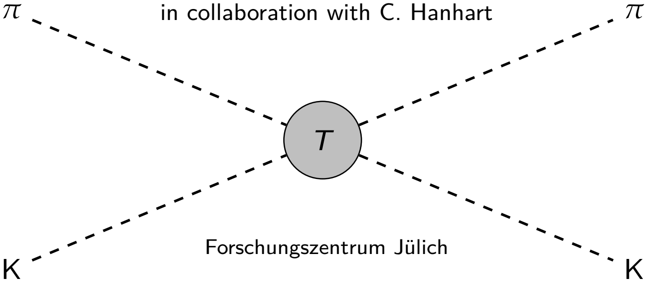
Extension of the N/D-method for Kπ scattering to energies above 1.4 GeV

Frederic Noël & Leon von Detten

in collaboration with C. Hanhart



Forschungszentrum Jülich

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Young Scientists forum at MISP 2020

Motivation

- want to describe $K\pi$ final state interactions (as in $B \to J/\psi K\pi$)
- need phase data for $K\pi$ scattering above 1.4 GeV consistent with unitarity and analyticity
- S-wave resonances in $K\pi$ are very broad and difficult to distinguish from background
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goal

theoretically sound description of $K\pi$ scattering consistent with unitarity and analyticity

Analyticity and unitarity

- S-matrix describing scattering processes:

$$|\text{out}\rangle = S\,|\text{in}\rangle$$

- T-matrix is non-trivial part of S-matrix:

$$S = 1 + iT$$

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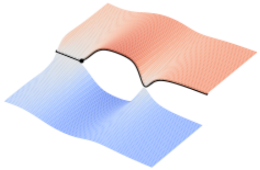
- analyticity up to cut(s) on the real axis
- Schwartz reflection principle:

$$T(s^*) = T(s)^* \;\Rightarrow\; \text{disc}\, T = 2i\, \text{Im}\, T$$



imag. part of 1st sheet (with right-hand cut (RHC))

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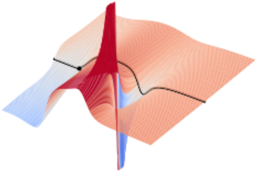

imag. part of 1st & 2nd sheet (with pole on 2nd sheet)

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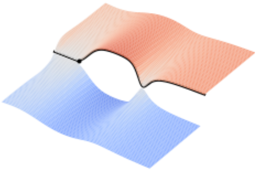

imag. part of 1[st] sheet
(with right-hand cut (RHC))

- resonance structures from poles on the second sheet
- unitarity of the S-Matrix (above threshold):

$$S^\dagger S = 1 \;\Rightarrow\; \text{Im}(T^{-1}) = -\sigma = -\frac{q_{K\pi}}{\sqrt{s}} \quad \text{(phase space)}$$

N/D method

[Chew and Mandelstam, 1960; Oller and Oset, 1999]

- general structure:

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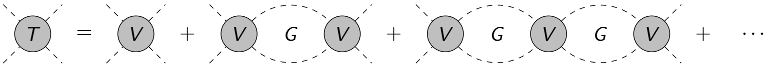

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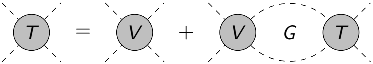

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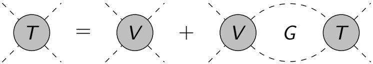

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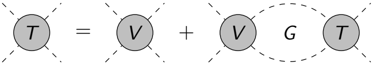

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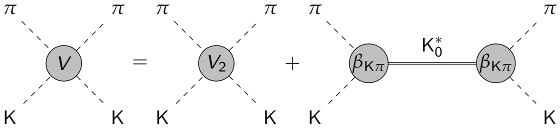
- in our case: $K\pi \to K\pi$ (S-Wave) with coupled channels $K\eta$ and $K\eta'$

LO χPT calculation

- V-matrix has contact and resonance parts: $V = V_2 + V_2^{\text{res}}$



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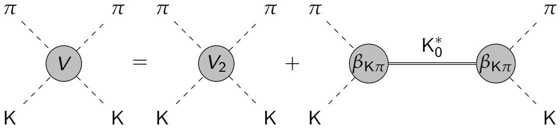


- contact potential from lowest order chiral perturbation theory (χPT):

$$V_2 = -\frac{3(m_K^2 - m_\pi^2)^2 + 2(m_K^2 + m_\pi^2)s - 5s^2}{8f^2 s}$$

- S-wave resonance exchange with couplings c_d, c_m and mass M_R:

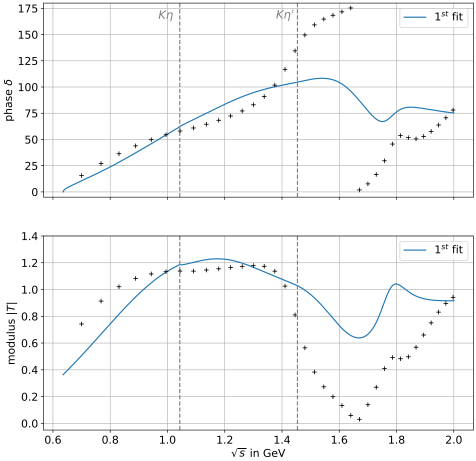
$$\beta_{K\pi} = \sqrt{\frac{3}{2}}\frac{(c_m - c_d)(m_K^2 + m_\pi^2) + c_d s}{f^2}; \quad K_0^*\text{-prop.: } \frac{i}{s - M_R{}^2}$$

[Ecker et al., 1989]

- analogous for $K\eta$, $K\eta'$ channels (η' as singlet consistent with large N_C)

Fit results

- we fit our model towards pseudo-data generated from a parametrisation constrained by forward dispersion relations [Peláez and Rodas, 2016]



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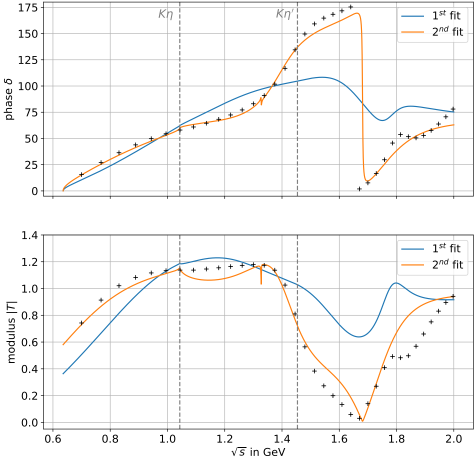

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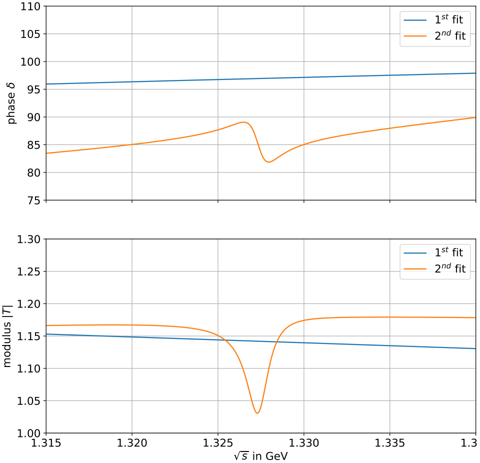

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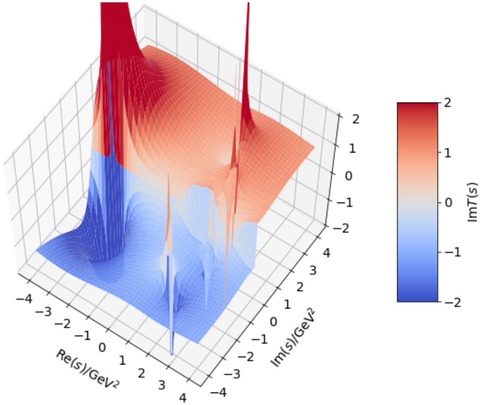
Complex plane plots

T-matrix with three channels ($K\pi$, $K\eta$, $K\eta'$) and two (explicit) resonances



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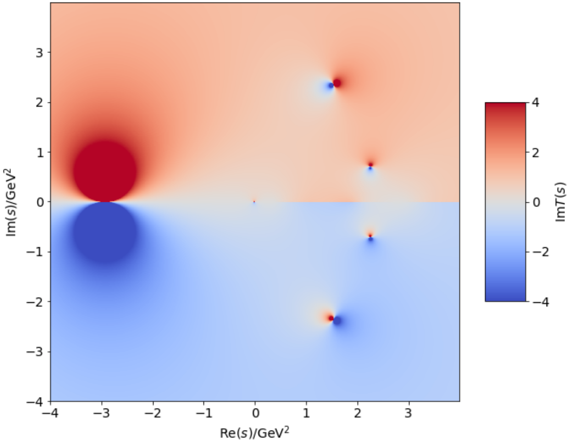

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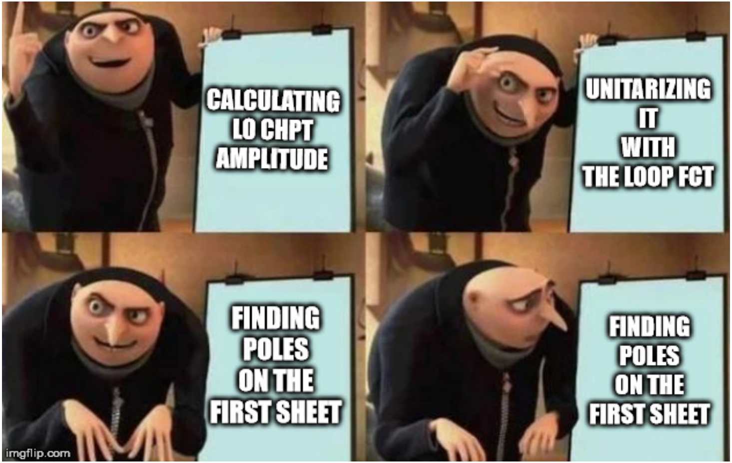
Our thesis so far



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- elastic phase [Peláez and Rodas, 2016] describes low energy region
- add resonances at higher energies while sustaining unitarity
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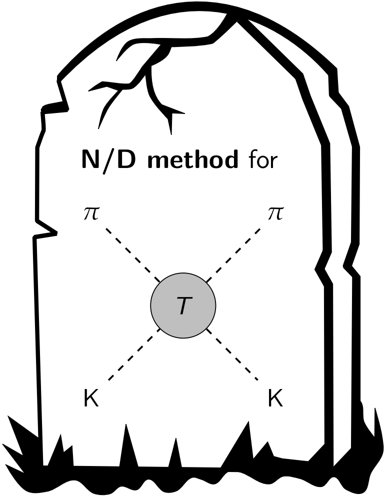
New approach (following [Ropertz, Hanhart, and Kubis, 2018] examining $B \to J/\psi\pi\pi$):

- elastic phase [Peláez and Rodas, 2016] describes low energy region
- add resonances at higher energies while sustaining unitarity
- coupled channels only added indirectly through resonances
- advantages:
  - no poles on the first sheet by construction
  - can calculate everything from the phase using dispersion relations
  - different K_0^* coupling constants as direct parameters

Thank you for your attention!



new approach

- basic idea (separate contact and resonance term):

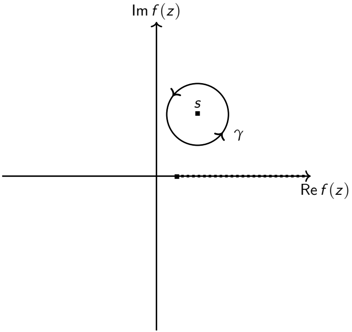
$$V = V_0 + V_\mathrm{R}$$
$$T = T_0 + T_\mathrm{R}$$

  - T_0 is unitarization of V_0 via $T_0 = V_0 + V_0 G T_0$
  - T is unitarization of V via $T = V + VGT$
- T_0 only depends on phase δ: $T_0 = \frac{\sin\delta e^{i\delta}}{\sigma}$
- T_R is dependent on T_0 and V_R: $T_R = \Omega(1 - V_\mathrm{R}\Sigma)^{-1} V_\mathrm{R}\Omega^\mathrm{T}$
  - Omnès function: $\Omega(s) = \exp\left(\frac{s}{\pi}\int_{s_\mathrm{thr}}^{\infty} \frac{\delta(s)}{z(z-s)}\right)$
  - selfenergy: $\Sigma(s) = \frac{s}{\pi}\int_{s_\mathrm{thr}}^{\infty} \frac{\Omega^\dagger(z)\sigma(z)\Omega(z)}{z(z-s)}$
- (subtracted) resonance term: $(V_\mathrm{R})_{ij} = \sum_r \frac{g_i^r \cdot s \cdot g_j^r}{M_r^2(M_r^2 - s)}$

Dispersion theory

- Cauchy's integral formula:

$$f(s) = \frac{1}{2\pi i} \oint_\gamma \frac{f(z)}{z-s}\,\mathrm{d}z$$



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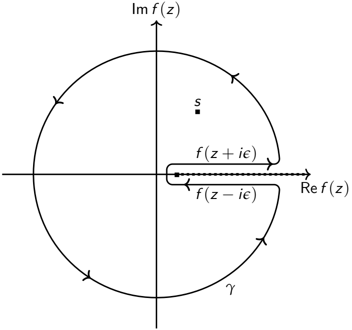

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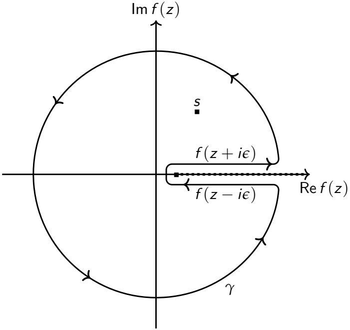
- if $f(z) \xrightarrow{|z|\to\infty} 0$



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- dispersion relation:

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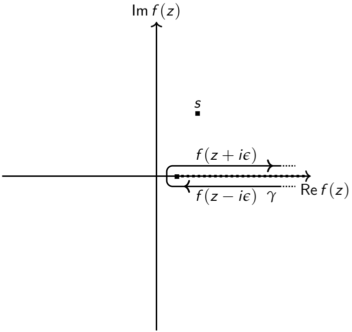

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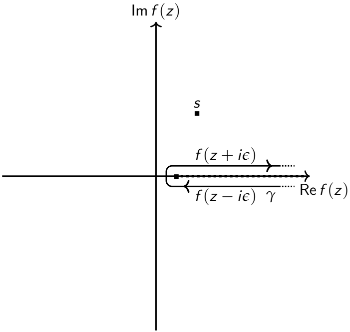
- if $\tilde{f}(z) = \frac{f(z)-f(\tilde{s})}{z-\tilde{s}} \xrightarrow{|z|\to\infty} 0 \quad \text{with} \quad \tilde{s} < s_0$



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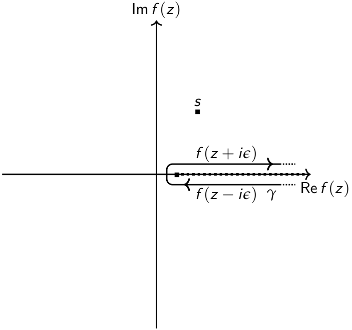
- once subtracted dispersion relation:

$$f(s) = f(\tilde{s}) + \frac{s-\tilde{s}}{2\pi i} \int_{s_0}^{\infty} \frac{\operatorname{disc} f(z)}{(z-s)(z-\tilde{s})}\, \mathrm{d}z$$



χPT calculation

- χPT is effective theory for low energies where QCD is non-perturbative
- arrange fields obeying the correct symmetry behaviour of SU(3) (pseudo-goldstones as octet and η_0 as singlet in $N_C \to \infty$ limit):

$$U = \exp\left(\frac{i}{\sqrt{2}}\frac{\Phi}{f}\right),\ \Phi = \frac{\eta_0}{\sqrt{3}}\mathbb{1}_3 + \lambda_i \varphi^i = \sqrt{2}\begin{pmatrix} \frac{\eta_0}{\sqrt{3}} + \frac{\eta_8}{\sqrt{6}} + \frac{\pi^0}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & \frac{\eta_0}{\sqrt{3}} + \frac{\eta_8}{\sqrt{6}} - \frac{\pi^0}{\sqrt{2}} & K^0 \\ K^- & \bar{K}^0 & \frac{\eta_0}{\sqrt{3}} - \frac{2\eta_8}{\sqrt{6}} \end{pmatrix}$$

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- expand in terms of powers of momenta/derivatives $(p/\Lambda_{\mathrm{QCD}})$
- contact potential from lowest order χPT-Lagrangian

$$\mathcal{L}_2 = \frac{1}{6f^2}\left(\langle \Phi\partial_\mu\Phi\Phi\partial^\mu\Phi\rangle + \langle\Phi\Phi\partial_\mu\Phi\partial^\mu\Phi\rangle\right) + \frac{1}{12f^2}\langle\Phi\Phi\Phi\Phi M\rangle + \mathcal{O}(\Phi^6/f^4)$$

$$\text{e.g.: } \mathrm{K}\pi \to \mathrm{K}\pi\text{(s-wave)}: \; T_{11}^{\left(I=\frac{1}{2}\right)}(s) = -\frac{3(m_\mathrm{K}^2 - m_\pi^2)^2 + 2(m_\mathrm{K}^2 + m_\pi^2)s - 5s^2}{8f^2 s}$$

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$$\mathcal{L}_2^{\text{res.}}\left[S(0^{++})\right] = \frac{2c_d}{f^2}\langle S\partial_\mu\Phi\partial^\mu\Phi\rangle - \frac{c_m}{f^2}\langle S\Phi M\Phi + \tfrac{1}{2}SM\Phi\Phi + \tfrac{1}{2}S\Phi\Phi M\rangle + \mathcal{O}(\Phi^3/f^3)$$

$$\text{e.g.: } \mathrm{K}\pi \to \mathrm{K}_0^*{:}\ \beta_{\mathrm{K}\pi}^{(\frac{1}{2})} = \sqrt{\frac{3}{2}}\frac{(c_m - c_d)(m_\mathrm{K}^2 + m_\pi^2 + c_d s)}{f^2};\ \mathrm{K}_0^*\text{-prop: } \frac{i}{s - {M_\mathrm{R}}^2}$$

References I

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