Monte Carlo event generators for neutrino-nucleus scattering in the few-GeV region

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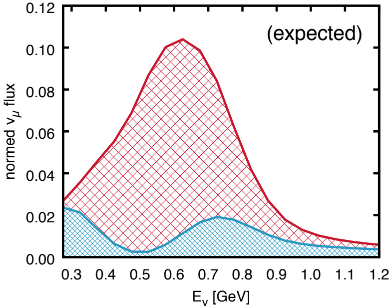
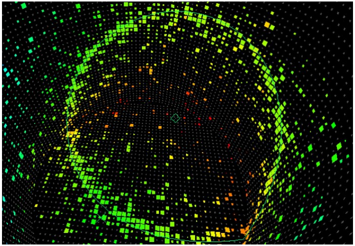
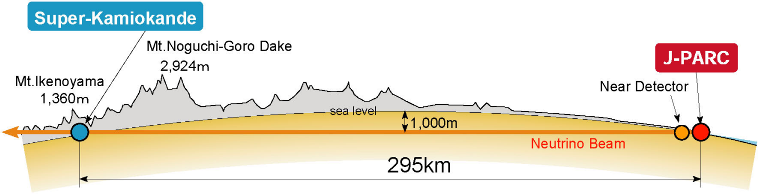
Neutrino oscillation experiments

$$P_{2\mathrm{f}}(\nu_\mu \to \nu_\mu) = 1 - \sin^2(2\theta)\sin^2\left(\frac{\Delta m^2 L}{4E_\nu}\right)$$

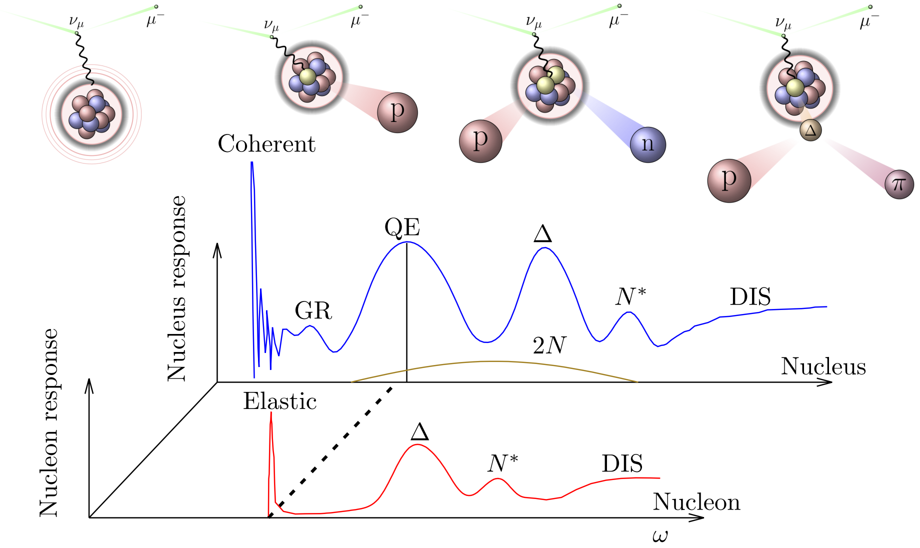




$$E_\nu^{\mathrm{rec}} = \frac{2(M_n - E_B)E_\mu - (E_B^2 - 2M_nE_B + m_\mu^2)}{2[M_n - E_B - E_\mu + |\vec{k}_\mu|\cos\theta_\mu]}$$

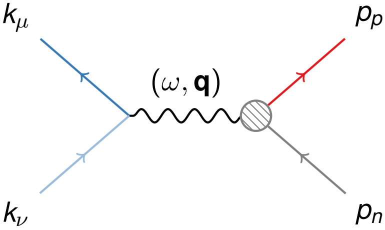


Nuclear response



T. Van Cuyck

Dimensionality of the problem



any binary scattering with on-shell particles

4 four-vectors = **16 variables**

- **4** : on-shell relations
- **4** : 4-mom. conservation
- **3** : nucleon rest frame
- **2** : neutrino along $\hat{z}$

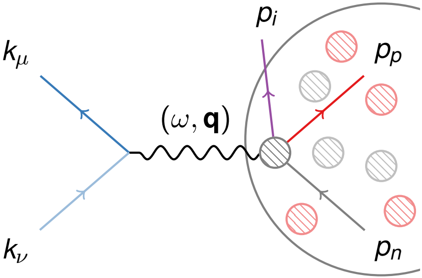
3 independent **variables**

$\rightarrow$ we can fix incoming energy (E_ν)

$\rightarrow$ the cross section is rotationally invariant (ϕ_μ)

$\rightarrow$ the final formula is 1-dimensional, e.g. $\mathrm{d}\sigma/\mathrm{d}q^2$

Dimensionality of the problem



scatterings including an off-shell target

3 independent **variables**

+ 3 : nucleus rest frame

+ 1 : off-shell nucleon

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7 independent **variables**

+ 3 : every on-shell particle

$\rightarrow$ we can fix incoming energy (E_ν)

$\rightarrow$ the cross section is rotationally invariant (ϕ_μ)

$\rightarrow$ the final formula is at least 5-dimensional

Computing νA cross section

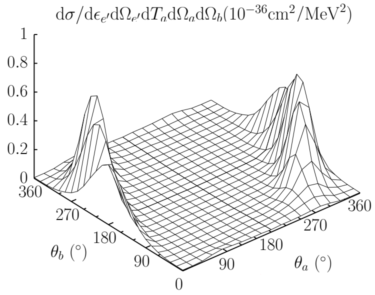
Monte Carlo generator

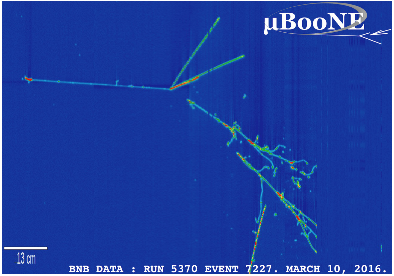
$\rightarrow$ generate **events**

$\rightarrow$ cover **whole phase space**

$\rightarrow$ useful but **approximated**

e.g. **NuWro**





Detailed calculation

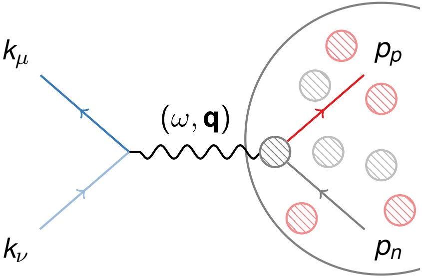
$\rightarrow$ compute **cross sections**

$\rightarrow$ **fixed kinematics**

$\rightarrow$ precise but **expensive**

e.g. **Ghent** group

Essential assumptions: Born approximation



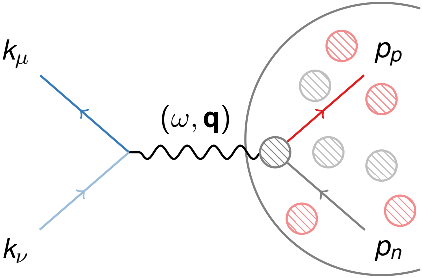
$$\sigma \propto L_{\mu\nu}(k_\nu, k_\mu) W^{\mu\nu}(p_n, q, p_p)$$

$$W^{\mu\nu} = J^{\mu\dagger} J^\nu$$

$$J^\mu = \int_X \mathrm{d}\mathbf{r}\; e^{i\mathbf{r}\cdot\mathbf{q}}\; \overline{\Psi}_f \mathcal{O}^\mu \Psi_i$$

- $\rightarrow$ only **one boson** exchange
- $\rightarrow$ **leptonic** ($L_{\mu\nu}(k_\nu, k_\mu)$) and **hadronic** ($W^{\mu\nu}(p_n, q, p_p)$) parts are **fully separable**
- $\rightarrow$ **nuclear modeling** deals with finding proper states Ψ_i, Ψ_f

Essential assumptions: Impulse approximation



$$\Psi_{i,f} = \sum \phi_N \otimes \phi_{A-1}$$

$$J^\mu = \int_X \mathrm{d}\mathbf{r}\; e^{i\mathbf{r}\cdot\mathbf{q}}\, \overline{\psi}_N \mathcal{O}^\mu \phi_i$$

$$J^\mu = \int \mathrm{d}\mathbf{p}'_N \int \frac{\mathrm{d}\mathbf{p}}{(2\pi)^{3/2}} \times \overline{\psi}_{sN}(\mathbf{p}'_N, \mathbf{p}_N) \mathcal{O}^\mu(q, p'_N) \phi_\kappa^{m_j}(\mathbf{p})$$

→ **interaction** with only **one particle** of a complex system

→ reduces the problem to finding only **single-particle states**

→ final wave functions are still under the effect of the nuclear potential

Essential assumptions: Plane waves

$\rightarrow$ no distortions, so one uses **asymptotic momenta** for final states:

$$J^{\mu} = \cancel{\int \mathrm{d}\mathbf{p}'_N} \int \frac{\mathrm{d}\mathbf{p}}{(2\pi)^{3/2}} \overline{\psi}_{sN}(\cancel{\mathbf{p}'_N}, \mathbf{p}_N) \mathcal{O}^{\mu}(q, \cancel{p'_N}) \phi_{\kappa}^{m_j}(\mathbf{p})$$

$$W^{\mu\nu} \propto \mathrm{Tr}(\phi_b(\mathbf{p}) \overline{\phi}_b(\mathbf{p}) \mathcal{O}^{\mu}(\not{p}_N + M) \mathcal{O}^{\nu})$$

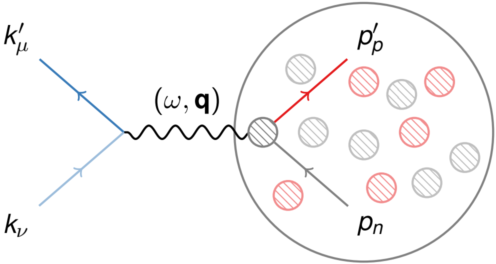
$\rightarrow$ **Relativistic Plane-wave** Impulse approximation

$\rightarrow$ one makes a projection to **positive energy states**

$$W^{\mu\nu} \propto |\phi_b(\mathbf{p})|^2 \mathrm{Tr}((\not{p} + M) \mathcal{O}^{\mu}(\not{p}_N + M) \mathcal{O}^{\nu})$$

$\rightarrow$ **Plane-wave** Impulse approximation

Plane-wave Impulse approximation



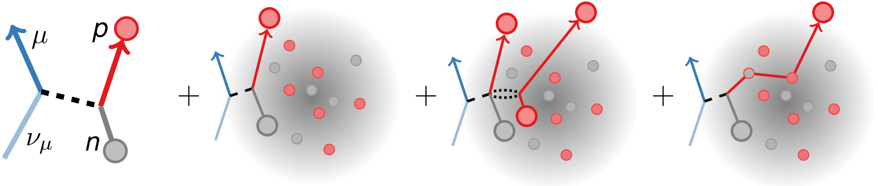
Factorization in the **absence of final-state interactions**:

$$\frac{\mathrm{d}^6\sigma^{\mathrm{PWIA}}}{\mathrm{d}\omega \mathrm{d}|\mathbf{q}|\mathrm{d}E_m \mathrm{d}\mathbf{p}_m} = \frac{G_F^2 \cos^2\theta_C |\mathbf{q}|}{4\pi E_{\mathbf{k}}^2 E_{\mathbf{p}} E_{\mathbf{p}'}} P_{(n)}(E_m, \mathbf{p}_m) L_{\mu\nu} \widetilde{H}^{\mu\nu} \delta(\omega + M - E_m - E_{\mathbf{p}'})$$

$P_{(n)}(E_m, \mathbf{p}_m)$ - probability density of initial nucleons

$L_{\mu\nu} \widetilde{H}^{\mu\nu} \delta(\omega + M - E_m - E_{\mathbf{p}'})$ - interaction dynamics for a given nucleon

Cross section in the factorized scheme



Neutrino-nucleon scattering | Initial nuclear state | Extra nuclear effects | Final state interactions

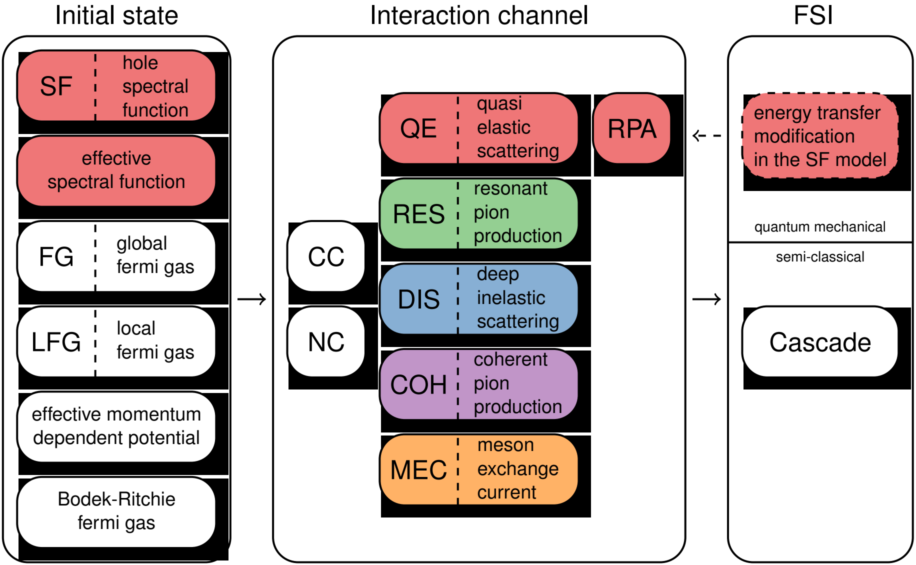
→ **Neutrino-nucleon scattering**: elementary interaction cross section

→ **Initial nuclear state**: modeling nucleons in the nuclear medium before the weak interaction

→ **Extra nuclear effects**: multiple-nucleon interactions or correlations

→ **Final state interactions**: in-medium outgoing particle propagation

NuWro blueprint



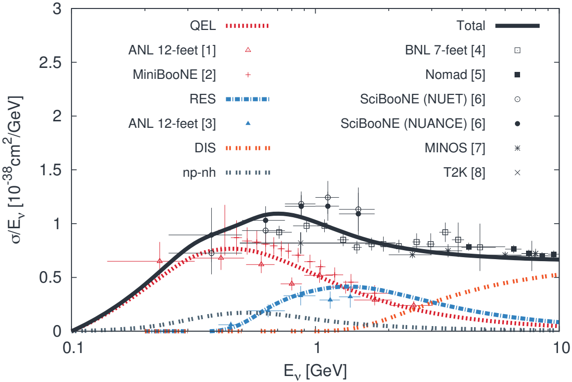
New models

User friendly

Wide applicability

Reliable results

Optimized code



[1] PRD 19 (1979) 2521
[2] PRD 81 (2010) 092005
[3] PRD 16 (1977) 3103
[4] PRD 25 (1982) 617
[5] PLB 660 (2008) 19
[6] PRD 83 (2011) 012005
[7] PRD 81 (2011) 072002
[8] PRD 87 (2013) 092003

Collaborators

Wrocław group

- Jan Sobczyk
  - Tomasz Bonus
  - Krzysztof Graczyk
  - Cezary Juszczak
  - Dmitry Zhuridov

Ghent group

- Natalie Jachowicz
  - Raúl González Jiménez
  - Alexis Nikolakopoulos
  - Jannes Nys
  - Vishvas Pandey
  - Tom Van Cuyck
  - Nils Van Dessel

and many more...

Backup slides

Detected rate of ν_α events

$$R_{\nu_\alpha} \sim \Phi_{\nu_\mu}(E_\nu) \times P_{\nu_\mu \to \nu_\alpha}(\{\Theta\}, E_\nu) \times \sigma_{\nu_\alpha}(E_\nu) \times \epsilon_{\mathrm{det.}}$$

Event **rate** | Incoming **flux** | Oscillation **probability** | **Cross section** | **Efficiency**

Knowledge of neutrino-nucleus **cross sections**:

→ allows to **reconstruct neutrino energy** from the detected **final states**,

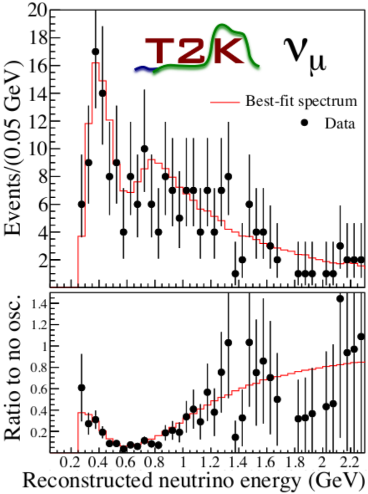
→ is the **crucial uncertainty** in **oscillation analyses**,

but...

→ is an **advanced** computational **problem**,

→ current **precision** is not exceeding **20%**,

→ **constraints** from **ND** are **not enough**.



K. Abe et al., arXiv:1807.07891 (edited)

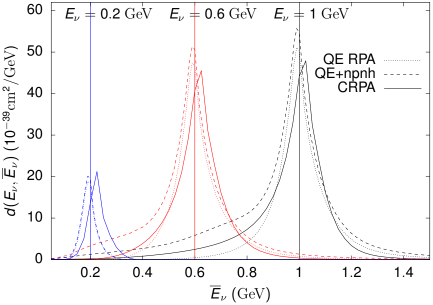
Uncertainty of neutrino energy reconstruction

$\rightarrow$ we need not only **inclusive** but also **exclusive** predictions

$\rightarrow$ energy is reconstructed using leptonic or hadronic information

"**Kinematic**" method

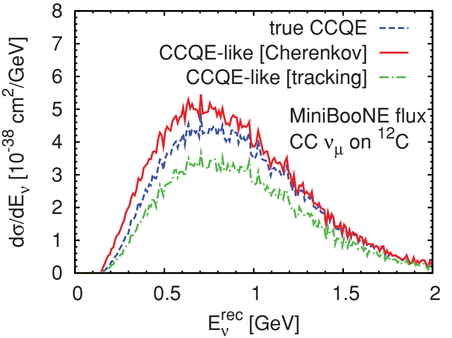
$$E_\nu^{\rm rec} = \frac{2(M_n - E_B)E_\mu - (E_B^2 - 2M_nE_B + m_\mu^2)}{2[M_n - E_B - E_\mu + |\vec{k}_\mu| \cos\theta_\mu]}$$



Alexis Nikolakopoulos

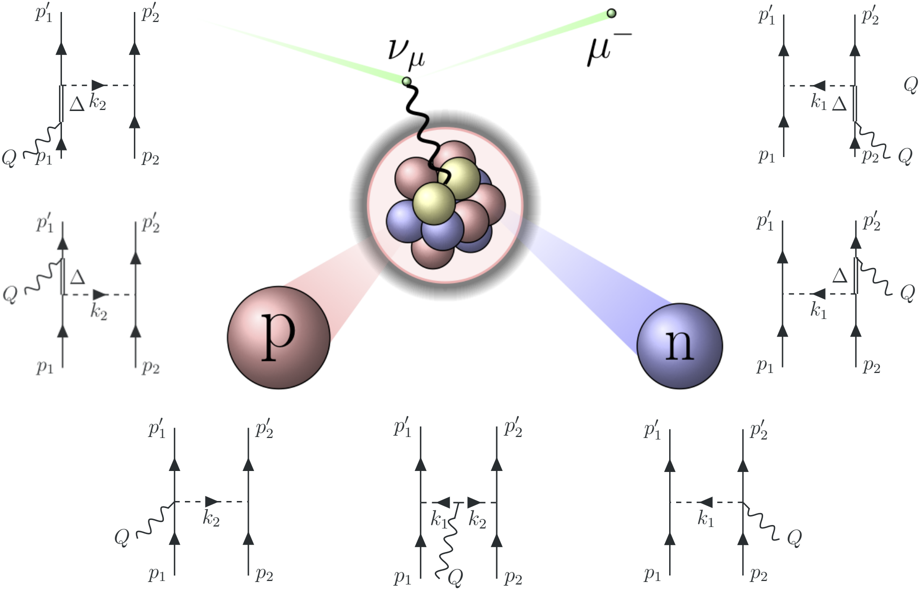
"**Calorimetric**" method

$$E_\nu^{\rm rec} = E_\mu - E_B + \sum_{nuc.}(E_i - M) + \sum_{mes.} E_j$$



T. Leitner, U. Mosel, Phys.Rev. C81 (2010) 064614

Two nucleon knock-out via meson exchange currents



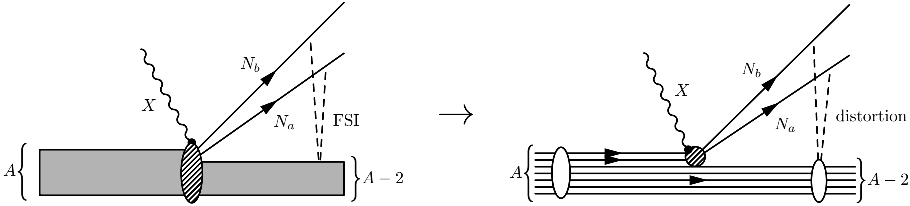
Ghent model

Nonrelativistic operators solved using **multipole expansion**

Bound and emitted **nucleons** are **Hartree-Fock** wave functions

Final state is accounted for the **elastic distortion**

Seagull and **pion-in-flight** diagrams are implemented
$\rightarrow$ **Δ-currents in progress**



T. Van Cuyck, N. Jachowicz, R. González-Jiménez et al., Phys.Rev. C95 (2017) 054611

Intranuclear cascade

Propagates particles through the nuclear medium

Probability of passing a distance λ:

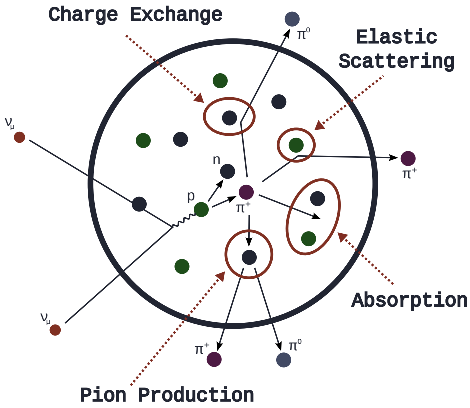
$$P(\lambda) = e^{-\lambda/\tilde{\lambda}}$$

where $\tilde{\lambda} \equiv (\rho\sigma)^{-1}$
ρ - local density
σ - cross section

Implemented for **nucleons** and **pions**

T. Golan, C. Juszczak, J.T. Sobczyk, Phys.Rev. C86 (2012) 015505

Semi-classical – neglects quantum mechanical effects



T. Golan