Extended Chern–Simons Model for a Vector Multiplet

based on arXiv:2105.01292

O. D. Nosyrev, D. S. Kaparulin, S. L. Lyakhovich

Moscow International School of Physics
National Reseach Tomsk State University
Faculty of Physics
Department of Quantum Field Theory

Dubna – 2022

Table of contents

Models with higher derivatives

+: Better convergence at the classical and quantum level.

+: Wide symmetry.

–: Instability of dynamics(ghost states).

–: Interaction is usually inconsistent with the stability of dynamics.

–: Models with a degenerate mass spectrum are usually unstable.

Goals of the study

Goals

- : the study of the extended Chern-Simons theory with higher derivatives, including the model with a resonance.
- : to study the structure of symmetries and conservation laws of the free theory from the stability view point;
- : to construct a new class of stable interactions admitting a Hamiltonian form of dynamics with a bounded Hamiltonian;
- : to propose a mechanism for stabilizing dynamics in the presence of resonance

ECS theory of the 3rd order

Strength vector:

$$A^a = A^a{}_\mu(x)dx^\mu, \qquad \mu = 0, 1, 2; \qquad a = 1, 2, \ldots, n. \tag{1}$$

Action functional:

$$S[A(x)] = \frac{1}{2}\int A^a{}_\mu(\alpha_1 F^{a\mu} + \alpha_2 G^{a\mu} + \alpha_3 K^{a\mu})d^3x\,.$$
$$F^a{}_\mu = \varepsilon_{\mu\nu\rho}\partial^\nu A^{a\rho}\,, \quad G^a{}_\mu = \varepsilon_{\mu\nu\rho}\partial^\nu F^{a\rho}\,, \quad K^a{}_\mu = \varepsilon_{\mu\nu\rho}\partial^\nu G^{a\rho}\,, \tag{2}$$

α_1, α_2, α_3 a parameters of the model;
$\epsilon_{\mu\nu\rho}$ is the 3d Levi-Civita symbol with $\epsilon_{012} = 1$.
Equation of motion:

$$\frac{\delta S}{\delta A^{a\mu}} \equiv \alpha_1 F^a{}_\mu + \alpha_2 G^a{}_\mu + \alpha_3 K^a{}_\mu = 0\,, \qquad a = 1, \ldots, n\,. \tag{3}$$

Symmetries and conservation laws

Series of symmetry transformations:

$$\delta_{\xi;\beta} A^a{}_\mu = -\varepsilon_{\mu\nu\rho}\xi^\rho(\beta^a{}_1 F^{a\rho} + \beta^a{}_2 G^{a\rho}) , \qquad a = 1, \dots, n,$$
$$\delta_{\xi;\beta} S[A(x)] = 0. \tag{4}$$

where ξ^μ and $\beta^a{}_k$, $a = 0, \dots, n$, $k = 1, 2$ - parameters of transformation. $2n$ parametric series of conserved tensors rank 2:

$$\Theta^{\mu\nu}(\beta;\alpha) = \sum_{a=1}^{n}(\beta^a{}_1\Theta^a{}_1(\alpha) + \beta^a{}_2\Theta^a{}_2(\alpha)) , \tag{5}$$

where

$$\Theta^{a\mu\nu}{}_1(\alpha) = \alpha_3(G^{a\mu}F^{a\nu} + G^{a\nu}F^{a\mu} - g^{\mu\nu}G^a{}_\rho F^{a\rho}) +$$
$$+\alpha_2(F^{a\mu}F^{a\nu} - \frac{1}{2}g^{\mu\nu}F^a{}_\rho F^{a\rho}) ; \tag{6}$$

$$
\Theta^{a\mu\nu}{}_2(\alpha) = \alpha_3(G^{a\mu}G^{a\nu} - \frac{1}{2}g^{\mu\nu}G^a{}_\rho G^{a\rho}) - \alpha_1(F^{a\mu}F^{a\nu} -
-\frac{1}{2}g^{\mu\nu}F^a{}_\rho F^{a\rho}) \,. \tag{7}
$$

00-component:

$$
\Theta^{00}(\beta;\alpha) = \frac{1}{2}\sum_{a=1}^{n}\Big\{\beta_2\alpha_3(G^{a0}G^{a0} + G^{ai}G^{ai}) +
+2\beta_1\alpha_3(G^{a0}F^{a0} + G^{ai}F^{ai}) + (\beta_1\alpha_2 - \beta_2\alpha_1)(F^{a0}F^{a0} + F^{ai}F^{ai})\Big\}\,, \tag{8}
$$

$$
i = 1, 2.
$$

where

$$
C(\beta^a;\alpha) = -(\beta^a{}_2)^2\alpha_1 + \beta^a{}_2\beta^a{}_1\alpha_2 - (\beta^a{}_1)^2\alpha_3\,. \tag{9}
$$

Stability conditions:

$$
\beta^a{}_2\alpha_3 > 0, \qquad C(\beta^a;\alpha) > 0\,, \qquad a = 1,\ldots,n\,. \tag{10}
$$

Inclusion of interaction

We assume that the dynamical fields take values in the Lie algebra of a semi-simple Lie group with the generators t^a, $a = 1, \dots, n$,

$$\mathcal{A}_\mu = A^a{}_\mu(x) t^a dx^\mu \,, \qquad [t^a, t^b] = f^{abc} t^c \,, \qquad \mathrm{tr}(t^a t^b) = \delta^{ab} \,. \tag{11}$$

Covariant derivative:

$$D_\mu = \partial_\mu + [\mathcal{A}_\mu, \cdot\,] \,. \tag{12}$$

Covariant strength vectors:

$$\mathcal{F}_\mu = \epsilon_{\mu\nu\rho}(\partial^\nu \mathcal{A}^\rho + \frac{1}{2}[\mathcal{A}^\nu, \mathcal{A}^\rho]) \tag{13}$$

$$\mathcal{G}_\mu = \epsilon_{\mu\nu\rho} D^\nu \mathcal{F}^\rho \tag{14}$$

$$\mathcal{K}_\mu = \epsilon_{\mu\nu\rho} D^\nu \mathcal{G}^\rho, \tag{15}$$

We seek for (non-Lagrangian) deformation of equations of motion

$$\mathbb{T}^\mu = \alpha_3 \mathcal{K}^\mu + \alpha_2 \mathcal{G}^\mu + \alpha_1 \mathcal{F}^\mu + \dots \tag{16}$$

that preserves gauge symmetries and gauge identities of the model.

Equation of motion:

$$\mathbb{T}^{\mu} = \alpha_3 \mathcal{K}^{\mu} + \alpha_2 \mathcal{G}^{\mu} + \alpha_1 \mathcal{F}^{\mu} - \\ - \frac{{\alpha_3}^2}{2C(\beta;\alpha)} \epsilon^{\mu\nu\rho} [\beta_1 \mathcal{F}_{\nu} + \beta_2 \mathcal{G}_{\nu}, \beta_1 \mathcal{F}_{\rho} + \beta_2 \mathcal{G}_{\rho}] = 0 \,. \tag{17}$$

Gauge identity:

$$\mathbb{D}_{\mu} \mathbb{T}^{\mu} = 0 \,, \qquad \mathbb{D}_{\mu} = D_{\mu} + \frac{\alpha_3}{C(\beta;\alpha)} [\beta_1 \mathcal{F}_{\mu} + \beta_2 \mathcal{G}_{\mu}, \cdot] \,. \tag{18}$$

Law of conservation:

$$\boldsymbol{\Theta}^{\mu\nu}(\beta;\alpha) = \mathrm{tr}\Big\{ \beta_2 \alpha_3 (\mathcal{G}^{\mu} \mathcal{G}^{\nu} - \frac{1}{2} g^{\mu\nu} \mathcal{G}_{\rho} \mathcal{G}^{\rho}) + \beta_1 \alpha_3 (\mathcal{G}^{\mu} \mathcal{F}^{\nu} + \\ + \mathcal{G}^{\nu} \mathcal{F}^{\mu} - g^{\mu\nu} \mathcal{G}_{\rho} \mathcal{F}^{\rho}) + (\beta_1 \alpha_2 - \beta_2 \alpha_1)(\mathcal{F}^{\mu} \mathcal{F}^{\nu} - \frac{1}{2} g^{\mu\nu} \mathcal{F}_{\rho} \mathcal{F}^{\rho}) \Big\} \,, \tag{19}$$

Stability conditions:

$$\beta_2 \alpha_3 > 0 \,, \qquad C(\beta;\alpha) > 0 \,. \tag{20}$$

Hamiltonian formalism

Hamiltonian system:

$$\dot{\varphi}^I(\mathbf{x}) = \{\varphi^I(\mathbf{x}), \textstyle\int \mathcal{H} d\boldsymbol{y}\,\}, \quad \theta_A(\varphi^I(\mathbf{x}), \boldsymbol{\nabla}\varphi^I(\mathbf{x}), \ldots) = 0\,;$$
$$\mathcal{H} = \mathcal{H}_0(\varphi^J(\mathbf{x}), \boldsymbol{\nabla}\varphi^J(\mathbf{x}), \ldots) + \lambda^A(\boldsymbol{x})\theta_A(\varphi^I(\mathbf{x}), \boldsymbol{\nabla}\varphi^I(\mathbf{x}), \ldots)\,. \tag{21}$$

Equations of evolution:

$$\dot{\mathcal{A}}_i = \partial_i \mathcal{A}_0 - [\mathcal{A}_0, \mathcal{A}_i] - \epsilon_{ij}\mathcal{F}_j\,; \tag{22}$$

$$\dot{\mathcal{F}}_i = \partial_i \mathcal{F}_0 + [\mathcal{A}_i, \mathcal{F}_0] - [\mathcal{A}_0, \mathcal{F}_i] - \epsilon_{ij}\mathcal{G}_j\,; \tag{23}$$

$$\dot{\mathcal{G}}_i = \partial_i \mathcal{G}_0 + \frac{\alpha_2}{\alpha_3}\epsilon_{ij}\mathcal{G}_j + \frac{\alpha_1}{\alpha_3}\epsilon_{ij}\mathcal{F}_j + [\mathcal{A}_i, \mathcal{G}_0] - [\mathcal{A}_0, \mathcal{G}_i] +$$
$$+ \frac{{\alpha_3}^2}{C(\beta;\alpha)}[\beta_1\mathcal{F}_0 + \beta_2\mathcal{G}_0, \beta_1\mathcal{F}_i + \beta_2\mathcal{G}_i]\,. \tag{24}$$

Constrains:

$$\Theta = \epsilon_{ij}(\alpha_1\partial_i\mathcal{A}_j + \alpha_2\partial_i\mathcal{F}_j + \alpha_3\partial_i\mathcal{G}_j + [\mathcal{A}_i, \frac{1}{2}\alpha_1\mathcal{A}_j + \alpha_2\mathcal{F}_j + \alpha_3\mathcal{G}_j] +$$
$$- \frac{\alpha_3}{2C(\beta;\alpha)}[\beta_1\mathcal{F}_i + \beta_2\mathcal{G}_i, \beta_1\mathcal{F}_j + \beta_2\mathcal{G}_j]) \approx 0\,. \tag{25}$$

Hamiltonian:

$$\mathcal{H}_0 = \frac{1}{2}\mathrm{tr}\Big\{\beta_2\alpha_3(\mathcal{G}^0\mathcal{G}^0 + \mathcal{G}^i\mathcal{G}^i) + 2\beta_1\alpha_3(\mathcal{G}^0\mathcal{F}^0 + \mathcal{G}^i\mathcal{F}^i) +$$
$$+(\beta_1\alpha_2 - \beta_2\alpha_1)(\mathcal{F}^0\mathcal{F}^0 + \mathcal{F}^i\mathcal{F}^i)\Big\} + \tag{26}$$
$$+\mathrm{tr}\Big[\frac{C(\beta;\alpha)}{\beta_2\alpha_2 - \beta_1\alpha_3}\mathcal{A}_0 - \frac{\beta_2\beta_1\alpha_3}{\beta_2\alpha_2 - \beta_1\alpha_3}\mathcal{F}_0 - \frac{{\beta_2}^2\alpha_3}{\beta_2\alpha_2 - \beta_1\alpha_3}\mathcal{G}_0\Big]\Theta\,.$$

Poisson brackets:

$$\{\mathcal{G}^a{}_i(\boldsymbol{x}), \mathcal{G}^b{}_j(\boldsymbol{y})\} = \frac{\beta_1{\alpha_2}^2 - \beta_1\alpha_1\alpha_3 - \beta_2\alpha_2\alpha_1}{{\alpha_3}^2 C(\beta;\alpha)}\epsilon_{ij}\delta^{ab}\delta^{(2)}(\boldsymbol{x}-\boldsymbol{y})\,; \tag{27}$$

$$\{\mathcal{F}^a{}_i(\boldsymbol{x}), \mathcal{G}^b{}_j(\boldsymbol{y})\} = \frac{\beta_2\alpha_1 - \beta_1\alpha_2}{\alpha_3 C(\beta;\alpha)}\epsilon_{ij}\delta^{ab}\delta^{(2)}(\boldsymbol{x}-\boldsymbol{y})\,; \tag{28}$$

$$\{\mathcal{F}^a{}_i(\boldsymbol{x}), \mathcal{F}^b{}_j(\boldsymbol{y})\} = \{\mathcal{A}^a{}_i(\boldsymbol{x}), \mathcal{G}^b{}_j(\boldsymbol{y})\} = \frac{\beta_1}{C(\beta;\alpha)}\epsilon_{ij}\delta^{ab}\delta^{(2)}(\boldsymbol{x}-\boldsymbol{y})\,; \tag{29}$$

$$\{\mathcal{A}^a{}_i(\boldsymbol{x}), \mathcal{F}^b{}_j(\boldsymbol{y})\} = \frac{-\beta_2}{C(\beta;\alpha)}\epsilon_{ij}\delta^{ab}\delta^{(2)}(\boldsymbol{x}-\boldsymbol{y})\,; \tag{30}$$

$$\{\mathcal{A}^a{}_i(\boldsymbol{x}), \mathcal{A}^b{}_j(\boldsymbol{y})\} = 0\,. \tag{31}$$

Poisson bracket between constrains:

$$\{\Theta^a(\boldsymbol{x}), \Theta^b(\boldsymbol{y})\} = \frac{\beta_2\alpha_2 - \beta_1\alpha_3}{C(\beta;\alpha)} f^{abc}\Theta^c(\boldsymbol{x})\delta(\boldsymbol{x}-\boldsymbol{y}). \tag{32}$$

Case of resonance

Action functional:

$$S[A(x)] = -\frac{1}{2}\int A^a{}_\mu K^{a\mu} d^3x\,. \tag{33}$$

Conservation law:

$$\begin{aligned}\mathbf{\Theta}^{\mu\nu}(\beta;\alpha) = -\beta_2(\mathcal{G}^\mu\mathcal{G}^\nu - \frac{1}{2}g^{\mu\nu}\mathcal{G}_\rho\mathcal{G}^\rho) - \beta_1(\mathcal{G}^\mu\mathcal{F}^\nu + \\ +\mathcal{G}^\nu\mathcal{F}^\mu - g^{\mu\nu}\mathcal{G}_\rho\mathcal{F}^\rho)\,.\end{aligned} \tag{34}$$

00-component:

$$\begin{aligned}\Theta^{00}(\beta;\alpha) = -\frac{1}{2}\sum_{a=1}^{n}\Big\{\beta_2(G^{a0}G^{a0} + G^{ai}G^{ai}) + 2\beta_1(G^{a0}F^{a0} + \\ +G^{ai}F^{ai})\Big\}\,.\end{aligned} \tag{35}$$

Stability conditions:

$$\beta^a{}_2 < 0, \qquad (\beta^a{}_1)^2 < 0\,, \qquad a = 1,\ldots,n\,. \tag{36}$$

Change parameters:

$$\alpha_1 = 0\,, \qquad \alpha_2 = \gamma^2\phi^2\,, \qquad \alpha_3 = \widetilde{\gamma}^2\phi^2 - 1 \qquad \widetilde{\gamma}^2 = \gamma^2\beta_2/\beta_1. \quad , \tag{37}$$

where $\phi(x)$ is new dynamic scalar field.
Equation of motion:

$$\mathbb{T}_\mu = \epsilon_{\mu\nu\rho}\Big\{D^\nu[(\widetilde{\gamma}^2\phi^2 - 1)\mathcal{G} + \gamma^2\phi^2\mathcal{F}]^\rho + \\ + \frac{(\widetilde{\gamma}^2\phi^2 - 1)^2}{2{\beta_1}^2}[\beta_1\mathcal{F}^\nu + \beta_2\mathcal{G}^\nu, \beta_1\mathcal{F}^\rho + \beta_2\mathcal{G}^\rho]\Big\} = 0\,, \tag{38}$$

$$\mathbb{T} = \Big\{\partial_\mu\partial^\mu - (m^2 - \widetilde{\gamma}^2\tfrac{(\beta_1\mathcal{F}_\rho + \beta_2\mathcal{G}_\rho)(\beta_1\mathcal{F}^\rho + \beta_2\mathcal{G}^\rho)}{{\beta_1}^2} + \phi^2\Big\}\phi = 0\,, \tag{39}$$

Hamiltonian:

$$
\begin{aligned}
\mathcal{H} = \mathrm{tr}\Big\{ & \frac{(\beta_2\mathcal{W}^0 - \beta_1\mathcal{F}^0)(\beta_2\mathcal{W}^0 - \beta_1\mathcal{F}^0)}{2(\widetilde{\gamma}^2\phi^2 - 1)\beta_2} + \\
& + \frac{(\beta_2\mathcal{W}^i - \beta_1\mathcal{F}^i)(\beta_2\mathcal{W}^i - \beta_1\mathcal{F}^i)}{2(\widetilde{\gamma}^2\phi^2 - 1)\beta_2} + \\
& + \frac{{\beta_1}^2}{2\beta_2}(\mathcal{F}^0\mathcal{F}^0 + \mathcal{F}^i\mathcal{F}^i) - (\mathcal{A}_0 - \beta_1\mathcal{F}_0 + \beta_2\mathcal{W}_0)\Theta\Big\} + \\
& + \frac{1}{2}(\pi\pi + \partial^i\phi\partial^i\phi) - \frac{1}{2}m^2\phi^2 + \frac{1}{4}\phi^4 \quad ,
\end{aligned}
\tag{40}
$$

where $\pi(\boldsymbol{x}) = \dot{\phi}(\boldsymbol{x})$,

$$
\mathcal{W}_\mu = (\widetilde{\gamma}^2\phi^2 - 1)\mathcal{G}_\mu + \gamma^2\phi^2\mathcal{F}_\mu \, . \tag{41}
$$

Stability conditions:

$$
\widetilde{\gamma}^2\phi^2 - 1 > 0 \, , \qquad \beta_2 > 0 \, . \tag{42}
$$

Poisson bracket:

$$\{\mathcal{W}^a{}_i(\boldsymbol{x}), \mathcal{W}^b{}_j(\boldsymbol{y})\} = \{\mathcal{F}^a{}_i(\boldsymbol{x}), \mathcal{W}^b{}_j(\boldsymbol{y})\} = \{\mathcal{A}^a{}_i(\boldsymbol{x}), \mathcal{F}^b{}_j(\boldsymbol{y})\} = 0\,; \tag{43}$$

$$\{\mathcal{A}^a{}_i(\boldsymbol{x}), \mathcal{W}^b{}_j(\boldsymbol{y})\} = -\{\mathcal{F}^a{}_i(\boldsymbol{x}), \mathcal{F}^b{}_j(\boldsymbol{y})\} = \frac{1}{\beta_1}\epsilon_{ij}\delta^{ab}\delta^{(2)}(\boldsymbol{x}-\boldsymbol{y})\,; \tag{44}$$

$$\{\mathcal{A}^a{}_i(\boldsymbol{x}), \mathcal{F}^b{}_j(\boldsymbol{y})\} = \frac{\beta_2}{{\beta_1}^2}\epsilon_{ij}\delta^{ab}\delta^{(2)}(\boldsymbol{x}-\boldsymbol{y})\,; \tag{45}$$

$$\{\phi(\boldsymbol{x}), \pi(\boldsymbol{y})\} = \delta^{(2)}(\boldsymbol{x}-\boldsymbol{y})\,. \quad . \tag{46}$$

Conclusion

- : In this presentation, an extended 3rd-order Chern Simons theory with non-Abelian gauge symmetry is constructed.
- : The model admits a Hamiltonian formulation of dynamics with a bounded Hamiltonian.
- : The case of resonance is considered, in which a Higgs-type mechanism is used to stabilize the dynamics.

THANK YOU FOR YOUR ATTENTION!