Hadronic molecular binding of $Z_b(10610)$ and $Z_b(10650)$

Simon León Krug

in collaboration with Christoph Hanhart

08.03.2020

Moscow International School of Physics 2020 (Young Scientist Forum)

Table of Contents

Why $B^{(*)}\bar{B}^{(*)}$-scattering?

- $Z_b(10610)/Z_b(10650)$:
  - Both masses close to BB^*/B^*B^* threshold.
  - $J^{PC} = 1^{+-}$ is possible with $B^{(*)}\bar{B}^{(*)}$ pair.

Why $B^{(*)}\bar{B}^{(*)}$-scattering?

- $Z_b(10610)/Z_b(10650)$:
  - Both masses close to BB^*/B^*B^* threshold.
  - $J^{PC} = 1^{+-}$ is possible with $B^{(*)}\bar{B}^{(*)}$ pair.
- Interested in potential of $B^{(*)}\bar{B}^{(*)}$ pair.
- Applicable to $D^{(*)}\bar{D}^{(*)}$ (any pair of heavy mesons).

Why $B^{(*)}\bar{B}^{(*)}$-scattering?

- $Z_b(10610)/Z_b(10650)$:
  - Both masses close to BB^*/B^*B^* threshold.
  - $J^{PC} = 1^{+-}$ is possible with $B^{(*)}\bar{B}^{(*)}$ pair.
- Interested in potential of $B^{(*)}\bar{B}^{(*)}$ pair.
- Applicable to $D^{(*)}\bar{D}^{(*)}$ (any pair of heavy mesons).

$\Rightarrow$ Is there hadronic binding between $B^{(*)}$ and $\bar{B}^{(*)}$?

Why $B^{(*)}\bar{B}^{(*)}$-scattering?

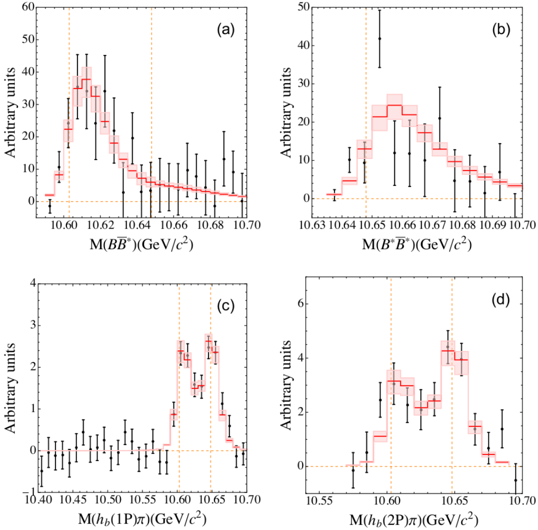
- $Z_b(10610)/Z_b(10650)$:
  - Both masses close to BB^*/B^*B^* threshold.
  - $J^{PC} = 1^{+-}$ is possible with $B^{(*)}\bar{B}^{(*)}$ pair.
- Interested in potential of $B^{(*)}\bar{B}^{(*)}$ pair.
- Applicable to $D^{(*)}\bar{D}^{(*)}$ (any pair of heavy mesons).

$\Rightarrow$ Is there hadronic binding between $B^{(*)}$ and $\bar{B}^{(*)}$?
$\Rightarrow$ T-matrix?

State of the art

Fit of LO + incomplete NLO to experimental data. Error bars represent a statistical error of 1σ.

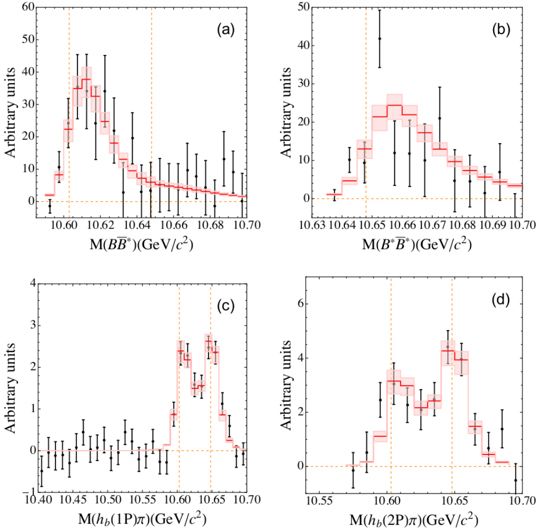
[Wang et al., 2018]



State of the art

Fit of LO + incomplete NLO to experimental data. Error bars represent a statistical error of 1σ. [Wang et al., 2018]

$\Rightarrow$ Is there evidence that the chiral expansion at NLO converges sufficiently fast?



Lippmann-Schwinger equation (1)

- From scattering theory: Extract T perturbatively with Lippmann-Schwinger equation:

$$T = V - VGT$$

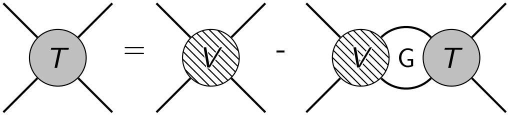
T: Scattering amplitude
G: Propagator
V: Potential

Lippmann-Schwinger equation (1)

- From scattering theory: Extract T perturbatively with Lippmann-Schwinger equation:

$$T = V - VGT$$

T: Scattering amplitude
G: Propagator
V: Potential

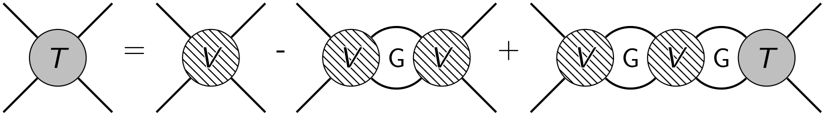


Lippmann-Schwinger equation (1)

- From scattering theory: Extract T perturbatively with Lippmann-Schwinger equation:

$$T = V - VGT$$

T: Scattering amplitude
G: Propagator
V: Potential



Lippmann-Schwinger equation (2)

Coupled-Channel Lippmann-Schwinger equation

$$T_{\alpha\beta}\left(E,\vec{p},\vec{p}'\right) = V^{\text{eff}}_{\alpha\beta}\left(\vec{p},\vec{p}'\right) - \sum_{\gamma}\int \frac{d^3\vec{q}}{(2\pi)^3} V^{\text{eff}}_{\alpha\gamma}\left(\vec{p},\vec{q}\right) G_{\gamma}\left(E,\vec{q}\right) T_{\gamma\beta}\left(E,\vec{q},\vec{p}'\right)$$

$$G_{\gamma}\left(E,\vec{q}\right) = \frac{1}{\frac{\vec{q}^{\,2}}{2\mu_{\gamma}} + m_{1,\gamma} + m_{2,\gamma} - E - i\epsilon}$$

E: Total energy of the system [Baru et al., 2019]

Two-particle basis

- Basis of the two-particle system:

$$1^{+-}: \quad \alpha,\beta,\gamma = \left\{ B\bar{B}^*(^3S_1), B\bar{B}^*(^3D_1), B^*\bar{B}^*(^3S_1), B^*\bar{B}^*(^3D_1) \right\}$$

Two-particle basis

- Basis of the two-particle system:

$$1^{+-}: \quad \alpha, \beta, \gamma = \left\{ B\bar{B}^*(^3S_1), B\bar{B}^*(^3D_1), B^*\bar{B}^*(^3S_1), B^*\bar{B}^*(^3D_1) \right\}$$

$$0^{++}: \quad \alpha, \beta, \gamma = \left\{ B\bar{B}(^1S_0), B^*\bar{B}^*(^1S_0), B^*\bar{B}^*(^5D_0) \right\}$$

$$1^{++}: \quad \alpha, \beta, \gamma = \left\{ B\bar{B}^*(^3S_1), B\bar{B}^*(^3D_1), B^*\bar{B}^*(^5D_1) \right\}$$

$$2^{++}: \quad \alpha, \beta, \gamma = \left\{ B\bar{B}(^1D_2), B\bar{B}^*(^3D_2), B^*\bar{B}^*(^5S_2), B^*\bar{B}^*(^1D_2), B^*\bar{B}^*(^5D_2), B^*\bar{B}^*(^5G_2) \right\}$$

Two-particle basis

- Basis of the two-particle system:

$$
\begin{aligned}
1^{+-}: &\quad \alpha,\beta,\gamma = \left\{ B\bar{B}^*({}^3S_1), B\bar{B}^*({}^3D_1), B^*\bar{B}^*({}^3S_1), B^*\bar{B}^*({}^3D_1) \right\} \\
0^{++}: &\quad \alpha,\beta,\gamma = \left\{ B\bar{B}({}^1S_0), B^*\bar{B}^*({}^1S_0), B^*\bar{B}^*({}^5D_0) \right\} \\
1^{++}: &\quad \alpha,\beta,\gamma = \left\{ B\bar{B}^*({}^3S_1), B\bar{B}^*({}^3D_1), B^*\bar{B}^*({}^5D_1) \right\} \\
2^{++}: &\quad \alpha,\beta,\gamma = \left\{ B\bar{B}({}^1D_2), B\bar{B}^*({}^3D_2), B^*\bar{B}^*({}^5S_2), B^*\bar{B}^*({}^1D_2), \right. \\
&\qquad\qquad\qquad \left. B^*\bar{B}^*({}^5D_2), B^*\bar{B}^*({}^5G_2) \right\}
\end{aligned}
$$

- Obtain one T-matrix for each $J^{PC} = \left\{1^{+-}, 0^{++}, 1^{++}, 2^{++}\right\}$.
- Only V^{eff} remains to be calculated...

EFT from QCD

- Lagrangian of QCD:

$$\mathcal{L}_{\mathrm{QCD}} = \bar{q}\left(i\gamma_\mu \mathcal{D}^\mu - M\right) q - \frac{1}{4}\mathcal{G}_{\mu\nu,a}\mathcal{G}_a^{\mu\nu}$$

$$\mathcal{D}_\mu = \partial_\mu - ig\frac{\lambda_a}{2}\mathcal{A}_{\mu,a}$$

$$\mathcal{G}_{\mu\nu,a} = \partial_\mu \mathcal{A}_{\nu,a} - \partial_\nu \mathcal{A}_{\mu,a} + gf_{abc}\mathcal{A}_{\mu,b}\mathcal{A}_{\nu,c}$$

$\Rightarrow$ Gluon as gauge bosons between quarks.

EFT from QCD

- Lagrangian of QCD:

$$\mathcal{L}_{\text{QCD}} = \bar{q}\left(i\gamma_\mu \mathcal{D}^\mu - M\right) q - \frac{1}{4}\mathcal{G}_{\mu\nu,a}\mathcal{G}_a^{\mu\nu}$$

$$\mathcal{D}_\mu = \partial_\mu - ig\frac{\lambda_a}{2}\mathcal{A}_{\mu,a}$$

$$\mathcal{G}_{\mu\nu,a} = \partial_\mu \mathcal{A}_{\nu,a} - \partial_\nu \mathcal{A}_{\mu,a} + g f_{abc}\mathcal{A}_{\mu,b}\mathcal{A}_{\nu,c}$$

$\Rightarrow$ Gluon as gauge bosons between quarks.

- Problem: Low-energy QCD ($\sqrt{s} < \Lambda_{QCD} \sim 200\,\text{MeV}$) is non-perturbative.

EFT from QCD

- Lagrangian of QCD:

$$\mathcal{L}_{\text{QCD}} = \bar{q}\left(i\gamma_\mu \mathcal{D}^\mu - M\right) q - \frac{1}{4}\mathcal{G}_{\mu\nu,a}\mathcal{G}_a^{\mu\nu}$$

$$\mathcal{D}_\mu = \partial_\mu - ig\frac{\lambda_a}{2}\mathcal{A}_{\mu,a}$$

$$\mathcal{G}_{\mu\nu,a} = \partial_\mu \mathcal{A}_{\nu,a} - \partial_\nu \mathcal{A}_{\mu,a} + gf_{abc}\mathcal{A}_{\mu,b}\mathcal{A}_{\nu,c}$$

$\Rightarrow$ Gluon as gauge bosons between quarks.

- Problem: Low-energy QCD ($\sqrt{s} < \Lambda_{QCD} \sim 200\,\text{MeV}$) is non-perturbative.
- Goal: Find effective field theory (EFT) with same underlying symmetry and perturbative low-energy behavior.

Heavy quark effective theory

- M includes quark masses:

$$\begin{aligned} m_u &\approx 2\,\mathrm{MeV} \\ m_d &\approx 5\,\mathrm{MeV} \\ m_s &\approx 93\,\mathrm{MeV} \\ m_c, m_b, m_t &\gg \Lambda_{QCD} \end{aligned}$$

Heavy quark effective theory

- M includes quark masses:

$$
\begin{aligned}
m_u &\approx 2\,\mathrm{MeV} \\
m_d &\approx 5\,\mathrm{MeV} \\
m_s &\approx 93\,\mathrm{MeV} \\
m_c, m_b, m_t &\gg \Lambda_{QCD}
\end{aligned}
$$

- b, $\bar{b}$ inside $B^{(*)}$, $\bar{B}^{(*)}$ are present. Use HQET:

Heavy quark effective theory

- M includes quark masses:

$$\begin{aligned} m_u &\approx 2\,\text{MeV} \\ m_d &\approx 5\,\text{MeV} \\ m_s &\approx 93\,\text{MeV} \\ m_c, m_b, m_t &\gg \Lambda_{QCD} \end{aligned}$$

- b, $\bar{b}$ inside $B^{(*)}$, $\bar{B}^{(*)}$ are present. Use HQET:
  - Assume b, $\bar{b}$ quasi-stable.
  - $m_b \longrightarrow \infty$: b, $\bar{b}$ static, only spectator quark.
  - $m_q \longrightarrow 0$: $\bar{q}$, q carries residual momentum, relativistic treatment.

Chiral perturbation theory

- Limit of vanishing quark masses:

$$\begin{aligned}\mathcal{L}_{\text{QCD}} \longrightarrow \mathcal{L}^{0}_{\text{QCD}} &= \bar{q} i \gamma_\mu \mathcal{D}^\mu q - \frac{1}{4} \mathcal{G}_{\mu\nu,a} \mathcal{G}^{\mu\nu}_a \\ &= \bar{q}_R i \gamma_\mu \mathcal{D}^\mu q_R + \bar{q}_L i \gamma_\mu \mathcal{D}^\mu q_L - \frac{1}{4} \mathcal{G}_{\mu\nu,a} \mathcal{G}^{\mu\nu}_a\end{aligned}$$

Chiral perturbation theory

- Limit of vanishing quark masses:

$$\mathcal{L}_{\text{QCD}} \longrightarrow \mathcal{L}^0_{\text{QCD}} = \bar{q} i\gamma_\mu \mathcal{D}^\mu q - \frac{1}{4}\mathcal{G}_{\mu\nu,a}\mathcal{G}^{\mu\nu}_a$$
$$= \bar{q}_R i\gamma_\mu \mathcal{D}^\mu q_R + \bar{q}_L i\gamma_\mu \mathcal{D}^\mu q_L - \frac{1}{4}\mathcal{G}_{\mu\nu,a}\mathcal{G}^{\mu\nu}_a$$

- $\text{SU}(2)_L \times \text{SU}(2)_R \longrightarrow \text{SU}(2)_V \times \text{SU}(2)_A \xrightarrow{\text{SSB}} \text{SU}(2)_V$

Chiral perturbation theory

- Limit of vanishing quark masses:

$$\mathcal{L}_{\text{QCD}} \longrightarrow \mathcal{L}^0_{\text{QCD}} = \bar{q} i \gamma_\mu \mathcal{D}^\mu q - \frac{1}{4}\mathcal{G}_{\mu\nu,a}\mathcal{G}^{\mu\nu}_a$$
$$= \bar{q}_R i \gamma_\mu \mathcal{D}^\mu q_R + \bar{q}_L i \gamma_\mu \mathcal{D}^\mu q_L - \frac{1}{4}\mathcal{G}_{\mu\nu,a}\mathcal{G}^{\mu\nu}_a$$

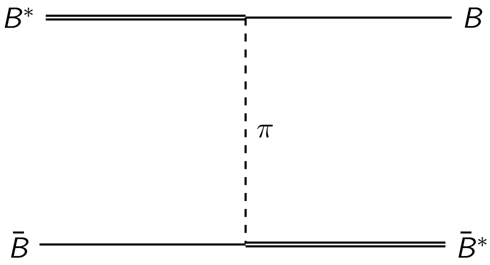
- $\text{SU}(2)_L \times \text{SU}(2)_R \longrightarrow \text{SU}(2)_V \times \text{SU}(2)_A \xrightarrow{\text{SSB}} \text{SU}(2)_V$
- Goldstone's theorem: A spontaneously broken global symmetery implies the existence of (massless) Goldstone bosons.
- π^+, π^-, π^0
- $m_u \neq m_d \neq 0$ as perturbative corrections.

EFT applied

- Effective field theory with $\mathrm{SU}(2)_{\mathrm{isospin}}$ symmetry, Goldstone bosons (pions) mediate between $B^{(*)}$, $\bar{B}^{(*)} \Rightarrow$ HMχPT.
- Isospin of $B^{(*)}$, $\bar{B}^{(*)}$ is sufficient.

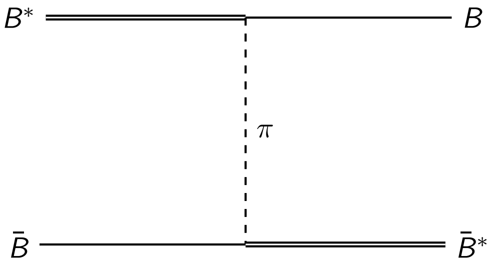
EFT applied

- Effective field theory with $SU(2)_{\text{isospin}}$ symmetry, Goldstone bosons (pions) mediate between $B^{(*)}$, $\bar{B}^{(*)} \Rightarrow$ HMχPT.
- Isospin of $B^{(*)}$, $\bar{B}^{(*)}$ is sufficient.



EFT applied

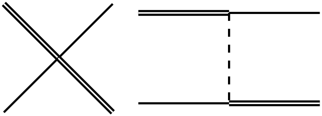
- Effective field theory with $SU(2)_{\text{isospin}}$ symmetry, Goldstone bosons (pions) mediate between $B^{(*)}$, $\bar{B}^{(*)}$ $\Rightarrow$ HMχPT.
- Isospin of $B^{(*)}$, $\bar{B}^{(*)}$ is sufficient.



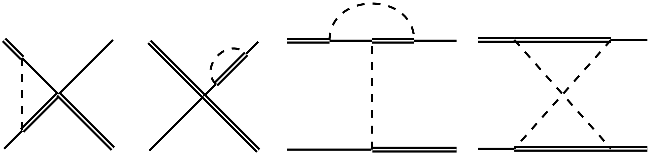
- Pions important for long-range part of interaction.
- Total ~ 200 diagrams at NLO.

Example

- Selection of diagrams of $B^* \bar{B} \longrightarrow B \bar{B}^*$ scattering:
  - LO (CT, 1PE)

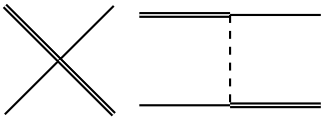


  - NLO (CCT, C1PE, 2PE)

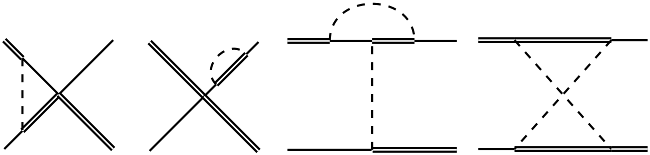


Example

- Selection of diagrams of $B^*\bar{B} \longrightarrow B\bar{B}^*$ scattering:
  - LO (CT, 1PE)



  - NLO (CCT, C1PE, 2PE)



- V^{eff}s are calculated but not yet partial-wave projected.

Thank you for your attention!

References

Wang, Q. et al. (Oct. 2018). "Line shapes of the $Z_b(10610)$ and $Z_b(10650)$ in the elastic and inelastic channels revisited". In: *Phys. Rev. D* 98 (7), p. 074023. DOI: 10.1103/PhysRevD.98.074023. URL: https://link.aps.org/doi/10.1103/PhysRevD.98.074023.

Baru, V. et al. (May 2019). "Spin partners W_{bJ} from the line shapes of the $Z_b(10610)$ and $Z_b(10650)$". In: *Physical Review D* 99.9. ISSN: 2470-0029. DOI: 10.1103/physrevd.99.094013. URL: http://dx.doi.org/10.1103/PhysRevD.99.094013.

Machleidt, R. and D. Entem (2011). "Chiral effective field theory and nuclear forces". In: *Physics Reports* 503.1, pp. 1–75. ISSN: 0370-1573. DOI: https://doi.org/10.1016/j.physrep.2011.02.001. URL: http://www.sciencedirect.com/science/article/pii/S0370157311000457.

Tanabashi, M. et al. (Aug. 2018). "Review of Particle Physics". In: *Phys. Rev. D* 98 (3), p. 030001. DOI: 10.1103/PhysRevD.98.030001. URL: https://link.aps.org/doi/10.1103/PhysRevD.98.030001.