Baryon Asymmetry and Holographic Composed Higgs

Early Universe First Order Phase Transition due to the Composite Higgs Boson Dynamics in the Soft-wall Holographic Model

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Baryon Asymmetry

Sakharov conditions
(necessary conditions to produce the antisymmetry of matter and antimatter):

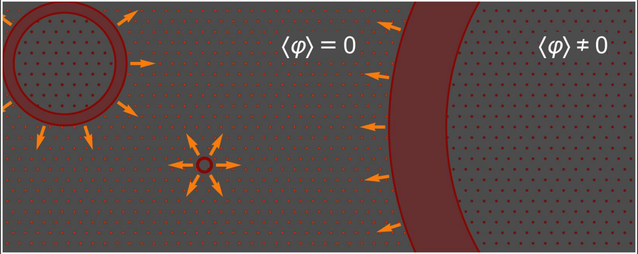
1. baryon number violation,
2. CP violation (particle - antiparticle),
3. CPT violation (violation of the thermodynamic equilibrium).

- Only the second is satisfied in perturbative Standard Model (CKM matrix; effect is too small).
- Counting non-perturbative effects (sphalerons) allows fulfilling the first two conditions.
- CTP theorem leads to "washing" of the asymmetry. It must be violated, for instance, with 1st order phase transition. In Standard Model ...

Electroweak Baryogenesis: Concept of "bubble" nucleation in SM

In SM: Phase Transition $\Rightarrow$ Electroweak Symmetry Breaking
(in the efficient potential of the electroweak model)

$$\mathrm{SU}(2)_\mathrm{L} \otimes \mathrm{U}(1)_\mathrm{Y} \to \mathrm{U}(1)_\mathrm{em}$$



$m_{\text{Higgs B.}} < 80$ GeV $\Rightarrow$ 1st order (wanted)

$m_{\text{Higgs B.}} = 125$ GeV $\Rightarrow$ crossover (in SM)

Baryon asymmetry cannot be explained in the framework of Standard Model.

Composed Higgs Model (CHM)

Problem: EW symmetry breaking doesn't disturb the Th-d equilibrium.
Possible solution: to consider *inner* symmetry braking, e.g.: CHM.

Ψ_I are fundamental fields with $\mathrm{SO}(5)$ inner symmetry

$$\langle \bar{\Psi}_I \Psi_J \rangle \xrightarrow[\text{low energy}]{\mathrm{SO}(5)\to\mathrm{SO}(4)} \begin{pmatrix} 0_{4\times 4} & 0 \\ 0 & \chi \end{pmatrix} \xRightarrow{\text{symmetry breaking}} \Sigma_{IJ} = \bar{\Psi}_I \Psi_J = \xi^\top \begin{pmatrix} 0_{4\times 4} & 0 \\ 0 & \chi \end{pmatrix} \xi$$

ξ are Goldstone bosons of the coset $\mathrm{SO}(5)/\mathrm{SO}(4)$. Higgs boson is one of them.

$$S_\chi = \int d^4x \left(\mathrm{tr}\, g^{\mu\nu} (\partial_\mu \Sigma)^\top (\partial_\nu \Sigma) - 2V_\chi(\Sigma) \right)$$

Extra motivation to use CHM: $\dfrac{\text{AdS/QCD}}{\text{“AdS/CHM”}}$ has the same tunes.

CHM $\sim$ QCD $\Rightarrow$ we can use the results of AdS/QCD.
(i.e. there is reason to suppose self-consistent; but we should not be lazy and check it anyway)

Effective Potential and Holography

$$\mathcal{Z}[J] = \int \mathcal{D}\varphi \, \exp\left(-S[\varphi] - \int d^4x \, \varphi(x)\, J(x)\right) \stackrel{\text{def}}{=} e^{-W[J]}$$

$$\langle\varphi\rangle = \left.\frac{\delta W[J]}{\delta J}\right|_{J=0}, \quad \Gamma[\langle\varphi\rangle] = W[J] - \int d^4x \, \frac{\delta W[J]}{\delta J(x)} J(x) \text{ – Effective Action}$$

$\boxed{\text{EoM: } \dfrac{\delta\Gamma}{\delta\langle\varphi\rangle} = J}$ Homogeneus Solution $\Rightarrow \langle\varphi\rangle = \text{const} \Rightarrow \Gamma = -\mathrm{Vol}_4 \, V_{\text{eff}}$ — Effective Potential

extrema condition

$$\mathcal{Z}[J] \xlongequal[\text{correspondence}]{\text{AdS/CFT}} \mathcal{Z}_{\text{AdS}} \xrightarrow[\text{limit}]{\text{classical}} \left. e^{-S_{\text{AdS}}} \right|_{\partial\text{AdS}}$$

$\boxed{V_{\text{eff}} = -\dfrac{1}{\mathrm{Vol}_4} \left. S_{\text{AdS}} \right|_{\partial\text{AdS}}}$ — boundary term of the bulk theory defines *quantum* effective potential

Holographic Model and Solutions

$$S_\chi = \int d^5x\,\sqrt{|g|}\,e^{\phi}\left(\operatorname{tr} g^{\mu\nu}(\partial_\mu\chi)(\partial_\nu\chi) - 2V_\chi(\chi)\right)$$ – is CHM action in dilaton AdS_5 only for the background field χ i.e. without fluctuations etc.

Potential parametrization : $V_\chi(\chi) = \frac{m^2}{2}\chi^2 - \frac{D}{4L^2}\lambda\chi^4 + \frac{\lambda^2\gamma}{6L^2}\chi^6 + o(\chi^8)$ is the expansion of a more general theory

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Fixed geometry: AdS with black hole horizon at $z = z_\mathrm{H}$

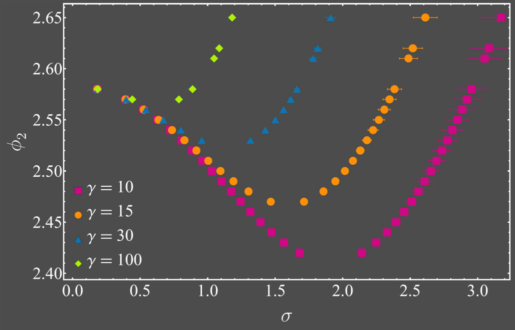
$$ds^2 = \frac{L^2}{z^2}\left(-f(z)\,dt^2 + \frac{dz^2}{f(z)} + d\vec{x}^2\right), \quad f(z) = 1 - \left(\frac{z}{z_\mathrm{H}}\right)^4; \quad \phi(z) = \phi_2 z^2.$$

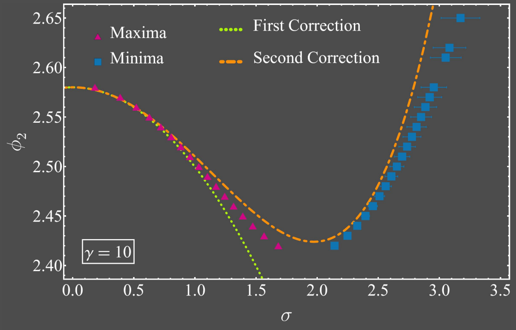
$$\boxed{z_\mathrm{H} = 1,\ T \sim \frac{1}{\sqrt{\phi_2}}} \quad \frac{\delta S_\chi}{\delta\chi} = 0 \Rightarrow \chi \xrightarrow{z\to 0} J\,z + \left(\sigma - \left(\frac{3}{2}J^3 + \phi_2 J\right)\log z\right)z^3 + o(z^5)$$

from EoM for effective action : $\mathrm{Vol}_4\frac{\delta V_\mathrm{eff}}{\delta\langle\varphi\rangle} = J$, without sources : $J = 0$ $\Rightarrow \overbrace{\chi \xrightarrow{z\to 0} \sigma z^3 + o(z^5)}^{\text{“extreme” solutions}} \Rightarrow \overbrace{\frac{\delta V_\mathrm{eff}}{\delta\langle\varphi\rangle} = 0}^{\text{extrema}}$

Extrema of the Effective Potential (“Extreme” Solutions)

“Extreme” curves defines the positions σ of the effective potential extrema as functions of the parameter γ and temperature ϕ_2 ($T \sim \frac{1}{\sqrt{\phi_2}}$)





σ “is” average of the Higgs background $\sigma \sim \langle \varphi \rangle$ on the border

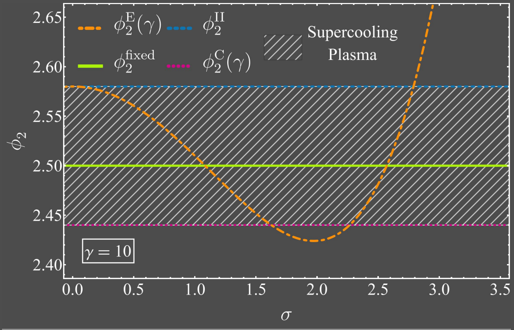
To define the proportional constant, we have to count interaction with SM.

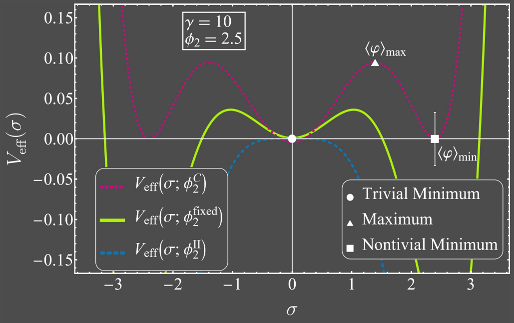
Without it, we can’t define it “in eV”.

Currently, all values are *in terms of the horizon coordinate* $z_{\mathrm{H}} = 1$.

First Order Phase Transition and Effective Potential

values of V_{eff} in the extrema $\Rightarrow$ $T = T_{\text{C}}$ $(\phi_2 = \phi_2^{\text{C}})$ $\Rightarrow$ nontrivial true vacuum appears.
At $T = T_{\text{II}}$ potential barrier vanishes.
First order PT is able in the range $T_{\text{II}} < T < T_{\text{C}}$ $(\phi_2^{\text{II}} > \phi_2 > \phi_2^{\text{C}}(\gamma))$





There is approximation $V_{\text{eff}} = a_0 + a_2\sigma^2 + a_4\sigma^4 + a_6\sigma^6$
with the points $(\sigma_{\text{max}}, V_{\text{max}}(\sigma_{\text{max}}))$ and $(\sigma_{\text{min}}, V_{\text{min}}(\sigma_{\text{min}}))$

Conclusion

What we have done?

- We suggested an interacting Composed Higgs Model in soft-wall holography.
- First-order phase transition that could lead to the desired breaking thermodynamic equilibrium was found as a quantum effect through holography.
- Extrema solutions give finite results. That allows us to approximate the effective potential by its extrema.

What should we do now?

- count interaction of CHM with SM to find critical temperature T_{C} “in eV”;
- find non-extrema values of the effective potential (here some extra UV renormalization is needed);
- count fluctuation and gauge interaction (adjoint representation of $\mathrm{SO}(5)$; it will give chemical potential).

What may we do with the model in the future? — search for PT in gravitational part “near-AdS”; — count gravity interaction (that breaks scale “invariance”).

Thank you for your attention!

List of the Backup Slides:

- Nucleation Ratio
- First Order Phase Transition Criteria
- A Bit More about"Extreme" Curve
- CHM Parametrization (fluctuations)
- Units ("$z_{\mathrm{H}} = 1$" and physical "in eV")
- "Symmetries" of the CHM Potential
- CHM potential isn't "Tuned"

Nucleation Ratio

The next step is to consider
Baryogenesis generates enough asymmetry (enough efficient) if
there is one bubble per Hubble volume

$$\underbrace{\text{Nucleation Ratio}:\quad AT^4 e^{-\frac{F_C}{T}}}_{\substack{\text{Bubbles produced}\\ \text{per time}\times\text{space volume}}} \sim \underbrace{H^4(T) = \left(\frac{T^2}{M_{\text{Pl}}}\right)^4 \quad \text{– Expanding of the Univerce}}_{1/\left(\text{Hubble time}\times\text{volume}\right)}$$

$F = F[\langle\varphi\rangle, R]$ – Free energy of the bubble; R is the radius of the bubble

Hubble horizon (time, volume, radius) — speed of receding object behind it is greater than the speed of light (Don't confuse with cosmological horizon)

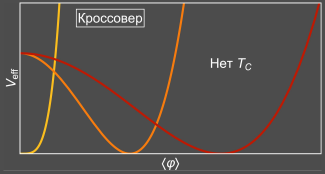
Bubble appears with a certain size. It defines with "micro-physics".
If its radius is grater, then critical one $\left.\frac{\partial F}{\partial R}\right|_{R_\mathrm{C} \overset{\text{def}}{=} R}$, the bubble grow. Otherwise, it bursts.

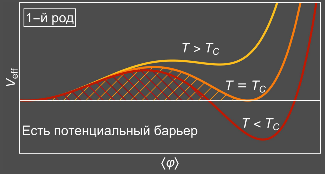
It gives $F_\mathrm{C} \overset{\text{def}}{=} F(R_\mathrm{C})$ and defines nucleation ratio and "viability of the model".

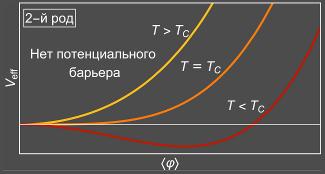
First Order Phase Transition Criteria

$$\frac{\delta\Gamma}{\delta\langle\varphi\rangle} = \mathsf{Vol}_d \frac{\delta V_{\text{eff}}}{\delta\langle\varphi\rangle} = J, \quad J = 0 \quad \Leftrightarrow \quad \text{condition for all extrema}$$

"Physical" potential V_{eff} should have the trivial minimum (we can take $V_{\text{eff}}\big|_{\langle\varphi\rangle=0} = 0$) and be even function $V_{\text{eff}}[\langle\varphi\rangle] = V_{\text{eff}}[-\langle\varphi\rangle]$ and give finite motion $V_{\text{eff}} \xrightarrow{\langle\varphi\rangle\to\pm\infty} \infty$







$$\frac{\delta V_{\text{eff}}}{\delta\langle\varphi\rangle}\bigg|_{\langle\varphi\rangle=0} = 0, \quad \frac{\delta V_{\text{eff}}}{\delta\langle\varphi\rangle}\bigg|_{\langle\varphi\rangle=\langle\varphi\rangle_{\max}>0} = 0, \quad \frac{\delta V_{\text{eff}}}{\delta\langle\varphi\rangle}\bigg|_{\langle\varphi\rangle=\langle\varphi\rangle_{\min}>\langle\varphi\rangle_{\max}} = 0,$$

$$T_C: \quad V_{\text{eff}}\Big|^{T=T_C}_{\langle\varphi\rangle=\langle\varphi\rangle_{\min}} = V_{\text{eff}}\Big|^{\forall T}_{\langle\varphi\rangle=0} \overset{\text{fix}}{=} 0, \quad V_{\text{eff}}\Big|^{T<T_C}_{\langle\varphi\rangle=\langle\varphi\rangle_{\max}} > V_{\text{eff}}\Big|^{T<T_C}_{\langle\varphi\rangle=\langle\varphi\rangle_{\min}}$$

A Bit More about"Extreme" Curve

$$\begin{array}{c}\text{EoM for the fields in the}\\\text{bulk gives all solutions}\end{array} : \ \frac{\delta S_\chi}{\delta \chi} = 0 \ \Rightarrow \ \chi = \chi_{\mathrm{Sol.}}(z; J, \sigma) \text{ – solutions}$$

$$S_\chi\left[\chi_{\mathrm{Sol.}}\right] = S_\chi\left[\chi_{\mathrm{Sol.}}\right]\Big|_{\partial \mathrm{AdS}} \ \begin{array}{c}\text{we can}\\\text{put it in}\end{array} : \ V_{\mathrm{eff}} = -\frac{1}{\mathrm{Vol}_4} S_\chi\left[\chi_{\mathrm{Sol.}}\right]\Big|_{\partial \mathrm{AdS}} = -\frac{1}{\mathrm{Vol}_4} S_\chi\left[\chi_{\mathrm{Sol.}}\right]$$

$$\begin{array}{c}\text{EoM for the}\\\text{effective action}\end{array} : \ \mathrm{Vol}_4 \frac{\delta V_{\mathrm{eff}}}{\delta \langle \varphi \rangle} = J, \quad \begin{array}{c}\text{without sources}\\ J = 0\end{array} \ \Rightarrow \ \frac{\delta V_{\mathrm{eff}}}{\delta \langle \varphi \rangle} = 0$$

σ is a condensate, that gives the Higgs field at the border $\sigma \sim \langle \varphi \rangle$

$$\frac{\delta V_{\mathrm{eff}}}{\delta \langle \varphi \rangle} = 0 = \frac{\delta}{\delta \sigma} S_\chi\left[\chi_{\mathrm{Sol.}}(z; J, \sigma)\right]\Big|_{J=0} \quad \Rightarrow \quad \{\sigma_1, \ldots, \sigma_n\} \text{ – extrema}$$

Parametrization of the Minimal Composed Higgs Model

$$\left(\mathcal{G}\ \begin{matrix}\text{invariant}\\ \text{vacuum}\end{matrix}\right) \xrightarrow[\text{spontaneous}]{\text{breaking}} \left(\mathcal{H}\ \begin{matrix}\text{invarian}\\ \text{vacuum}\end{matrix}\right) \Rightarrow \text{Goldstone bosons} \ni \text{Higgs boson}$$

$$\begin{matrix}\text{physical model}\\ \text{must include EW}\end{matrix} : \ \mathcal{G} \supset \mathcal{H} \supseteq \mathsf{SU}(2)_\mathsf{L} \otimes \mathsf{U}(1)_\mathsf{Y} \ \Rightarrow \begin{matrix}\text{minimal}\\ \text{model}\end{matrix} : \ \mathcal{G} = \mathsf{SO}(5),\ \mathcal{H} = \mathsf{SO}(4)$$

$$\hat{\chi}(x) = \xi\,\Sigma\,\xi^\top, \quad \Sigma(x) = \begin{pmatrix} 0_{4\times4} & 0 \\ 0 & \chi \end{pmatrix} + T^i\sigma^i, \quad \xi(x) = \exp(i\pi^a T^a),$$

ξ are massless Goldstone bosons ($T^i \in \mathcal{G}/\mathcal{H}$),
σ are massive "radial" (by analogy with Mexican hat) fluctuations ($T^a \in \mathcal{H}$),
χ is background field. It gives the main contribution in the effective potential.

What Units are Used?

The scale has been fixed with the horizon coordinate in AdS $z_{\mathrm{H}} = 1$.
This fixed the energy/scale units of the all values. z_{H} recovering is needed to define "physical" units (eV, K, etc.)

"physical" dilaton parameter $\tilde{\phi}_2 = \dfrac{\phi_2}{z_{\mathrm{H}}^2}$;   "physical" temperature $T = \dfrac{|f'(z_{\mathrm{H}})|}{4\pi} = \dfrac{1}{\pi z_{\mathrm{H}}}$

$$\tilde{\phi}_2 = \text{const} \;\Rightarrow\; z_{\mathrm{H}} = \frac{\phi_2}{\tilde{\phi}_2} \;\Rightarrow\; \text{“Temperatute” in “horizon” units } T = \frac{1}{\pi}\sqrt{\frac{\tilde{\phi}_2}{\phi_2}} \sim \frac{1}{\sqrt{\phi_2}}$$

To recover $\tilde{\phi}_2$, we have to include gauge interaction of $\mathcal{G}$

$$\mathcal{D}_\mu \Sigma = \partial_\mu \Sigma + [A_\mu, \Sigma]$$

and include terms of the interaction CHM with SM.

“Symmetries” of the CHM Potential

$V_\chi(\chi) = \frac{m^2}{2}\chi^2 - \frac{D}{4L^2}\lambda\chi^4 + \frac{\lambda^2\gamma}{6L^2}\chi^6$ is the expantion of a more general theory

Suggestions:

- The potential V_χ always has true vacuum with $E_{\min}$ ($V_\chi \xrightarrow{\chi\to\pm\infty} \infty$). So we may use any even power χ^n instead of the last term χ^6.
- The expansion of V_χ has certain sign of the second term $\lambda > 0$ (the first one m^2 chosen for the theory to be conformal in AdS).
- Higher orders of the expansion don't give new minima at the considered temperatures.

The certain parametrization has been chosen with respect to the “symmetries”

“Scale invariace”, defining the coefficents : $\begin{matrix} L \to L' \\ \chi \to \sqrt{\lambda}\chi \end{matrix}$; Conformality near the AdS border (“correct” conformal weights) : $\begin{matrix} \Delta_- = 1 \\ \Delta_+ = 3 \end{matrix} \Rightarrow m^2 = -\frac{D}{3L^2}$

D is for *the Large D limit*. But its usage doesn't give any results.
(to keep interaction constants finite at $D \to \infty$)

CHM potential isn't "Tuned"

$$V_\chi = a_2\chi^2 + a_4\chi^4 + a_6\chi^6, \quad a_2 < 0,\ a_4 < 0,\ a_6 > 0$$
$$V_{\text{eff}} = b_2\langle\varphi\rangle^2 + b_4\langle\varphi\rangle^4 + b_6\langle\varphi\rangle^6, \quad b_2 > 0,\ b_4 < 0,\ b_6 > 0$$

in details:

- $V_{\text{eff}} = V_{\text{eff}}[\langle\varphi\rangle]$ describes a quantum objects at the border. V_χ is a dual classical potential in the bulk.
- $V_{\text{eff}} = -\frac{1}{\text{Vol}_4} S_{\text{AdS}}\Big|_{\partial\text{AdS}}$ includes the solutions of the EoM $\frac{\delta S_\chi}{\delta\chi=0}$ in bulk. In other words, V_{eff} includes physics of AdS