Self-Tuning Inflation

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We develop an inflationary model without small parameters on the basis of multidimensional $f(R)$ gravity with a minimally coupled scalar field. The model is described by two stages. The first one begins at energy scales about the D-dimensional Planck mass and ends with the de Sitter metric of our space and the maximally symmetric extra dimensions. In the following, the quantum fluctuations produce a wide set of inhomogeneous extra metrics in causally disconnected regions quickly generated in the de Sitter space. We find a specific extra space metric that leads to effective model that fits the observational data.

Set up

Consider the quadratic $f(R)$ theory with a minimally coupled scalar field φ in $\mathrm{D} = 4 + \mathrm{n}$ dimensions

$$S = \frac{m_{\mathrm{D}}^{\mathrm{D}-2}}{2} \int d^{\mathrm{D}}x \sqrt{|g_{\mathrm{D}}|} \Big(f(R) + \partial^{\mathrm{M}}\varphi\, \partial_{\mathrm{M}}\varphi - 2V(\varphi) \Big), \quad f(R) = aR^2 + R + c$$

for the chosen metric of $M_1 \times M_3 \times M_{\mathrm{n}}$ manifold as

$$ds^2 = dt^2 - \mathrm{e}^{2\alpha(t)}\delta_{ij}dx^i dx^j - \mathrm{e}^{2\beta(t)} m_{\mathrm{D}}^{-2} \left(\left(dx^4\right)^2 + r^2(x^4)\left(dx^5\right)^2 + \ldots + r^2(x^4) \prod_{\mathrm{k}=5}^{\mathrm{D}-2} \left(\sin^2 x^{\mathrm{k}}\right) \left(dx^{\mathrm{k}+1}\right)^2 \right)$$

The first stage of inflation: high energies

At the first stage, all dynamical variables depend only on time. At the end of this stage, one can show that the extra absolutely symmetrical space, i.e. $r(x^4) = \sin x^4$, size and the Hubble parameter are constants.

The asymptotic behavior of the metric and the scalar field from equations is: $\varphi_{as} = 0$ and

$$H_{as}^2 = \frac{-(\mathrm{n}+2) \pm \sqrt{(\mathrm{n}+2)^2 - 4a\mathrm{n}(\mathrm{n}+4)c}}{6a\mathrm{n}(\mathrm{n}+4)}; \quad \mathrm{e}^{-2\beta_{as}} = \frac{-(\mathrm{n}+2) \pm \sqrt{(\mathrm{n}+2)^2 - 4a\mathrm{n}(\mathrm{n}+4)c}}{2a\mathrm{n}(\mathrm{n}+4)(\mathrm{n}-1)\, m_{\mathrm{D}}^2} \equiv \mathrm{e}^{-2\beta_{\mathrm{c}}}.$$

Action takes the following form after integration over the extra coordinates

$$S_{eff} = \frac{m_{Pl}^2}{2} \int d^4x \sqrt{|g_4|} \left(a_{eff} R_4^2 + R_4 + c_{eff} \right)$$

with the effective parameters

$$\frac{a_{eff} m_{\mathrm{Pl}}^2}{2} = \frac{\pi^{\frac{\mathrm{n}+1}{2}} m_{\mathrm{D}}^{-\mathrm{n}} \mathrm{e}^{\mathrm{n}\beta_{\mathrm{c}}}}{\Gamma\left(\frac{\mathrm{n}+1}{2}\right)} f_{RR}(R_{\mathrm{n}}), \qquad \frac{m_{\mathrm{Pl}}^2}{2} = \frac{2\pi^{\frac{\mathrm{n}+1}{2}} m_{\mathrm{D}}^{-\mathrm{n}} \mathrm{e}^{\mathrm{n}\beta_{\mathrm{c}}}}{\Gamma\left(\frac{\mathrm{n}+1}{2}\right)} f_R(R_{\mathrm{n}}), \qquad \frac{c_{eff} m_{\mathrm{Pl}}^2}{2} = \frac{2\pi^{\frac{\mathrm{n}+1}{2}} m_{\mathrm{D}}^{-\mathrm{n}} \mathrm{e}^{\mathrm{n}\beta_{\mathrm{c}}}}{\Gamma\left(\frac{\mathrm{n}+1}{2}\right)} f(R_{\mathrm{n}})$$

which are related to the Ricci scalar $R_{\mathrm{n}} = \mathrm{n}(\mathrm{n}-1)\, m_{\mathrm{D}}^2 \mathrm{e}^{-2\beta_{\mathrm{c}}}$ of the extra dimensions.

The space expansion and the metric or scalar field fluctuations can last for an arbitrarily long time due to the Hubble parameter being equal to a constant. Their scale quickly overcomes the present horizon. The fluctuations within the extra space are of most interest. Some of them can deform the extra dimensions significantly. It leads to an alternation of the Lagrangian parameters a_{eff} and c_{eff} and launches the second, low energy step of inflation.

The second stage of inflation: low energy

The high-energy stage is finished with the de-Sitter 4-dim metric and maximally symmetrical extra space. As usual, the space expands exponentially, producing more and more causally disconnected volume, each of which is characterized by a specific extra space metric and a scalar field distribution.

Let us consider an inhomogeneous n-dim extra metric with the renaming of the coordinate $x^4 \equiv u$ and taking $\mathrm{e}^{2\beta_{\mathrm{c}}}$ for convenience

$$ds^2 = dt^2 - \mathrm{e}^{2Ht}\left(\left(dx^1\right)^2 + \left(dx^2\right)^2 + \left(dx^3\right)^2 \right) - \mathrm{e}^{2\beta_{\mathrm{c}}} m_{\mathrm{D}}^{-2} \left(du^2 + r^2(u)\, d\Omega_{\mathrm{n}-1}^2 \right).$$

The combination of equations $\big((tt) - (x^5x^5)\big) \cdot (\mathrm{n}-1) - (uu) - R \cdot f_R$ has only the first-order derivatives and can be used as a restriction on the boundary conditions of the coupled second-order differential equations:

$$\Big(3(\mathrm{n}+3)H^2 - R\Big) f_R + \frac{f(R)}{2} = -\frac{(\varphi')^2}{2} m_{\mathrm{D}}^2 \mathrm{e}^{-2\beta_{\mathrm{c}}} + V(\varphi).$$

In case of homogeneous scalar field distribution there is a solution R_{c} of constant curvature for any form of $f(R)$ function. For constant curvature the difference of $(tt) - (uu)$-components of equations allows to find analytically the function $r(u)$:

$$r(u) = m_{\mathrm{D}} \mathrm{e}^{-\beta_{\mathrm{c}}} \frac{\sqrt{(\mathrm{n}-1)}}{\sqrt{3}H} \sin\left(\frac{\sqrt{3}H}{\sqrt{(\mathrm{n}-1)}} \mathrm{e}^{\beta_{\mathrm{c}}} m_{\mathrm{D}}^{-1} u \right)$$

$\forall f_R(R_{\mathrm{c}}) \neq 0$ and then $R_{\mathrm{c}} = 12H^2 + 3\mathrm{n}H^2$. Our reasoning does not dependent on the specific form of $f(R)$.

We obtain numerical solution of a system of equations in case of inhomogeneous scalar field distribution resolved with respect to unknown functions $r(u)$, $\varphi(u)$, and $R(u)$ with regular boundary conditions (for details see [2204.04647])

$$r(u_0) = 0, \quad r'(u_0) = 1, \quad R(u_0) = R_0, \quad \varphi(u_0) = \varphi_0, \quad R'(u_0) = \varphi'(u_0) = 0$$

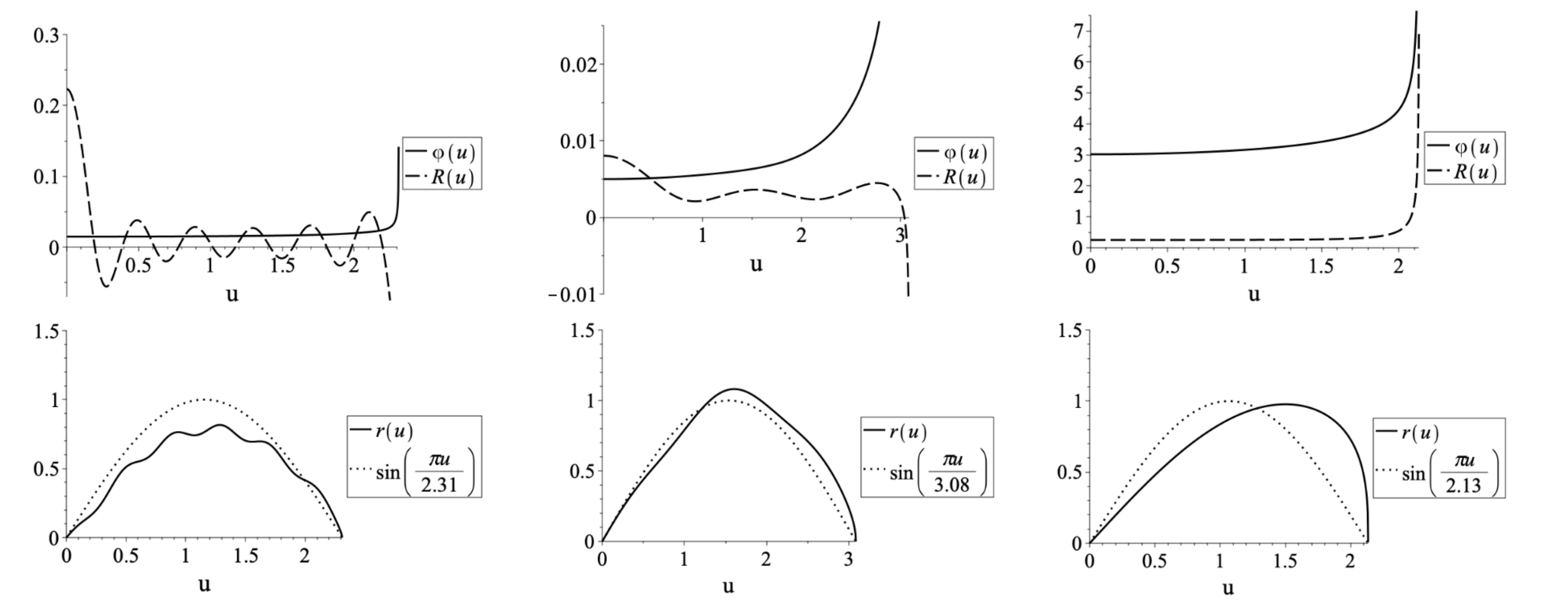
and R_0 and φ_0 values are linked by the constraint equation above. The deformed configuration leads to an alternation of the Lagrangian parameters a_{eff} and c_{eff} and launches the second, low-energy step of inflation. After integration over extra coordinates, action turns to the effective theory with the effective values of the parameters determined by the following expressions:

$$a_{eff} = \frac{2\pi^{\frac{\mathrm{n}}{2}} \mathrm{e}^{\mathrm{n}\beta_{\mathrm{c}}}}{\Gamma\left(\frac{\mathrm{n}}{2}\right)} \frac{m_{\mathrm{D}}^2}{m_{\mathrm{Pl}}^2} \int_{u_{min}}^{u_{max}} f_{RR}\big(R_{\mathrm{n}}(u)\big) r^{\mathrm{n}-1}(u) du,$$

$$\frac{m_{\mathrm{Pl}}^2}{m_{\mathrm{D}}^2} = \frac{4\pi^{\frac{\mathrm{n}}{2}} \mathrm{e}^{\mathrm{n}\beta_{\mathrm{c}}}}{\Gamma\left(\frac{\mathrm{n}}{2}\right)} \mathrm{e}^{\mathrm{n}\beta_{\mathrm{c}}} \int_{u_{min}}^{u_{max}} f_R\big(R_{\mathrm{n}}(u)\big) r^{\mathrm{n}-1}(u) du,$$

$$c_{eff} = \frac{4\pi^{\frac{\mathrm{n}}{2}} \mathrm{e}^{\mathrm{n}\beta_{\mathrm{c}}}}{\Gamma\left(\frac{\mathrm{n}}{2}\right)} \frac{m_{\mathrm{D}}^2}{m_{\mathrm{Pl}}^2} \int_{u_{min}}^{u_{max}} \Big(f\big(R_{\mathrm{n}}(u)\big) - \big(\varphi'(u)\big)^2 m_{\mathrm{D}}^2 \mathrm{e}^{-2\beta_{\mathrm{c}}} - 2V\big(\varphi(u)\big) \Big) r^{\mathrm{n}-1}(u)\, du.$$

[1] Starobinsky, Phys. Lett. B (1980).
[2] Mukhanov, Chibisov, ZhETF Pisma (1981).
[3] Planck 2018 results. X. Constraints on inflation, Astron. & Astrophys. (2020).
[4] Tristram et al., Phys. Rev. D (2022).



(a) : $a = -1.5$, $c = -0.005$, $\mathrm{n} = 3$, $m = 0.01$, $\beta_{\mathrm{c}} = 3.80$, $H = 0.018$, $\varphi_0 = 0.015$ and $R_0 \simeq 0.223$.

(b) : $a = -45$, $c = -0.0021$, $\mathrm{n} = 4$, $m = 0.01$, $\beta_{\mathrm{c}} = 4.48$, $H = 0.011$, $\varphi_0 = 0.005$ and $R_0 \simeq 0.008$.

(c) : $a = 20$, $c = -0.95$, $\mathrm{n} = 6$, $m = 0.05$, $\beta_{\mathrm{c}} = 2.65$, $H = 0.092$, $\varphi_0 = 3.02$ and $R_0 \simeq 0.254$.

Fig. 1: Numerical solution for the functions $r(u)$, $\varphi(u)$, $R(u)$ the given parameters and boundary conditions.

Fitting to observational data

We assume that it is the second step of inflation that is responsible for such observable parameters as the power spectra of scalar curvature perturbations and tensor perturbations. Our model should not contradict observations and the most economic way to find appropriate parameters is to notice that the structure of our effective action coincides with that of the Starobinsky model[1]. The latter is known to fit observations quite well. Therefore, it remains only to match the parameters of the Starobinsky model with the parameters of our model. The inflationary predictions originally calculated to the lowest order[2] and R^2 multiplier from the COBE normalization[3]

$$n_s \simeq 1 - \frac{2}{N_{\mathrm{e}}} \simeq 0.9649 \pm 0.0042, \quad r \simeq \frac{12}{N_{\mathrm{e}}^2} < 0.032, \quad \text{and} \quad a_{\mathrm{Starob}} \simeq 1.12 \cdot 10^9 \left(\frac{N_{\mathrm{e}}}{60}\right)^2 m_{\mathrm{Pl}}^{-2}$$

are in good agreement with the Planck 2018 data with a combination of BICEP/Keck Array'18 and BAO[4] for the number of e-folds in the range of $50 < N_{\mathrm{e}} < 60$.

There are some metrics that are responsible for the observed, secondary stage of inflation. In this case, they should restore the Starobinsky model. Our analysis shows that the parameter values of our model $a = 20$, $c = -0.95$, $\mathrm{n} = 6$, $m = 0.05$ suit our aims. The effective value of $a_{eff} m_{\mathrm{Pl}}^2 \simeq a_{\mathrm{Starob}} m_{\mathrm{Pl}}^2 \sim 10^9$, the D-dimensional mass becomes $m_{\mathrm{D}} \sim 10^{14}$ GeV, c_{eff} is negligibly small, $H \sim 10^{13}$ GeV and the average size of the extra space $\sim 10^{-27}$ cm at the boundary value of the scalar field $\varphi(0) = \varphi_0 \simeq 3.015$.

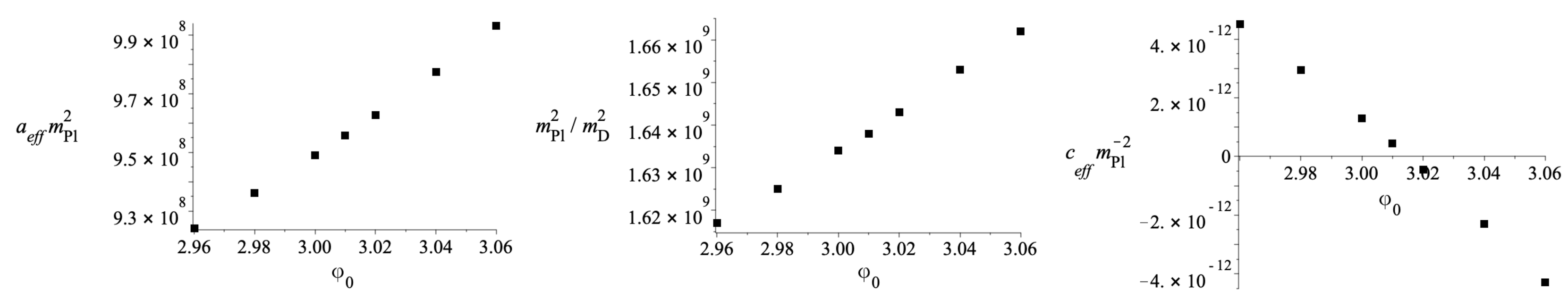


Fig. 2: The dependence of the effective coefficients a_{eff}, c_{eff} and the ratio of the 4-dim Planck mass to the D-dim Planck mass on the boundary value of the scalar field φ_0 for the given parameters and boundary conditions. An example of a solution for such a set of parameters is shown in Fig. 1(c) above.

$H \to 0$ limit

As we can see, at the first stage, there is a ratio independent of the choice of a specific form of $f(R)$ function $3H^2 = (\mathrm{n}-1) m_{\mathrm{D}}^2 \mathrm{e}^{-2\beta_{\mathrm{c}}}$. In the $H \to 0$ limit, obtained expression leads to an infinitely large size of extra dimensions $\mathrm{e}^{\beta_{\mathrm{c}}}$, which obviously contradicts observations.

Whereas in the case of an inhomogeneous internal subspace, integrating the (tt)-component

$$\int_{u_{min}}^{u_{max}} \left[\cancel{\left((R')^2 f_{RRR} + \left(R'' + (\mathrm{n}-1)\frac{r'}{r} R' \right) f_{RR} \right) m_{\mathrm{D}}^2 \mathrm{e}^{-2\beta_{\mathrm{c}}}} + 3H^2 f_R - \frac{f(R)}{2} + \frac{(\varphi')^2}{2} m_{\mathrm{D}}^2 \mathrm{e}^{-2\beta_{\mathrm{c}}} + V(\varphi) \right] r^{\mathrm{n}-1}(u)\, du = 0$$

comparing with the above expression of c_{eff} leads to

$$\frac{c_{eff} m_{\mathrm{Pl}}^2}{2} = \frac{\pi^{\frac{\mathrm{n}}{2}} m_{\mathrm{D}}^2 \mathrm{e}^{\mathrm{n}\beta_{\mathrm{c}}}}{\Gamma\left(\frac{\mathrm{n}}{2}\right)} \int_0^{u_{max}} \Big(6H^2 f_R(R_{\mathrm{D}}) - f(R_{\mathrm{D}}) + f(R_{\mathrm{n}}) \Big) r^{\mathrm{n}-1}(u)\, du, \quad \text{here } R_{\mathrm{D}} = 12H^2 + R_{\mathrm{n}}.$$

So that $c_{eff} \xrightarrow{H \to 0} 0$, that looks reasonable for the effective action as the Starobinsky model and we can find a solution for a compact subspace at $H \simeq 0$, in contrast to the case of a maximally symmetric one.

Conclusions

- Many inflationary models explain observational data at the cost of using a small parameter to account for the smallness of the Hubble parameter $H \sim 10^{-6} m_{\mathrm{Pl}}$. Also, it is implicitly assumed that one of the model parameters related to the cosmological constant is extremely small (fine-tuned).
- In this work, we elaborate the inflationary model without unacceptably small or large parameters of the Lagrangian: $a = 20 m_{\mathrm{D}}^{-2}$, $c = -0.95 m_{\mathrm{D}}^2$, $m = 0.05 m_{\mathrm{D}}$, and $\mathrm{n} = 6$. The effective parameters $a_{eff} m_{\mathrm{Pl}}^2 \sim 10^9$, $c_{eff} m_{\mathrm{Pl}}^{-2} \simeq 0$ suitable for the experimental data are formed by the inhomogeneous extra metric. We also show the way to a significant decrease in the cosmological constant.
- We plan to investigate the stability of the obtained solutions of inhomogeneous extra space and explore their effects on inflationary predictions and observable low-energy physics, e.g. on the electroweak scale.