Moscow International School of Physics 2022

# Canonical description for formulation of embedding gravity as General Relativity with additional matter

Taisiia Zaitseva Supervisor: Sergey Paston

Saint Petersburg University

28 July 2022



Embedding theory: approach to describe gravity.



embedding function y<sup>a</sup>(x<sup>μ</sup>) : M → ℝ<sup>N<sub>+</sub>,N<sub>−</sub>
μ = 0, 1, 2, 3 a = 0, 1, ..., N − 1
ambient space metric η<sub>ab</sub> is flat
surface metric g<sub>μν</sub> is induced: g<sub>μν</sub> = (∂<sub>μ</sub>y<sup>a</sup>)(∂<sub>ν</sub>y<sup>b</sup>)η<sub>a</sub>
</sup>

Einstein-Hilbert action:  $S = \int d^4x \sqrt{-g} (\frac{R}{2\varkappa} + \mathcal{L}_m)$ ndependent variable:  $g_{\mu\nu}$  (OTO)  $\rightarrow y^a(x)$  (embedding theory) Equations of motion (Regge-Teitelboim equations):  $D_{\mu} ((G^{\mu\nu} - \varkappa T^{\mu\nu})\partial_{\nu}y^a)$ 

Embedding theory: approach to describe gravity.

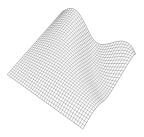


embedding function y<sup>a</sup>(x<sup>μ</sup>) : M → ℝ<sup>N<sub>+</sub>,N<sub>-</sub>
μ = 0, 1, 2, 3 a = 0, 1, ..., N - 1
ambient space metric η<sub>ab</sub> is flat
surface metric g<sub>μν</sub> is induced: g<sub>μν</sub> = (∂<sub>μ</sub>y<sup>a</sup>)(∂<sub>ν</sub>y<sup>b</sup>)η<sub>ab</sub>
</sup>

Einstein-Hilbert action:  $S = \int d^*x \sqrt{-g(\frac{X}{2\varkappa} + \mathcal{L}_m)}$ Independent variable:  $g_{\mu\nu}$  (OTO)  $\rightarrow y^a(x)$  (embedding theory)

Equations of motion (Regge-Teitelboim equations):  $D_\muig((G^{\mu
u}-arkappa T^{\mu
u})\partial_
u y^aig)=0$ 

Embedding theory: approach to describe gravity.



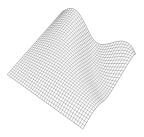
embedding function y<sup>a</sup>(x<sup>μ</sup>) : M → ℝ<sup>N<sub>+</sub>,N<sub>-</sub></sub>
μ = 0, 1, 2, 3 a = 0, 1, ..., N - 1
ambient space metric η<sub>ab</sub> is flat
surface metric g<sub>μν</sub> is induced: g<sub>μν</sub> = (∂<sub>μ</sub>y<sup>a</sup>)(∂<sub>ν</sub>y<sup>b</sup>)η<sub>ab</sub>
</sup>

Einstein-Hilbert action:  $S = \int d^4x \sqrt{-g} (\frac{R}{2\varkappa} + \mathcal{L}_m)$ 

Independent variable:  $g_{\mu\nu}$  (OTO)  $\rightarrow y^a(x)$  (embedding theory)

Equations of motion (Regge-Teitelboim equations):  $D_{\mu} ((G^{\mu
u} - \varkappa T^{\mu
u}) \partial_{
u} y^a) = 0$ 

Embedding theory: approach to describe gravity.



embedding function y<sup>a</sup>(x<sup>μ</sup>) : M → ℝ<sup>N<sub>+</sub>,N<sub>-</sub>
μ = 0, 1, 2, 3 a = 0, 1, ..., N - 1
ambient space metric η<sub>ab</sub> is flat
surface metric g<sub>μν</sub> is induced: g<sub>μν</sub> = (∂<sub>μ</sub>y<sup>a</sup>)(∂<sub>ν</sub>y<sup>b</sup>)η<sub>ab</sub>
</sup>

Einstein-Hilbert action:  $S = \int d^4x \sqrt{-g} (\frac{R}{2\varkappa} + \mathcal{L}_m)$ Independent variable:  $g_{\mu\nu}$  (OTO)  $\rightarrow y^a(x)$  (embedding theory)

Embedding theory: approach to describe gravity.



embedding function y<sup>a</sup>(x<sup>μ</sup>) : M → ℝ<sup>N<sub>+</sub>,N<sub>-</sub></sub>
μ = 0, 1, 2, 3 a = 0, 1, ..., N - 1
ambient space metric η<sub>ab</sub> is flat
surface metric g<sub>μν</sub> is induced: g<sub>μν</sub> = (∂<sub>μ</sub>y<sup>a</sup>)(∂<sub>ν</sub>y<sup>b</sup>)η<sub>ab</sub>
</sup>

Einstein-Hilbert action:  $S = \int d^4x \sqrt{-g} (\frac{R}{2\varkappa} + \mathcal{L}_m)$ Independent variable:  $g_{\mu\nu}$  (OTO)  $\rightarrow y^a(x)$  (embedding theory) Equations of motion (Regge-Teitelboim equations):  $D_{\mu} ((G^{\mu\nu} - \varkappa T^{\mu\nu})\partial_{\nu}y^a) = 0$ 

$$D_{\mu}\Big((G^{\mu
u}-arkappa T^{\mu
u})\partial_{
u}y^{a}\Big)=0$$

We introduce the notation  $\varkappa au^{\mu
u} = (G^{\mu
u} - \varkappa T^{\mu
u})$ , then

• Einstein equations with the contribution of some additional (fictitious) matter with the energy-momentum tensor  $\tau^{\mu\nu}$ ::

$$G^{\mu
u}-arkappa(T^{\mu
u}+ au^{\mu
u})=0;$$

$$D_{\mu}(\tau^{\mu\nu}\partial_{\nu}y^{a})=0.$$

$$D_{\mu}\Big((G^{\mu
u}-arkappa T^{\mu
u})\partial_{
u}y^{a}\Big)=0$$

#### We introduce the notation $\varkappa \tau^{\mu \nu} = (G^{\mu \nu} - \varkappa T^{\mu \nu})$ , then

• Einstein equations with the contribution of some additional (fictitious) matter with the energy-momentum tensor  $\tau^{\mu\nu}$  ::

$$G^{\mu\nu} - \varkappa (T^{\mu\nu} + \tau^{\mu\nu}) = 0;$$

$$D_{\mu}(\tau^{\mu\nu}\partial_{\nu}y^{a})=0.$$

$$D_{\mu}\Big((G^{\mu
u}-arkappa T^{\mu
u})\partial_{
u}y^{a}\Big)=0$$

We introduce the notation  $\varkappa \tau^{\mu \nu} = (G^{\mu \nu} - \varkappa T^{\mu \nu})$ , then

• Einstein equations with the contribution of some additional (fictitious) matter with the energy-momentum tensor  $\tau^{\mu\nu}$ ::

$$G^{\mu
u}-arkappa(T^{\mu
u}+ au^{\mu
u})=$$
0;

$$D_{\mu}(\tau^{\mu\nu}\partial_{\nu}y^{a})=0.$$

$$D_{\mu}\Big((G^{\mu
u}-arkappa T^{\mu
u})\partial_{
u}y^{a}\Big)=0$$

We introduce the notation  $\varkappa \tau^{\mu \nu} = (G^{\mu \nu} - \varkappa T^{\mu \nu})$ , then

• Einstein equations with the contribution of some additional (fictitious) matter with the energy-momentum tensor  $\tau^{\mu\nu}$ ::

$$G^{\mu
u}-arkappa(T^{\mu
u}+ au^{\mu
u})=0;$$

$$D_{\mu}(\tau^{\mu\nu}\partial_{\nu}y^{a})=0.$$

Reformulating the embedding theory as GR with embedding matter at the level of action

The equivalence between embedding theory and GR with fictitious embedding matter at the level of equations of motion:

$$\begin{array}{c} D_{\mu} \Big( (G^{\mu\nu} - \varkappa T^{\mu\nu}) \partial_{\nu} y^{a} \Big) = 0 \end{array} \Leftrightarrow \begin{array}{c} G^{\mu\nu} - \varkappa (T^{\mu\nu} + \tau^{\mu\nu}) = 0 \\ D_{\mu} \big( \tau^{\mu\nu} \partial_{\nu} y^{a} \big) = 0 \end{array}$$

The same equivalence at the level of action:

$$S[y^{a}] = \frac{1}{2\varkappa} \int d^{4}x \sqrt{-g}R \Leftrightarrow S = S^{\mathsf{EH}} + S_{\mathsf{m}} + S^{\mathsf{add}}$$
$$S^{\mathsf{add}} = ?$$
$$S^{\mathsf{add}} = \frac{1}{2} \int d^{4}x \sqrt{-g} \left(g_{\mu\nu} - (\partial_{\mu}y^{a})(\partial_{\nu}y_{a})\right) \tau^{\mu\nu}$$

Reformulating the embedding theory as GR with embedding matter at the level of action

The equivalence between embedding theory and GR with fictitious embedding matter at the level of equations of motion:

$$D_{\mu}\Big((G^{\mu
u} - \varkappa T^{\mu
u})\partial_{
u}y^{a}\Big) = 0 \ \Leftrightarrow \ egin{array}{c} G^{\mu
u} - \varkappa (T^{\mu
u} + au^{\mu
u}) = 0 \ D_{\mu}ig( au^{\mu
u}\partial_{
u}y^{a}ig) = 0 \end{array}$$

The same equivalence at the level of action:

$$S[y^{a}] = \frac{1}{2\varkappa} \int d^{4}x \sqrt{-g}R \Leftrightarrow S = S^{\mathsf{EH}} + S_{\mathsf{m}} + S^{\mathsf{add}}$$
$$S^{\mathsf{add}} = ?$$
$$S^{\mathsf{add}} = \frac{1}{2} \int d^{4}x \sqrt{-g} \left(g_{\mu\nu} - (\partial_{\mu}y^{a})(\partial_{\nu}y_{a})\right) \tau^{\mu\nu}$$

Reformulating the embedding theory as GR with embedding matter at the level of action

The equivalence between embedding theory and GR with fictitious embedding matter at the level of equations of motion:

$$D_{\mu}\Big((G^{\mu
u} - \varkappa T^{\mu
u})\partial_{
u}y^{a}\Big) = 0 \ \Leftrightarrow \ egin{array}{c} G^{\mu
u} - \varkappa (T^{\mu
u} + au^{\mu
u}) = 0 \ D_{\mu}ig( au^{\mu
u}\partial_{
u}y^{a}ig) = 0 \end{array}$$

The same equivalence at the level of action:

$$S[y^{a}] = \frac{1}{2\varkappa} \int d^{4}x \sqrt{-g}R \Leftrightarrow S = S^{\mathsf{EH}} + S_{\mathsf{m}} + S^{\mathsf{add}}$$
$$S^{\mathsf{add}} = ?$$
$$S^{\mathsf{add}} = \frac{1}{2} \int d^{4}x \sqrt{-g} \Big( g_{\mu\nu} - (\partial_{\mu}y^{a})(\partial_{\nu}y_{a}) \Big) \tau^{\mu\nu}$$

Arnowitt-Deser-Mizner variables (ADM):

$$\beta_{ik} = g_{ik}, \qquad N_k = g_{0k}, \qquad N = \frac{1}{\sqrt{-g^{00}}}.$$

The ADM action:

$$S^{\text{ADM}} = \int d^4 x \Big( 2N K_{ik} L^{ik,lm} K_{lm} + \frac{1}{2\varkappa} N \sqrt{\beta} R^3 \Big).$$

Notation:

$$\begin{split} & \mathcal{K}_{ik} = \frac{1}{2N} \Big( \overset{3}{D}_{i} N_{k} + \overset{3}{D}_{k} N_{i} - \partial_{0} \beta_{ik} \Big); \\ & \mathcal{L}^{ik,lm} = \frac{\sqrt{\beta}}{8\varkappa} \Big( \beta^{il} \beta^{km} + \beta^{im} \beta^{kl} - 2\beta^{ik} \beta^{lm} \Big); \\ & \overline{\mathcal{L}}_{ik,lm} = \frac{2\varkappa}{\sqrt{\beta}} \Big( \beta_{il} \beta_{km} + \beta_{im} \beta_{kl} - \beta_{ik} \beta_{lm} \Big). \end{split}$$

Arnowitt-Deser-Mizner variables (ADM):

$$\beta_{ik} = g_{ik}, \qquad N_k = g_{0k}, \qquad N = \frac{1}{\sqrt{-g^{00}}}.$$

The ADM action:

$$S^{\text{ADM}} = \int d^4x \Big( 2NK_{ik}L^{ik,lm}K_{lm} + \frac{1}{2\varkappa}N\sqrt{\beta}R^3 \Big).$$

Notation:

$$\begin{split} & \mathcal{K}_{ik} = \frac{1}{2N} \Big( \overset{3}{D}_{i} \mathcal{N}_{k} + \overset{3}{D}_{k} \mathcal{N}_{i} - \partial_{0} \beta_{ik} \Big); \\ & \mathcal{L}^{ik,lm} = \frac{\sqrt{\beta}}{8\varkappa} \Big( \beta^{il} \beta^{km} + \beta^{im} \beta^{kl} - 2\beta^{ik} \beta^{lm} \Big); \\ & \overline{\mathcal{L}}_{ik,lm} = \frac{2\varkappa}{\sqrt{\beta}} \Big( \beta_{il} \beta_{km} + \beta_{im} \beta_{kl} - \beta_{ik} \beta_{lm} \Big). \end{split}$$

From  $\tau^{\mu\nu}$  to  $\phi$ ,  $\phi^k$ ,  $\phi^{ij}$ :

$$\begin{split} \phi &= -\frac{1}{2} N^2 \sqrt{\beta} \tau^{00}; \\ \phi^k &= -N \sqrt{\beta} \tau^{k0} - \frac{\phi}{N} (N^k + \beta^{ik} e^b_i e_{b0}); \\ \phi^{ij} &= -\frac{1}{2} N \sqrt{\beta} \tau^{ij} + \beta^{ik} \overline{\beta}^{jm} \frac{\phi}{N} e^b_k e^b_m e_{0a} e_{0b}. \end{split}$$

Notation:

$$e^{a}_{\mu} = \partial_{\mu}y^{a};$$
  
 $\overline{\beta}_{ik} = e^{a}_{i}e_{ak},$ 

$$S^{\text{add}} = \int d^4 x \left( (\overline{\beta}_{ij} - \beta_{ij}) \phi^{ij} + (e_i^a e_{a0} - N_i) \phi^i + \left( N + \frac{1}{N} e_{a0} \overset{\mathfrak{a}}{\Pi}^{ab}_{\perp} e_{b0} \right) \phi \right),$$

Here  $\Pi_{\perp b}^{a} = \delta_{b}^{a} - e_{i}^{a} e_{bj}\overline{\beta}^{ij}$ .

From  $\tau^{\mu\nu}$  to  $\phi$ ,  $\phi^k$ ,  $\phi^{ij}$ :

$$\begin{split} \phi &= -\frac{1}{2} N^2 \sqrt{\beta} \tau^{00}; \\ \phi^k &= -N \sqrt{\beta} \tau^{k0} - \frac{\phi}{N} (N^k + \beta^{ik} e^b_i e_{b0}); \\ \phi^{ij} &= -\frac{1}{2} N \sqrt{\beta} \tau^{ij} + \beta^{ik} \overline{\beta}^{jm} \frac{\phi}{N} e^a_k e^b_m e_{0a} e_{0b}. \end{split}$$

Notation:

$$e^a_\mu = \partial_\mu y^a;$$
  
 $\overline{\beta}_{ik} = e^a_i e_{ak}.$ 

$$\boxed{S^{\mathsf{add}} = \int d^4 x \left( (\overline{\beta}_{ij} - \beta_{ij}) \phi^{ij} + (e^a_i e_{a0} - N_i) \phi^i + \left( N + \frac{1}{N} e_{a0} \overset{\mathfrak{s}_{ab}}{\Pi^{ab}_{\perp}} e_{b0} \right) \phi \right),}$$

Here  $\Pi^{\mathbf{3}}_{\perp b} = \delta^{\mathbf{a}}_{b} - e^{\mathbf{a}}_{i} e_{bj} \overline{\beta}^{ij}$ .

Momenta corresponding to the ADM variables  $\beta_{ik}$ ,  $N_k$ , N:

$$\pi^{ik} = \frac{\delta S}{\delta \dot{\beta}_{ik}} = -2L^{ik,lm} K_{lm}, \qquad \pi_N^k = 0, \qquad \pi_N = 0.$$
$$\Phi = \pi_N \approx 0 \qquad \Phi^k = \pi_N^k \approx 0$$

Momenta corresponding to  $\phi$ ,  $\phi^i$ ,  $\phi^{ik}$ :

$$\Psi=\pi^{\phi}pprox 0$$
  $\Psi_{i}=\pi^{\phi}_{i}pprox 0$   $\Psi_{ik}=\pi^{\phi}_{ik}pprox 0$ 

$$p_{a} = \frac{\delta S}{\delta \dot{y}^{a}} = \phi^{k} e_{ak} + \frac{2\phi}{N} \mathring{\Pi}_{\perp a}^{b} \dot{y}_{b}.$$
$$\Omega_{j} = p_{a} e_{j}^{a} - \phi^{k} \overline{\beta}_{kj} \approx 0$$

Momenta corresponding to the ADM variables  $\beta_{ik}$ ,  $N_k$ , N:

$$\pi^{ik} = \frac{\delta S}{\delta \dot{\beta}_{ik}} = -2L^{ik,lm} \mathcal{K}_{lm}, \qquad \pi^k_N = 0, \qquad \pi_N = 0.$$
$$\Phi = \pi_N \approx 0 \qquad \Phi^k = \pi^k_N \approx 0$$

Momenta corresponding to  $\phi$ ,  $\phi^i$ ,  $\phi^{ik}$ :

$$\Psi=\pi^{\phi}pprox 0$$
  $\Psi_{i}=\pi^{\phi}_{i}pprox 0$   $\Psi_{ik}=\pi^{\phi}_{ik}pprox 0$ 

$$p_{a} = \frac{\delta S}{\delta \dot{y}^{a}} = \phi^{k} e_{ak} + \frac{2\phi}{N} \overset{a}{\Pi}_{\perp a} \overset{b}{\dot{y}_{b}}.$$
$$\Omega_{j} = p_{a} e_{j}^{a} - \phi^{k} \overline{\beta}_{kj} \approx 0$$

Momenta corresponding to the ADM variables  $\beta_{ik}$ ,  $N_k$ , N:

$$\pi^{ik} = \frac{\delta S}{\delta \dot{\beta}_{ik}} = -2L^{ik,lm} \mathcal{K}_{lm}, \qquad \pi_N^k = 0, \qquad \pi_N = 0.$$
$$\Phi^k = \pi_N^k \approx 0$$

Momenta corresponding to  $\phi$ ,  $\phi^i$ ,  $\phi^{ik}$ :

$$\Psi = \pi^{\phi} pprox 0$$
  $\Psi_i = \pi^{\phi}_i pprox 0$   $\Psi_{ik} = \pi^{\phi}_{ik} pprox 0$ 

$$p_{a} = \frac{\delta S}{\delta \dot{y}^{a}} = \phi^{k} e_{ak} + \frac{2\phi}{N} \vec{\Pi}_{\perp a}^{\ b} \dot{y}_{b}.$$
$$\Omega_{j} = p_{a} e_{j}^{a} - \phi^{k} \overline{\beta}_{kj} \approx 0$$

Momenta corresponding to the ADM variables  $\beta_{ik}$ ,  $N_k$ , N:

$$\pi^{ik} = \frac{\delta S}{\delta \dot{\beta}_{ik}} = -2L^{ik,lm} \mathcal{K}_{lm}, \qquad \pi_N^k = 0, \qquad \pi_N = 0.$$
$$\Phi^k = \pi_N^k \approx 0$$

Momenta corresponding to  $\phi$ ,  $\phi^i$ ,  $\phi^{ik}$ :

$$p_{a} = \frac{\delta S}{\delta \dot{y}^{a}} = \phi^{k} e_{ak} + \frac{2\phi}{N} \vec{\Pi}_{\perp a}^{\ b} \dot{y}_{b}.$$

Momenta corresponding to the ADM variables  $\beta_{ik}$ ,  $N_k$ , N:

$$\pi^{ik} = \frac{\delta S}{\delta \dot{\beta}_{ik}} = -2L^{ik,lm} K_{lm}, \qquad \pi^k_N = 0, \qquad \pi_N = 0.$$
$$\Phi^k = \pi^k_N \approx 0$$

Momenta corresponding to  $\phi$ ,  $\phi^i$ ,  $\phi^{ik}$ :

$$p_{a} = \frac{\delta S}{\delta \dot{y}^{a}} = \phi^{k} e_{ak} + \frac{2\phi}{N} \vec{\Pi}_{\perp a}^{b} \dot{y}_{b}.$$
$$\Omega_{j} = p_{a} e_{j}^{a} - \phi^{k} \overline{\beta}_{kj} \approx 0$$

Momenta corresponding to the ADM variables  $\beta_{ik}$ ,  $N_k$ , N:

$$\pi^{ik} = \frac{\delta S}{\delta \dot{\beta}_{ik}} = -2L^{ik,lm} K_{lm}, \qquad \pi^k_N = 0, \qquad \pi_N = 0.$$
$$\Phi^k = \pi^k_N \approx 0$$

Momenta corresponding to  $\phi$ ,  $\phi^i$ ,  $\phi^{ik}$ :

$$p_{a} = \frac{\delta S}{\delta \dot{y}^{a}} = \phi^{k} e_{ak} + \frac{2\phi}{N} \mathring{\Pi}_{\perp a}^{b} \dot{y}_{b}.$$
$$\Omega_{j} = p_{a} e_{j}^{a} - \phi^{k} \overline{\beta}_{kj} \approx 0$$

#### ${\cal H}=\pi^{ik}\dot{eta}_{ik}+{\it p}_a\dot{y}^a-{\cal L}+{ m primary}$ constraints with Lagrange multipliers

Hamiltonian density  ${\mathcal H}$  can be conveniently broken down into two terms

$$\mathcal{H} = \mathcal{H}^{\mathsf{ADM}} + \mathcal{H}^{\mathsf{add}},$$

$$\mathcal{H}^{\mathsf{ADM}} = \frac{1}{2} N \pi^{ik} \overline{L}_{ik,lm} \pi^{lm} + \pi^{ik} \left( \overset{\mathfrak{s}}{D}_i N_k + \overset{\mathfrak{s}}{D}_k N_i \right) - \frac{1}{2\varkappa} N \sqrt{\beta} \overset{\mathfrak{s}}{R} + \lambda \Phi + \lambda_k \Phi^k,$$

$$\mathcal{H}^{add} = rac{N}{4\phi} \rho_a \mathring{\Pi}^{ab}_{\perp} \rho_b + \phi^{ij} (\beta_{ij} - \overline{\beta}_{ij}) + \rho_a e^a_i \overline{\beta}^{ik} N_k - \phi N + \chi \Psi + \chi^i \Psi_i + \chi^{ij} \Psi_{ij} + \xi^i \Omega_i.$$

 ${\cal H}=\pi^{ik}\dot{eta}_{ik}+{\it p}_a\dot{y}^a-{\cal L}+{
m primary}$  constraints with Lagrange multipliers

Hamiltonian density  ${\mathcal H}$  can be conveniently broken down into two terms

 $\mathcal{H} = \mathcal{H}^{\mathsf{ADM}} + \mathcal{H}^{\mathsf{add}},$ 

$$\mathcal{H}^{\text{ADM}} = \frac{1}{2} N \pi^{ik} \overline{L}_{ik,lm} \pi^{lm} + \pi^{ik} \left( \overset{\mathfrak{s}}{D}_{i} N_{k} + \overset{\mathfrak{s}}{D}_{k} N_{i} \right) - \frac{1}{2\varkappa} N \sqrt{\beta} \overset{\mathfrak{s}}{R} + \lambda \Phi + \lambda_{k} \Phi^{k},$$

$$\mathcal{H}^{\mathsf{add}} = rac{N}{4\phi} p_a \overset{\mathfrak{s}}{\Pi}^{ab}_{\perp} p_b + \phi^{ij} (eta_{ij} - \overline{eta}_{ij}) + p_a e^a_i \overline{eta}^{ik} N_k - \phi N + \chi \Psi + \chi^i \Psi_i + \chi^{ij} \Psi_{ij} + \xi^i \Omega_i.$$

 ${\cal H}=\pi^{ik}\dot{eta}_{ik}+{\it p}_a\dot{y}^a-{\cal L}+$  primary constraints with Lagrange multipliers

Hamiltonian density  ${\mathcal H}$  can be conveniently broken down into two terms

$$\mathcal{H} = \mathcal{H}^{\mathsf{ADM}} + \mathcal{H}^{\mathsf{add}},$$

$$\mathcal{H}^{ADM} = \frac{1}{2} N \pi^{ik} \overline{L}_{ik,lm} \pi^{lm} + \pi^{ik} \left( \overset{3}{D}_i N_k + \overset{3}{D}_k N_i \right) - \frac{1}{2\varkappa} N \sqrt{\beta} \overset{3}{R} + \lambda \Phi + \lambda_k \Phi^k,$$
$$\mathcal{H}^{add} = \frac{N}{4\phi} \rho_a \overset{3}{\Pi}_{\perp}^{ab} \rho_b + \phi^{ij} (\beta_{ij} - \overline{\beta}_{ij}) + \rho_a e_i^a \overline{\beta}^{ik} N_k - \phi N + \chi \Psi + \chi^i \Psi_i + \chi^{ij} \Psi_{ij} + \xi^i \Omega$$

 ${\cal H}=\pi^{ik}\dot{eta}_{ik}+{\it p}_a\dot{y}^a-{\cal L}+{
m primary}$  constraints with Lagrange multipliers

Hamiltonian density  ${\mathcal H}$  can be conveniently broken down into two terms

$$\mathcal{H} = \mathcal{H}^{\mathsf{ADM}} + \mathcal{H}^{\mathsf{add}},$$

$$\mathcal{H}^{\text{ADM}} = \frac{1}{2} N \pi^{ik} \overline{L}_{ik,lm} \pi^{lm} + \pi^{ik} \left( \overset{3}{D}_{i} N_{k} + \overset{3}{D}_{k} N_{i} \right) - \frac{1}{2\varkappa} N \sqrt{\beta} \overset{3}{R} + \lambda \Phi + \lambda_{k} \Phi^{k},$$

$$\mathcal{H}^{\mathsf{add}} = \frac{N}{4\phi} p_{\mathfrak{a}} \Pi_{\perp}^{\mathfrak{ab}} p_{\mathfrak{b}} + \phi^{ij} (\beta_{ij} - \overline{\beta}_{ij}) + p_{\mathfrak{a}} e_{i}^{\mathfrak{a}} \overline{\beta}^{ik} N_{k} - \phi N + \chi \Psi + \chi^{i} \Psi_{i} + \chi^{ij} \Psi_{ij} + \xi^{i} \Omega_{i}$$

# Secondary constraints

#### All the primary constraints $T_{\alpha}$ , $\alpha = 1...M$ must be preserved: $\dot{T}_{\alpha} = \{H, T_{\alpha}\} \approx 0.$

Conditions for the conservation of primary constraints give the first generation of secondary constraints:

$$\begin{split} \Xi &= \phi + \frac{\zeta}{2} p_{\perp} \approx 0; \\ \mathcal{H}_{0} &= \mathcal{H}_{0}^{\text{ADM}} + \zeta p_{\perp} \approx 0; \\ \mathcal{H}_{k} &= \mathcal{H}_{k}^{\text{ADM}} + e_{k}^{a} p_{a} \approx 0; \\ \Sigma_{ij} &= \beta_{ij} - \overline{\beta}_{ij} \approx 0. \end{split}$$

Next generation of secondary constraints:

$$\Lambda_{ik} = \overline{L}_{ik,lm} \pi^{lm} - 2\zeta n_a \overset{3}{b}_{ik}^a \approx 0;$$
  
$$\Upsilon_{ij} = \overline{\Upsilon}_{ij} + \phi^{km} \Upsilon_{ij,km}^{(1)} \approx 0.$$

Notation:  

$$\mathcal{H}_{0}^{\text{ADM}} = \frac{1}{2} \pi^{ik} \overline{L}_{ik,lm} \pi^{lm} - \frac{1}{2\varkappa} \sqrt{\beta} \overset{3}{R};$$

$$\mathcal{H}_{i}^{\text{ADM}} = -2\beta_{im} \sqrt{\beta} \overset{3}{D}_{j} \frac{\pi^{jm}}{\sqrt{\beta}};$$

$$p_{\perp} = \sqrt{-p_{a}} \overset{3}{\Pi}_{\perp}^{ab} p_{b};$$

$$\zeta = \pm 1.$$

$$p_{\perp}^{a} = \overset{3}{\Pi}_{\perp}^{ab} p_{b};$$

$$n^{a} = \frac{p_{\perp}^{a}}{p_{\perp}};$$

$$\overset{3}{b}_{ik}^{a} = \overset{3}{D}_{i} \overset{3}{D}_{k} y^{a}.$$

# Secondary constraints

All the primary constraints  $T_{\alpha}$ ,  $\alpha = 1...M$  must be preserved:  $\dot{T}_{\alpha} = \{H, T_{\alpha}\} \approx 0$ .

Conditions for the conservation of primary constraints give the first generation of secondary constraints:

$$\begin{split} \Xi &= \phi + \frac{\zeta}{2} p_{\perp} \approx 0; \\ \mathcal{H}_0 &= \mathcal{H}_0^{\text{ADM}} + \zeta p_{\perp} \approx 0; \\ \mathcal{H}_k &= \mathcal{H}_k^{\text{ADM}} + e_k^a p_a \approx 0; \\ \Sigma_{ij} &= \beta_{ij} - \overline{\beta}_{ij} \approx 0. \end{split}$$

Next generation of secondary constraints:

$$\Lambda_{ik} = \overline{L}_{ik,lm} \pi^{lm} - 2\zeta n_a \overset{3}{b}_{ik}^a \approx 0;$$
  
$$\Upsilon_{ij} = \overline{\Upsilon}_{ij} + \phi^{km} \Upsilon_{ij,km}^{(1)} \approx 0.$$

Notation:  

$$\mathcal{H}_{0}^{\text{ADM}} = \frac{1}{2} \pi^{ik} \overline{L}_{ik,lm} \pi^{lm} - \frac{1}{2\varkappa} \sqrt{\beta} \overset{3}{R};$$

$$\mathcal{H}_{i}^{\text{ADM}} = -2\beta_{im} \sqrt{\beta} \overset{3}{D}_{j} \frac{\pi^{jm}}{\sqrt{\beta}};$$

$$p_{\perp} = \sqrt{-p_{a}} \overset{3}{\Pi}_{\perp}^{ab} p_{b};$$

$$\zeta = \pm 1.$$

$$p_{\perp}^{a} = \overset{3}{\Pi}_{\perp}^{ab} p_{b};$$

$$n^{a} = \frac{p_{\perp}^{a}}{p_{\perp}};$$

$$\overset{3}{b}_{ik}^{a} = \overset{3}{D}_{i} \overset{3}{D}_{k} y^{a}.$$

# Secondary constraints

All the primary constraints  $T_{\alpha}$ ,  $\alpha = 1...M$  must be preserved:  $\dot{T}_{\alpha} = \{H, T_{\alpha}\} \approx 0$ .

Conditions for the conservation of primary constraints give the first generation of secondary constraints:

$$\begin{split} \Xi &= \phi + \frac{\zeta}{2} p_{\perp} \approx 0; \\ \mathcal{H}_0 &= \mathcal{H}_0^{\text{ADM}} + \zeta p_{\perp} \approx 0; \\ \mathcal{H}_k &= \mathcal{H}_k^{\text{ADM}} + e_k^a p_a \approx 0; \\ \Sigma_{ij} &= \beta_{ij} - \overline{\beta}_{ij} \approx 0. \end{split}$$

Next generation of secondary constraints:

$$\Lambda_{ik} = \overline{L}_{ik,lm} \pi^{lm} - 2\zeta n_a \mathring{b}^a_{ik} \approx 0;$$
  
$$\Upsilon_{ij} = \overline{\Upsilon}_{ij} + \phi^{km} \Upsilon^{(1)}_{ij,km} \approx 0.$$

Notation:  

$$\mathcal{H}_{0}^{\text{ADM}} = \frac{1}{2} \pi^{ik} \overline{L}_{ik,lm} \pi^{lm} - \frac{1}{2\varkappa} \sqrt{\beta} \overset{3}{R};$$

$$\mathcal{H}_{i}^{\text{ADM}} = -2\beta_{im} \sqrt{\beta} \overset{3}{D}_{j} \frac{\pi^{jm}}{\sqrt{\beta}};$$

$$p_{\perp} = \sqrt{-p_{a}} \overset{3}{\Pi}_{\perp}^{ab} p_{b};$$

$$\zeta = \pm 1.$$

$$p_{\perp}^{a} = \overset{3}{\Pi}_{\perp}^{ab} p_{b};$$

$$n^{a} = \frac{p_{\perp}^{a}}{p_{\perp}};$$

$$\overset{3}{b}_{ik}^{a} = \overset{3}{D}_{i} \overset{3}{D}_{k} y^{a}.$$

It is convenient to write the Hamiltonian density as a linear combination of the constraints already introduced:

$$\mathcal{H} = \mathcal{N}\mathcal{H}_{0} + \mathcal{N}^{k}\mathcal{H}_{k} + \Sigma_{ij}(\phi^{ij} + \beta^{ik}\overline{\beta}^{jm}e_{k}^{a}p_{a}\mathcal{N}_{m}) + \lambda\Phi + \lambda_{k}\Phi^{k} + \chi\Psi + \chi^{i}\Psi_{i} + \chi^{ij}\Psi_{ij} + \xi^{i}\Omega_{i}.$$

#### First class constraints

First class constraints are constraints whose Poisson brackets with all other constraints are just constraints linear combinations:

$$\{T_{\alpha}, T_{\beta}\} = C_{\alpha\beta}^{\gamma} T_{\gamma} \approx 0.$$

Quantizing the theory with first class constraints  $T_{\alpha}$ ,  $\alpha = 1 \dots M$ :

$$egin{aligned} [\widehat{ au}_lpha, \widehat{ au}_eta] &= \widehat{ au}^\gamma_{lphaeta} \widehat{ au}_\gamma; \ \widehat{ au}_lpha \ket{arphi} &= \mathsf{0}. \end{aligned}$$

This does not work with second class constraints!

- First class constraints:  $\mathcal{H}_k$ .
- Others are second class.

- First class constraints:  $\mathcal{H}_k$ .
- Others are second class.

- First class constraints:  $\mathcal{H}_k$ .
- Others are second class.

- First class constraints:  $\mathcal{H}_k$ .
- Others are second class.

$$\begin{aligned} \mathcal{H}_{0} &= \mathcal{H}_{0}^{\text{ADM}} + \zeta p_{\perp} \approx 0 & \text{variables:} \\ \mathcal{H}_{k} &= \mathcal{H}_{k}^{\text{ADM}} + e_{k}^{a} p_{a} \approx 0 \\ \Sigma_{ij} &= \beta_{ij} - \overline{\beta}_{ij} \approx 0 & y^{a} \qquad p_{a} \\ \Lambda_{ik} &= \overline{L}_{ik,lm} \pi^{lm} - 2\zeta n_{a} \overset{3}{b}_{ik}^{a} \approx 0 & \beta_{ik} \qquad \pi^{ik} \end{aligned}$$

Hamiltonian:

$$\mathcal{H} = \widetilde{N}\mathcal{H}_{0} + \widetilde{N}^{k}\mathcal{H}_{k} + \Sigma_{ij}\left(\phi^{ij} + \beta^{ik}\overline{\beta}^{jm}e_{k}^{a}p_{a}N_{m}\right).$$

Two options:

• Eliminate the embedding function  $y^a$  and its conjugate momentum  $p_a$ .

• Eliminate  $\beta_{ij}$  and  $\pi^{ij}$ .

. . .

$$\begin{aligned} \mathcal{H}_{0} &= \mathcal{H}_{0}^{\text{ADM}} + \zeta p_{\perp} \approx 0 & \text{variables:} \\ \mathcal{H}_{k} &= \mathcal{H}_{k}^{\text{ADM}} + e_{k}^{a} p_{a} \approx 0 \\ \Sigma_{ij} &= \beta_{ij} - \overline{\beta}_{ij} \approx 0 & y^{a} \qquad p_{a} \\ \Lambda_{ik} &= \overline{L}_{ik,lm} \pi^{lm} - 2\zeta n_{a} \overset{3}{b}_{ik}^{a} \approx 0 & \beta_{ik} & \pi^{ik} \end{aligned}$$

#### Hamiltonian:

$$\mathcal{H} = \widetilde{N}\mathcal{H}_{0} + \widetilde{N}^{k}\mathcal{H}_{k} + \Sigma_{ij}\big(\phi^{ij} + \beta^{ik}\overline{\beta}^{jm}e_{k}^{a}p_{a}N_{m}\big).$$

Two options:

• Eliminate the embedding function  $y^a$  and its conjugate momentum  $p_a$ .

• Eliminate  $\beta_{ij}$  and  $\pi^{ij}$ .

$$\begin{aligned} \mathcal{H}_{0} &= \mathcal{H}_{0}^{\text{ADM}} + \zeta p_{\perp} \approx 0 & \text{variables:} \\ \mathcal{H}_{k} &= \mathcal{H}_{k}^{\text{ADM}} + e_{k}^{a} p_{a} \approx 0 \\ \Sigma_{ij} &= \beta_{ij} - \overline{\beta}_{ij} \approx 0 & y^{a} \quad p_{a} \\ \Lambda_{ik} &= \overline{L}_{ik,lm} \pi^{lm} - 2\zeta n_{a} \overset{3}{b}_{ik}^{a} \approx 0 & \beta_{ik} \quad \pi^{ik} \end{aligned}$$

Hamiltonian:

$$\mathcal{H} = \widetilde{N}\mathcal{H}_{0} + \widetilde{N}^{k}\mathcal{H}_{k} + \Sigma_{ij}(\phi^{ij} + \beta^{ik}\overline{\beta}^{jm}e_{k}^{a}p_{a}N_{m}).$$

Two options:

• Eliminate the embedding function  $y^a$  and its conjugate momentum  $p_a$ .

• Eliminate  $\beta_{ij}$  and  $\pi^{ij}$ .

. . .

$$\begin{aligned} \mathcal{H}_{0} &= \mathcal{H}_{0}^{\mathrm{ADM}} + \zeta p_{\perp} \approx 0 & \text{variables:} \\ \mathcal{H}_{k} &= \mathcal{H}_{k}^{\mathrm{ADM}} + e_{k}^{a} p_{a} \approx 0 \\ \Sigma_{ij} &= \beta_{ij} - \overline{\beta}_{ij} \approx 0 & y^{a} \qquad p_{a} \\ \Lambda_{ik} &= \overline{L}_{ik,lm} \pi^{lm} - 2\zeta n_{a} \overset{3}{b}_{ik}^{a} \approx 0 \end{aligned}$$

Hamiltonian:

$$\mathcal{H} = \widetilde{N}\mathcal{H}_{0} + \widetilde{N}^{k}\mathcal{H}_{k} + \Sigma_{ij}(\phi^{ij} + \beta^{ik}\overline{\beta}^{jm}e_{k}^{a}p_{a}N_{m}).$$

Two options:

- Eliminate the embedding function  $y^a$  and its conjugate momentum  $p_a$ . Problem: equation  $\partial_i y^a \partial_k y_a = \beta_{ik}$ .
- Eliminate  $\beta_{ij}$  and  $\pi^{ij}$ .

$$\begin{aligned} \mathcal{H}_{0} &= \mathcal{H}_{0}^{\mathrm{ADM}} + \zeta p_{\perp} \approx 0 & \text{variables:} \\ \mathcal{H}_{k} &= \mathcal{H}_{k}^{\mathrm{ADM}} + e_{k}^{a} p_{a} \approx 0 \\ \Sigma_{ij} &= \beta_{ij} - \overline{\beta}_{ij} \approx 0 & y^{a} \qquad p_{a} \\ \Lambda_{ik} &= \overline{L}_{ik,lm} \pi^{lm} - 2\zeta n_{a} \overset{3}{b}_{ik}^{a} \approx 0 \end{aligned}$$

Hamiltonian:

$$\mathcal{H} = \widetilde{\mathcal{N}}\mathcal{H}_{0} + \widetilde{\mathcal{N}}^{k}\mathcal{H}_{k} + \Sigma_{ij} \big(\phi^{ij} + \beta^{ik}\overline{\beta}^{jm}e_{k}^{a}p_{a}N_{m}\big).$$

Two options:

- Eliminate the embedding function  $y^a$  and its conjugate momentum  $p_a$ . Problem: equation  $\partial_i y^a \partial_k y_a = \beta_{ik}$ .
- Eliminate  $\beta_{ij}$  and  $\pi^{ij}$ .  $\leftarrow$  we choose this way

## Solving the remaining second class constraints

Solution:

$$\beta_{ij} = \overline{\beta}_{ij}; \qquad \pi^{ij} = 2\zeta n_a b^a_{lk} L^{lk,ij}.$$

First order action:

$$S^{(1)} = \int dt \int d^3x \Big( \pi^{ik} \dot{\beta}_{ik} + p_{\partial} \dot{y}^{\partial} - N\mathcal{H}_0 - N^k \mathcal{H}_k \Big).$$

Then

$$\mathcal{L}^{(1)} = -\zeta \Big( B_{ab} n^b + \frac{1}{2} (n_c B^{cb} n_b + B^c_c) n_a \Big) \dot{y}^a - N \mathcal{H}_0 - N^k \mathcal{H}_k,$$

here  $B^{ab} = 4 \overset{3}{b}_{ik}^{a} \overset{3}{b}_{lm}^{b} L^{ik,lm}$ .

Solution:

$$\beta_{ij} = \overline{\beta}_{ij}; \qquad \pi^{ij} = 2\zeta n_a b^a_{lk} L^{lk,ij}.$$

First order action:

$$S^{(1)} = \int dt \int d^3x \Big( \pi^{ik} \dot{\beta}_{ik} + p_{\vartheta} \dot{y}^{\vartheta} - N\mathcal{H}_0 - N^k \mathcal{H}_k \Big).$$

Then

$$\mathcal{L}^{(1)} = -\zeta \Big( B_{ab} n^b + \frac{1}{2} (n_c B^{cb} n_b + B^c_c) n_a \Big) \dot{y}^a - N \mathcal{H}_0 - N^k \mathcal{H}_k,$$

here  $B^{ab} = 4 \overset{3}{b}_{ik}^{a} \overset{3}{b}_{lm}^{b} L^{ik,lm}$ .

Solution:

$$\beta_{ij} = \overline{\beta}_{ij}; \qquad \pi^{ij} = 2\zeta n_a b^a_{lk} L^{lk,ij}.$$

First order action:

$$S^{(1)} = \int dt \int d^3x \Big( \pi^{ik} \dot{\beta}_{ik} + p_a \dot{y}^a - N\mathcal{H}_0 - N^k \mathcal{H}_k \Big).$$

Then

$$\mathcal{L}^{(1)} = -\zeta \Big( B_{ab} n^b + rac{1}{2} (n_c B^{cb} n_b + B^c_c) n_a \Big) \dot{y}^a - N \mathcal{H}_0 - N^k \mathcal{H}_k,$$

here  $B^{ab} = 4 \overset{3}{b}_{ik}^{a} \overset{3}{b}_{lm}^{b} L^{ik,lm}$ .

$$\mathcal{L}^{(1)} = -\zeta \Big( B_{ab} n^b + \frac{1}{2} (n_c B^{cb} n_b + B^c_c) n_a \Big) \dot{y}^a - N \mathcal{H}_0 - N^k \mathcal{H}_k$$

Primary constraints:

$$\begin{split} \widehat{\Phi}_{i} &= \pi_{a}e_{i}^{a} \approx 0; \\ \widehat{\Phi}_{4} &= n(y^{a}, \pi_{a})^{2} + 1 \approx 0; \\ \widehat{\Psi}^{a} &= \frac{n^{a}(y^{a}, \pi_{a})}{\sqrt{-n^{2}}} - \frac{p_{\perp}^{a}}{p_{\perp}} \approx 0; \\ \pi_{p}^{a} &\approx 0; \\ \pi_{N}^{k} &\approx 0. \end{split}$$

Secondary constraints:

$$\begin{split} \mathcal{H}_{0} &= \mathcal{H}_{0}^{\text{ADM}}(y^{a}, \pi_{a}) + \zeta p_{\perp} \approx 0; \\ \mathcal{H}_{k} &= \mathcal{H}_{k}^{\text{ADM}}(y^{a}, \pi_{a}) + e_{k}^{a} p_{a} \approx 0; \\ N &\approx 0; \\ N_{k} &\approx 0. \end{split}$$

$$\mathcal{H} = \widehat{\chi}^i \widehat{\Phi}_i + \widehat{\chi} \widehat{\Phi}_4.$$

$$\mathcal{L}^{(1)} = -\zeta \Big( B_{ab} n^b + \frac{1}{2} (n_c B^{cb} n_b + B^c_c) n_a \Big) \dot{y}^a - N \mathcal{H}_0 - N^k \mathcal{H}_k$$

Primary constraints:

$$\begin{split} \widehat{\Phi}_{i} &= \pi_{a}e_{i}^{a} \approx 0; \\ \widehat{\Phi}_{4} &= n(y^{a},\pi_{a})^{2} + 1 \approx 0; \\ \widehat{\Psi}^{a} &= \frac{n^{a}(y^{a},\pi_{a})}{\sqrt{-n^{2}}} - \frac{p_{\perp}^{a}}{p_{\perp}} \approx 0; \\ \pi_{p}^{a} &\approx 0; \\ \pi_{N}^{k} &\approx 0. \end{split}$$

Secondary constraints:

$$\begin{split} \mathcal{H}_{0} &= \mathcal{H}_{0}^{\text{ADM}}(y^{a}, \pi_{a}) + \zeta p_{\perp} \approx 0; \\ \mathcal{H}_{k} &= \mathcal{H}_{k}^{\text{ADM}}(y^{a}, \pi_{a}) + e_{k}^{a} p_{a} \approx 0; \\ N &\approx 0; \\ N_{k} &\approx 0. \end{split}$$

$$\mathcal{H} = \widehat{\chi}^i \widehat{\Phi}_i + \widehat{\chi} \widehat{\Phi}_4.$$

$$\mathcal{L}^{(1)} = -\zeta \Big( B_{ab} n^b + \frac{1}{2} (n_c B^{cb} n_b + B^c_c) n_a \Big) \dot{y}^a - N \mathcal{H}_0 - N^k \mathcal{H}_k$$

Primary constraints:

$$\begin{split} \widehat{\Phi}_{i} &= \pi_{a}e_{i}^{a} \approx 0; \\ \widehat{\Phi}_{4} &= n(y^{a},\pi_{a})^{2} + 1 \approx 0; \\ \widehat{\Psi}^{a} &= \frac{n^{a}(y^{a},\pi_{a})}{\sqrt{-n^{2}}} - \frac{p_{\perp}^{a}}{p_{\perp}} \approx 0; \\ \pi_{p}^{a} &\approx 0; \\ \pi_{N}^{k} &\approx 0. \end{split}$$

Secondary constraints:

$$\begin{split} \mathcal{H}_{0} &= \mathcal{H}_{0}^{\text{ADM}}(y^{a}, \pi_{a}) + \zeta p_{\perp} \approx 0; \\ \mathcal{H}_{k} &= \mathcal{H}_{k}^{\text{ADM}}(y^{a}, \pi_{a}) + e_{k}^{a} p_{a} \approx 0; \\ N &\approx 0; \\ N_{k} &\approx 0. \end{split}$$

$$\mathcal{H} = \widehat{\chi}^i \widehat{\Phi}_i + \widehat{\chi} \widehat{\Phi}_4.$$

$$\mathcal{L}^{(1)} = -\zeta \Big( B_{ab} n^b + \frac{1}{2} (n_c B^{cb} n_b + B^c_c) n_a \Big) \dot{y}^a - N \mathcal{H}_0 - N^k \mathcal{H}_k$$

Primary constraints:

$$\begin{split} \widehat{\Phi}_{i} &= \pi_{a}e_{i}^{a} \approx 0; \\ \widehat{\Phi}_{4} &= n(y^{a}, \pi_{a})^{2} + 1 \approx 0; \\ \widehat{\Psi}^{a} &= \frac{n^{a}(y^{a}, \pi_{a})}{\sqrt{-n^{2}}} - \frac{p_{\perp}^{a}}{p_{\perp}} \approx 0; \\ \pi_{p}^{a} &\approx 0; \\ \pi_{N}^{k} &\approx 0. \end{split}$$

Secondary constraints:

$$\begin{split} \mathcal{H}_{0} &= \mathcal{H}_{0}^{\text{ADM}}(y^{a}, \pi_{a}) + \zeta p_{\perp} \approx 0; \\ \mathcal{H}_{k} &= \mathcal{H}_{k}^{\text{ADM}}(y^{a}, \pi_{a}) + e_{k}^{a} p_{a} \approx 0; \\ N &\approx 0; \\ N_{k} &\approx 0. \end{split}$$

$$\mathcal{H} = \widehat{\chi}^i \widehat{\Phi}_i + \widehat{\chi} \widehat{\Phi}_4.$$

For the complete embedding theory formulated in the form of GR with an additional contribution of the so-called *embedding matter*, the canonical description of the theory is constructed.

- All the constraints are found.
- Some of the constraints are second class.
- Second class constraints are solved by eliminating the variables  $\beta_{ik}$ ,  $\pi^{ik}$ .
- The constructed canonical system passes into the canonical formulation of the complete embedding theory.

- Solving the constraints by excluding canonical variables in such a way that the variables  $\beta_{ik}$ ,  $\pi^{ik}$ , as well as variables describing embedding matter remain.
- Studying the Einstein limit.

For the complete embedding theory formulated in the form of GR with an additional contribution of the so-called *embedding matter*, the canonical description of the theory is constructed.

#### • All the constraints are found.

- Some of the constraints are second class.
- Second class constraints are solved by eliminating the variables  $\beta_{ik}$ ,  $\pi^{ik}$ .
- The constructed canonical system passes into the canonical formulation of the complete embedding theory.

- Solving the constraints by excluding canonical variables in such a way that the variables  $\beta_{ik}$ ,  $\pi^{ik}$ , as well as variables describing embedding matter remain.
- Studying the Einstein limit.

For the complete embedding theory formulated in the form of GR with an additional contribution of the so-called *embedding matter*, the canonical description of the theory is constructed.

- All the constraints are found.
- Some of the constraints are second class.
- Second class constraints are solved by eliminating the variables  $eta_{ik},\,\pi^{ik}.$
- The constructed canonical system passes into the canonical formulation of the complete embedding theory.

- Solving the constraints by excluding canonical variables in such a way that the variables  $\beta_{ik}$ ,  $\pi^{ik}$ , as well as variables describing embedding matter remain.
- Studying the Einstein limit.

For the complete embedding theory formulated in the form of GR with an additional contribution of the so-called *embedding matter*, the canonical description of the theory is constructed.

- All the constraints are found.
- Some of the constraints are second class.
- Second class constraints are solved by eliminating the variables  $\beta_{ik}$ ,  $\pi^{ik}$ .
- The constructed canonical system passes into the canonical formulation of the complete embedding theory.

- Solving the constraints by excluding canonical variables in such a way that the variables  $\beta_{ik}$ ,  $\pi^{ik}$ , as well as variables describing embedding matter remain.
- Studying the Einstein limit.

For the complete embedding theory formulated in the form of GR with an additional contribution of the so-called *embedding matter*, the canonical description of the theory is constructed.

- All the constraints are found.
- Some of the constraints are second class.
- Second class constraints are solved by eliminating the variables  $\beta_{ik}$ ,  $\pi^{ik}$ .
- The constructed canonical system passes into the canonical formulation of the complete embedding theory.

- Solving the constraints by excluding canonical variables in such a way that the variables  $\beta_{ik}$ ,  $\pi^{ik}$ , as well as variables describing embedding matter remain.
- Studying the Einstein limit.

For the complete embedding theory formulated in the form of GR with an additional contribution of the so-called *embedding matter*, the canonical description of the theory is constructed.

- All the constraints are found.
- Some of the constraints are second class.
- Second class constraints are solved by eliminating the variables  $\beta_{ik}$ ,  $\pi^{ik}$ .
- The constructed canonical system passes into the canonical formulation of the complete embedding theory.

- Solving the constraints by excluding canonical variables in such a way that the variables  $\beta_{ik}$ ,  $\pi^{ik}$ , as well as variables describing embedding matter remain.
- Studying the Einstein limit.

For the complete embedding theory formulated in the form of GR with an additional contribution of the so-called *embedding matter*, the canonical description of the theory is constructed.

- All the constraints are found.
- Some of the constraints are second class.
- Second class constraints are solved by eliminating the variables  $\beta_{ik}$ ,  $\pi^{ik}$ .
- The constructed canonical system passes into the canonical formulation of the complete embedding theory.

- Solving the constraints by excluding canonical variables in such a way that the variables  $\beta_{ik}$ ,  $\pi^{ik}$ , as well as variables describing embedding matter remain.
- Studying the Einstein limit.