=4 SYM in the limit of strong coupling with type IIB supergravity against the background of $AdS_5 \times S^5$ can be written as [3]:

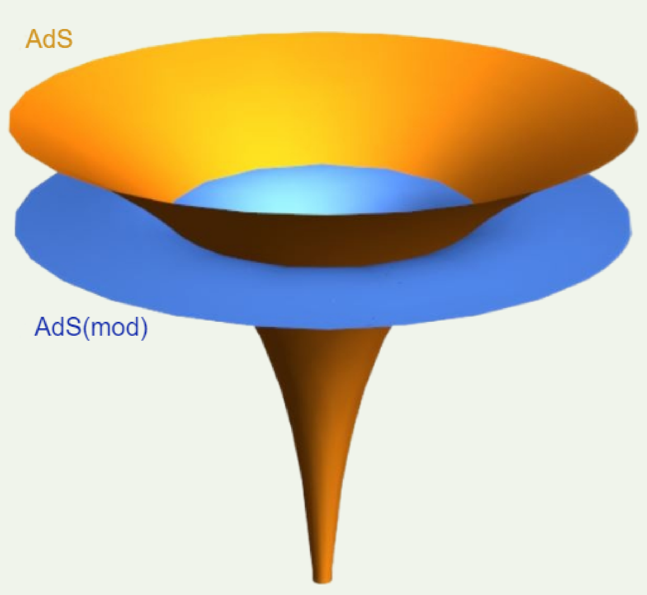
$$\left\langle exp\left\{-i\sum_k \int d^4x J_0^k O^k\right\}\right\rangle = exp\left\{-iS_{IIB}[AdS_5 \times S^5]|_{J^k(\epsilon)=J_0^k}\right\}$$

The mean in the left part is calculated in the strongly coupled $\mathcal{N}$=4 SYM in the limit $N \rightarrow \infty$, while $\{O^i\}$ has a complete set of local operators in this theory. The action on the right side is a type IIB supergravity action on $AdS_5 \times S^5$, it is minimal on classical solutions of J^k for all fields included in it. These classical solutions take the values $J^k = J_0^k$ on a four-dimensional hypersurface $z = \epsilon$ and must have polynomial behavior when aiming $\epsilon \rightarrow 0$. At the same time, the values of J_0 serve as sources for the left side of this ratio.

Modification in the second order by r/R

The near-throat metric of the black D3 brane in the second order in r/R has the form:

$$ds^2 = \underbrace{\frac{R^2}{z^2}\left(dx^\mu dx_\mu + \left(1 + \frac{R^4}{2z^4}\right)dz^2\right)}_{AdS_5(m)} + \underbrace{R^2\left(1 + \frac{R^4}{2z^4}\right)d\Omega_5^2}_{S^5}$$



AdS_5 limit: It is simply to see that $AdS_5(m) \times S^5$ is a minimal extension of $AdS_5 \times S^5$ with both flat multipliers on the boundary $z \rightarrow 0$:

$$\mathcal{R}_{S^5}(\partial) = \mathcal{R}_{AdS_5(m)}(\partial) = \mathcal{R}_{AdS_5(m)\times S^5}(\partial) = 0$$

where R is a Ricci-scalar, and ∂ is denoted by the conformal boundary $z \rightarrow 0$. On the other hand, it is obvious that there is a trivial limit $AdS_5(m) \rightarrow AdS_5$ for $z \rightarrow \infty$. It follows that our approach leads to a UV modification of the theory at the boundary

S^5 reduction: Standard method of KK gravity reduction:

$$\frac{1}{k^2}\int d^{10}x\sqrt{-g_{10}}\mathcal{R}_{10} = \frac{V_5}{k^2}\int d^5x\sqrt{-g_5}\mathcal{R}_5$$

When $V_5 = \pi^3 R^5$ is S^5 volume, $k^2 = 16\pi G_N$ and one can show that:

$$G_{N_5} = \frac{G_N}{V_5}, \quad S = \frac{1}{16\pi G_{N_5}}\int d^5x(\mathcal{L}_{gr} + \mathcal{L}_{matter})$$

But in fact we can't do S^5 reduction on $AdS_5(m) \times S^5$ $\rightarrow$ There is non-trivial "gravity faze" and supersymmetry $\mathcal{N} = 4 \rightarrow \mathcal{N} = 1$ reduction for the boundary theory. For these reasons we consider only on a zero- mode in field expansion: $\Phi(x^\mu, z, \Omega_5) = \sum_l \Phi_l(x^\mu, z)Y_l(\Omega_5)$.

Scalar field EoM: If we introduce a convenient designation $B(z) = \sqrt{1 + \frac{R^4}{2z^4}}$ and doing 4D Fourier transform $\phi(x_\mu, z) = \int \frac{d^4k}{(2\pi)^4}\Phi_k(z)\phi(k_\mu)e^{ikx}$, then we can represent (for simplicity, considering a scalar field) the equation of motion in the following Schrödinger form:

$$-\Psi_k''(z) + V(z)\Psi_k(z) = k^2B^2(z)\Psi_k(z), \quad \Phi_k(z) = \left(\frac{R^3}{z^3B(z)}\right)^{-1/2}\Psi_k(z)$$

Field equation factorization

The potential in the previous equation has the form

$$V(z) = \frac{3z^2B'(z)^2 - 2zB(z)\left(zB''(z) - 3B'(z)\right) + 4m_5^2R^2B(z)^4 + 15B(z)^2}{4z^2B(z)^2}$$

And for $B(z) \rightarrow 1$ there is a famous GPKW solution [4] : $\psi_k^{GPKW}(z \rightarrow \infty) = \frac{z^{1/2}K_\nu(kz)}{R^{1/2}K_\nu(kR)}$

when the order of modified Bessel function K_ν is $\nu = \sqrt{4 + (m_5R)^2}$

A rather unexpected possibility of explicit factorization of the correction part

Using the following transformation, one can get reduce of the nontrivial spectral density $B(z)$ in the equations of motion:

$$x(z) = \int_R^z \sqrt{1 + \frac{R^4}{2t^4}}dt\Big|_{R\rightarrow 0(4)} \approx -\frac{R^4}{12z^3} - \frac{11R}{12} + z,$$

$$\psi_k(x(z)) = \sqrt{B(z)}\Psi_k(z),$$

$$Q(x(z)) = \frac{V(z(x))}{B^2(z(x))} - \frac{\left(B^{-1/2}(z(x))\right)''}{B^{3/2}(z(x))} = \frac{6zB'(z) + 4m_5^2R^2B(z)^3 + 15B(z)}{4z^2B(z)^3}.$$

when $x(z)$ can be easily approximated without an elliptic functions up to 4 order with rescaling $R \rightarrow \alpha'$.

After using this transformation one can show that:

- $\rightarrow$ We can clearly identify the potential for GPKW solution:

$$Q(x) = Q_0(x) + \frac{6zB'(z) + 15B(z)(1 - B^2(z))}{4z^2B^3(z)}$$

$$Q_0(x) = \frac{m_5^2R^2 + \frac{15}{4}}{z(x)^2} \Rightarrow \psi_k^{GPKW}(z(x))$$

- $\rightarrow$ Resulting solution can be constructed in the factorized form:

$$\boxed{\Phi_k(z) = \sqrt{\frac{z^3}{R^3}}\psi_k^{GPKW}(x(z))\chi_k(x(z))}$$

when $\chi_k(x)$ is a correction and $x(z = \epsilon) = 0$ on the boundary.

- $\rightarrow$ The behavior of the GPSW solution at the boundary is well defined:

$$\psi_k^{GPKW}(x(z))|_{x\rightarrow 0} \approx \left(\frac{5}{4}\right)^{\Delta_-}(z - \epsilon)^{\Delta_-}$$

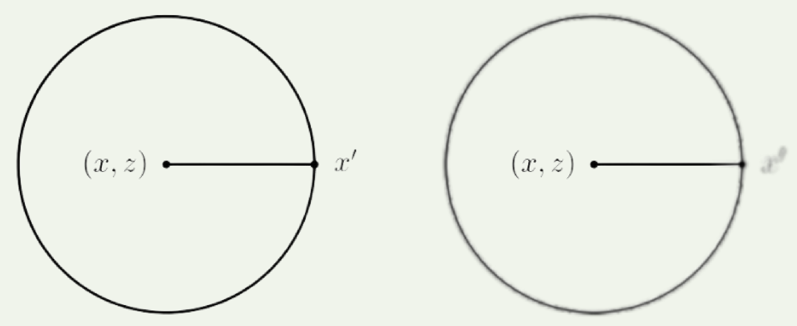
when $\Delta_\pm = 2 \pm \nu$ the dimension of the corresponding SYM operator (scaling exponents).

- $\rightarrow$ EoM for the correction has the form:

$$\boxed{\begin{aligned} &-\chi_k'' - 2\chi_k'\frac{\psi_k'}{\psi_k} + \hat{Q}\chi_k = 0 \\ &\hat{Q} = \frac{6zB'(z) + 15B(z)(1 - B^2(z))}{4z^2B^3(z)} \end{aligned}}$$

Boundary conditions for corrections: $\chi_k(\infty) = 1$ (deep in the throat) and $\chi_k(0) > 1$ (on the boundary).

Geometric interpretation: smeared Witten diagrams



The complete solution of the equations of motion we can write now in the form:

$$\phi(z, x) = \epsilon^{\Delta_-}\left(\frac{z}{\epsilon}\right)^2\int \frac{d^4k}{(2\pi)^4}\chi_k(x(z))\frac{K_\nu(kz)}{K_\nu(k\epsilon)}$$

Then the boundary-bulk propagator for a point source located at the boundary will be modified as:

$$K_m(z; x^\mu, \tilde{x}^\mu) = \epsilon^{\Delta_-}\left(\frac{z}{\epsilon}\right)^2\int \frac{d^4k}{(2\pi)^4}\chi_k(x(z))\frac{K_\nu(kz)}{K_\nu(k\epsilon)}e^{ik_\mu(x^\mu - \tilde{x}^\mu)}.$$

It seems like a smeared Witten diagrams, with "momentum-blure" of point sources on the boundary. Using a boundary-bulk propagator, it is also possible to investigate the blurring of bulk-bulk propagators by the ratio:

$$K_m(z; x^\mu, \tilde{x}^\mu) = \epsilon^3 \lim_{z'\rightarrow\epsilon} z'(z')^{-\Delta_+}\frac{1}{B(z')}\partial_{z'}G_m(z, x; z', x')$$

Correlation function modification

$\mathcal{N} = 4$ SYM	IIB supergravity
Lagrangian: $\mathcal{L}_{\mathcal{N}=4}$	dilaton: Φ
$SO(6)$: J_μ^a	g. field red. from S^5: A_μ^a
stress tensor: $T^{\mu\nu}$	metric pert.: h_{MN}

- $\rightarrow$ S-wave dilaton solution interact with $\mathcal{N} = 4SYM$ Lagrangian :

$$O_{\Phi,0}(x) \sim \mathcal{L}_{SYM}[\{h\}, x]$$

- $\rightarrow$ The action for the general solution is written in the form:

$$S^{min} \sim N^2 \int d^4xd^4y\frac{e^{ik(x+y)}\chi_k^2(0)}{(\epsilon^2 + |x - y|^2)^4}$$

- $\rightarrow$ Then the modification of the correlator of two such primitive operators will be:

$$\langle O_{\Phi,0}(x)O_{\Phi,0}(y)\rangle_m \sim \frac{N^2H[\chi_k^2(0)]}{(\epsilon^2 + |x - y|^2)^4}$$

$H[\chi_k^2(0)] < 0$ there is some functional, that could be found after a full solving of correction function. Correction in correlator means that we can define a non-planar extension of our boundary theory.

Conclusions

We formulated EoM upto second order R/z for the AdS_5 modification for s-wave dilaton. Also shown is the possibility to determine the supersymmetry breaking in this model. We are going to consider the problem in our future research. The direct factorization of corrections to the solution against the background of AdS_5 is performed. We consider EoM for the corrections and suggested the modification of the bulk-bulk and bulk-boundary propagators. Besides, the correlator of two primary operators related to the Lagrangian $\mathcal{N} = 4SYM$ is obtained. The method can be identified with the non-planar correction for $\mathcal{N} = 4SYM$.

References

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