Neutrino physics (3)

Evgeny Akhmedov

Max-Planck Institute für Kernphysik, Heidelberg

Neutrino oscillations in matter

The MSW effect (Wolfenstein, 1978; Mikheyev & Smirnov, 1985)

Matter can change the pattern of neutrino oscillations drastically

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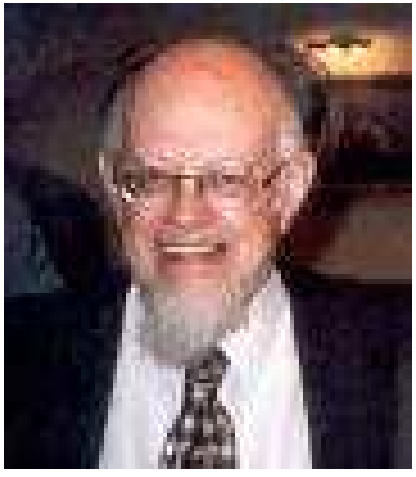


How can matter affect neutrino oscillations?

For $E \sim 1$ MeV neutrinos mean free path in lead is ~ 1 l.y. !

$$\diamondsuit \quad \text{mean free path} \;=\; \langle \sigma n v \rangle^{-1} \,,$$

For incoherent processes (capture, finite-angle scattering)

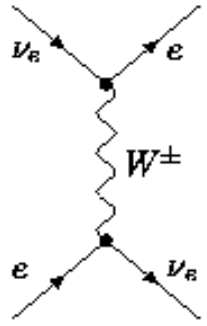
$$\diamondsuit \qquad \sigma \;\propto\; (G_F)^2$$

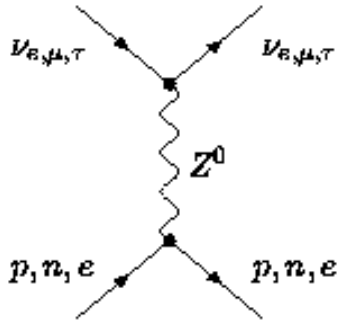
Coherent forward scattering: effects $\sim G_F$, i.e. much stronger!

Lead to effective potentials for neutrinos in matter $\sim G_F N$.

Neutrino oscillations in matter

Coherent forward scattering on the particles in matter





$$V_e^{\mathrm{CC}} \equiv V = \sqrt{2}\, G_F\, N_e$$

2f neutrino evolution equation $(x \simeq t)$:

$$\diamond \quad i\frac{d}{dx}\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} -\frac{\Delta m^2}{4E}\cos 2\theta + V(x) & \frac{\Delta m^2}{4E}\sin 2\theta \\ \frac{\Delta m^2}{4E}\sin 2\theta & \frac{\Delta m^2}{4E}\cos 2\theta \end{pmatrix}\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}$$

For antineutrinos $V(x) \to -V(x)$.

Neutrino potential in matter

At low neutrino energies the effective Hamiltonian CC interactions

$$H_{\rm CC} = \frac{G_F}{\sqrt{2}}\,[\bar{e}\gamma_\mu(1-\gamma_5)\nu_e][\bar{\nu}_e\gamma^\mu(1-\gamma_5)e] = \frac{G_F}{\sqrt{2}}\,[\bar{e}\gamma_\mu(1-\gamma_5)e][\bar{\nu}_e\gamma^\mu(1-\gamma_5)\nu_e]\,,$$

(Fierz transformation used). To obtain the matter-induced potential for ν_e fix the variables corresponding to ν_e and integrate over the electron variables:

$$H_{\rm eff}(\nu_e) = \langle H_{\rm CC}\rangle_{electron} \equiv \bar{\nu}_e V_e \nu_e\,.$$

We have:

$$\langle\bar{e}\gamma_0 e\rangle = \langle e^\dagger e\rangle = N_e\,, \qquad \langle\bar{e}\boldsymbol{\gamma} e\rangle = \langle\mathbf{v}_e\rangle\,, \quad \langle\bar{e}\gamma_0\gamma_5 e\rangle = \langle\frac{\boldsymbol{\sigma}_e\mathbf{p}_e}{E_e}\rangle\,, \quad \langle\bar{e}\boldsymbol{\gamma}\gamma_5 e\rangle = \langle\boldsymbol{\sigma}_e\rangle\,,$$

For unpolarized medium of zero total momentum only the first term survives

$\Rightarrow$

$$\diamondsuit \qquad \boxed{(V_e)_{\rm CC} \equiv V = \sqrt{2}\,G_F N_e}\,.$$

Oscillations in matter of constant density

$$\diamondsuit \qquad \boxed{P_{\rm tr} \;=\; \sin^2 2\theta_m \, \sin^2\left(\pi \frac{L}{l^m_{\rm osc}}\right)}$$

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Osc. length: $l^m_{\rm osc} = l_{\rm osc}(\sin 2\theta_m / \sin 2\theta)$.

Oscillations in matter of constant density

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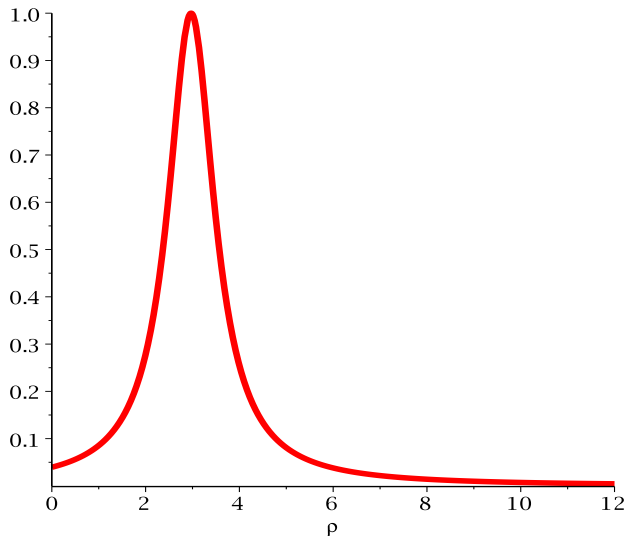

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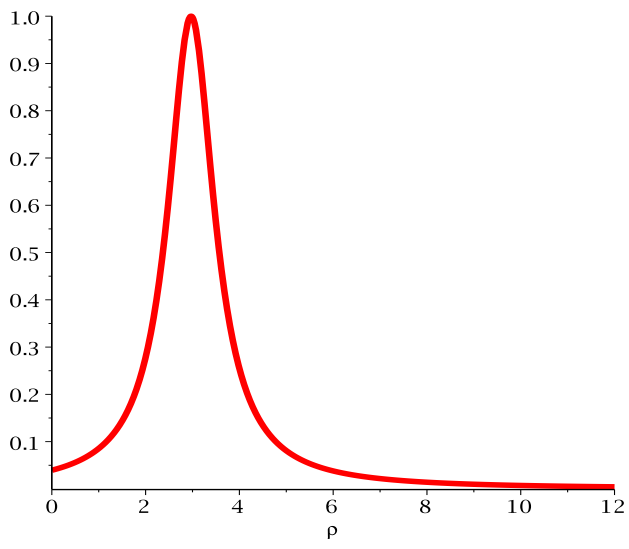


MSW resonance:

$$\diamondsuit \quad \boxed{\sqrt{2}G_F N_e = \frac{\Delta m^2}{2E}\cos 2\theta} \quad \Rightarrow$$

$$\theta_m = 45^\circ$$

independently of θ !

$$(l^m_{\rm osc})_{\rm res} = l_{\rm osc}/\sin 2\theta.$$

The MSW resonance condition

$$\Diamond \qquad \boxed{\pm\sqrt{2}G_F N_e = \frac{\Delta m^2}{2E}\cos 2\theta}$$

For given E yields $(N_e)_{res}$ (or vice versa).

For neutrinos LHS > 0 $\Rightarrow$ can only be satisfied if RHS > 0:

$$\Delta m^2 \cos 2\theta \;=\; (m_2^2 - m_1^2)(\cos^2\theta - \sin^2\theta) \;>\; 0$$

$\Rightarrow$ If ν_2 is heavier than ν_1, one needs $\cos^2\theta \;>\; \sin^2\theta$ and vice versa.

$\Leftrightarrow$ Lighter mass eigenstate must have larger ν_e component.

If one chooses $\cos 2\theta > 0$, the resonance for neutrinos occurs when

$$\Delta m_{21}^2 \;>\; 0.$$

For $\Delta m_{21}^2 \;<\; 0$ $\Rightarrow$ res. takes place for antineutrinos.

Matter of varying density

At any point x eff. Hamiltonian $H_m(x)$ can be diagonalized by unitary transf. $U_m = U_m(x)$ with the mixing angle $\theta_m = \theta_m(x)$:

$$\diamondsuit \qquad \tan 2\theta_m(x) = \frac{\sin 2\theta \cdot \frac{\Delta m^2}{2E}}{\frac{\Delta m^2}{2E}\cos 2\theta - \sqrt{2} G_F N_e(x)}$$

In general osc. probability cannot be found in closed form.

$|\nu_{1m}\rangle$, $|\nu_{2m}\rangle$ – local (at point x) eigenstates of H_m (matter eigenstates):

$$|\nu_{1m}\rangle = \cos\theta_m |\nu_e\rangle - \sin\theta_m |\nu_\mu\rangle$$

$$|\nu_{2m}\rangle = \sin\theta_m |\nu_e\rangle + \cos\theta_m |\nu_\mu\rangle$$

$$N_e \gg (N_e)_{\rm res}: \quad \theta_m \approx 90^\circ$$

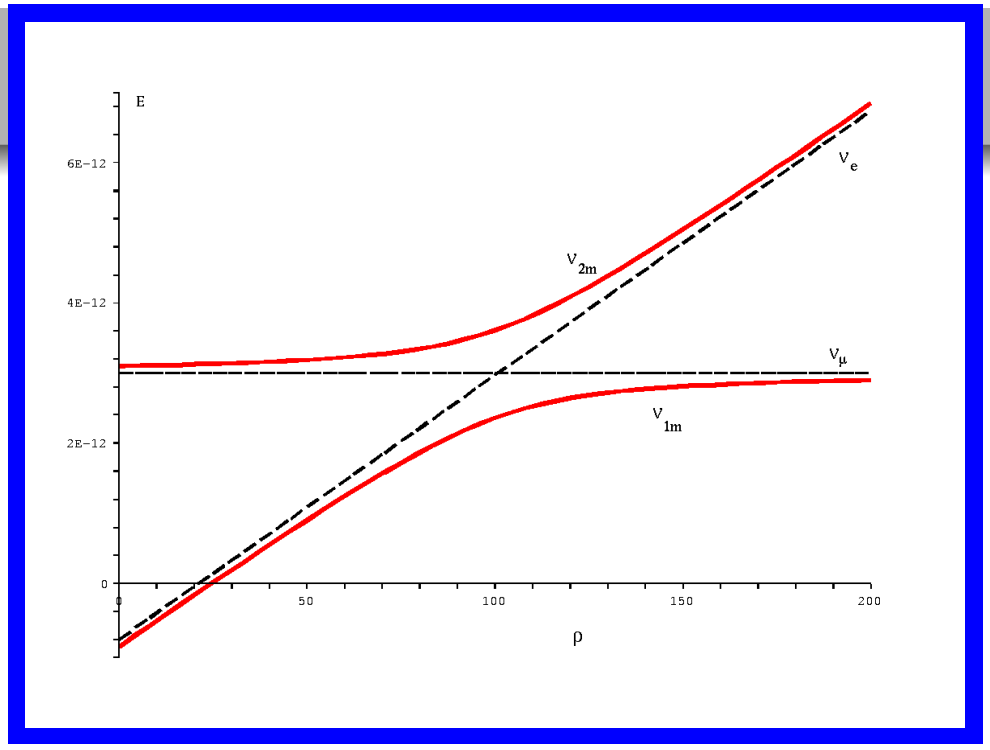
$$N_e = (N_e)_{\rm res}: \quad \theta_m = 45^\circ$$

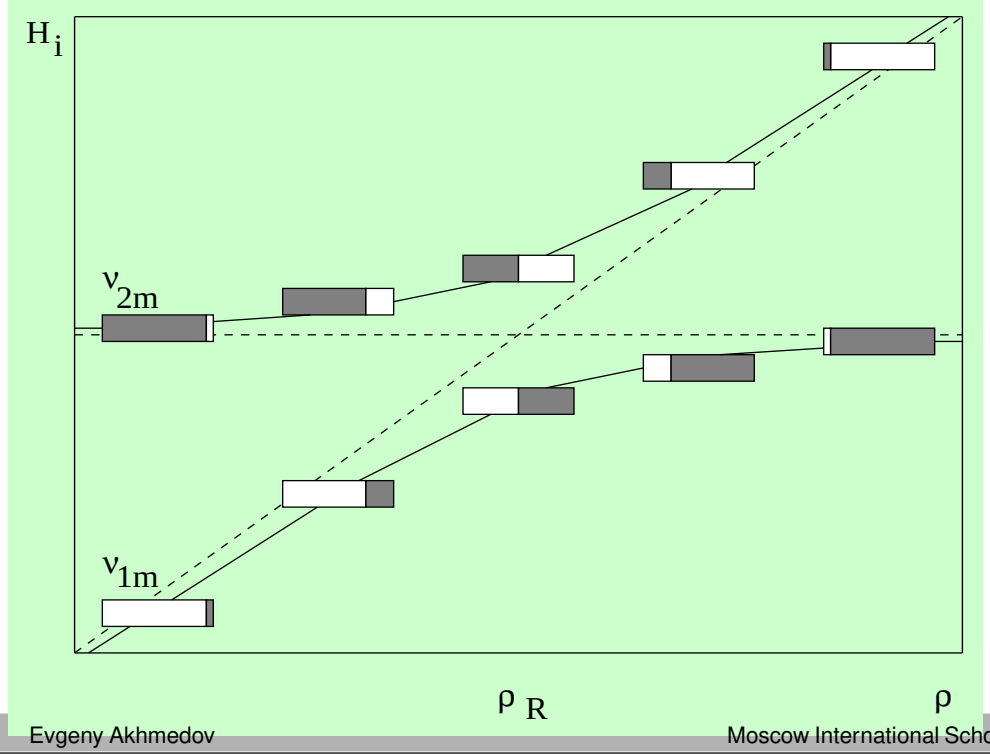
$$N_e \ll (N_e)_{\rm res}: \quad \theta_m \approx \theta$$

In the adiabatic regime: ν_{1m} and ν_{2m} do not go into each other $\Rightarrow$

ν_e born at high density will remain ν_e at small N_e with probability $\sin^2\theta$ and go to ν_μ with probability $\cos^2\theta$ independently of L !

Adiabatic flavour conversion





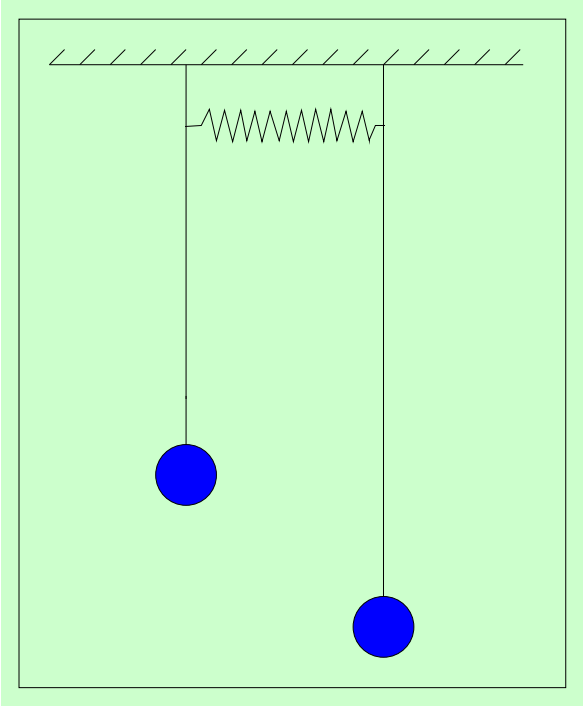
Adiabaticity: slow density change along the neutrino path

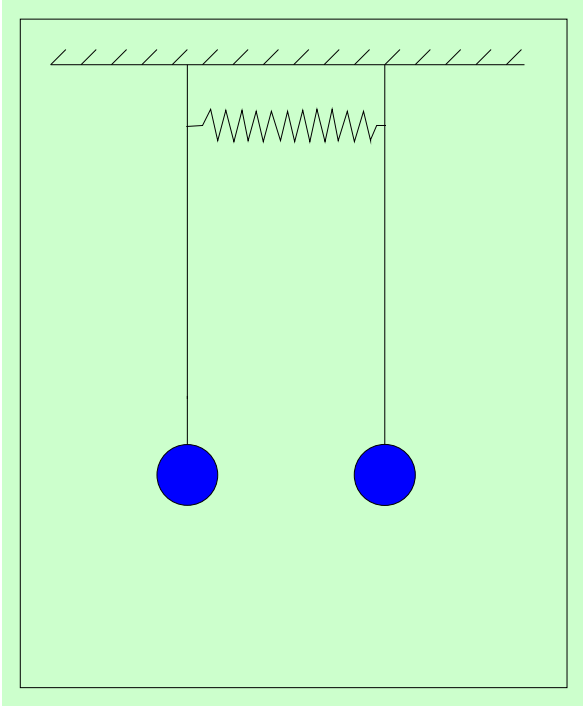
$$\frac{\sin^2 2\theta}{\cos 2\theta}\frac{\Delta m^2}{2E}L_\rho \gg 1$$

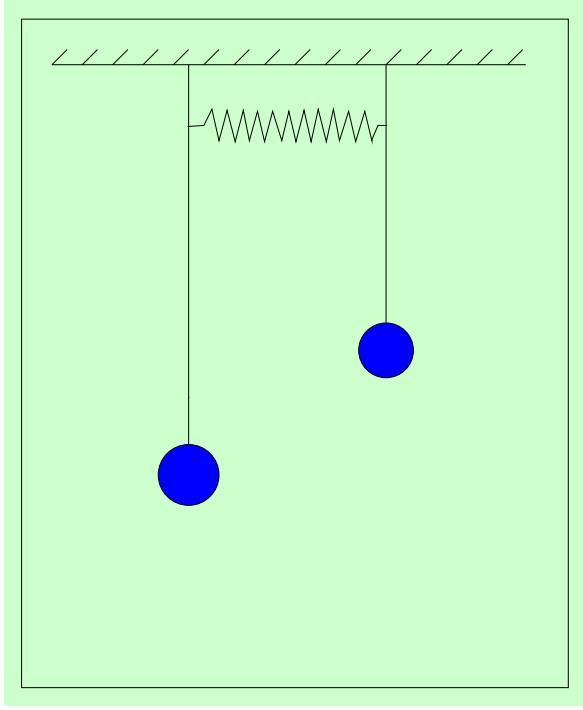
L_ρ – electron density scale hight:

$$L_\rho = \left|\frac{1}{N_e}\frac{dN_e}{dx}\right|^{-1}$$

Analogy: Two coupled pendula







Mechanical model of the MSW effect

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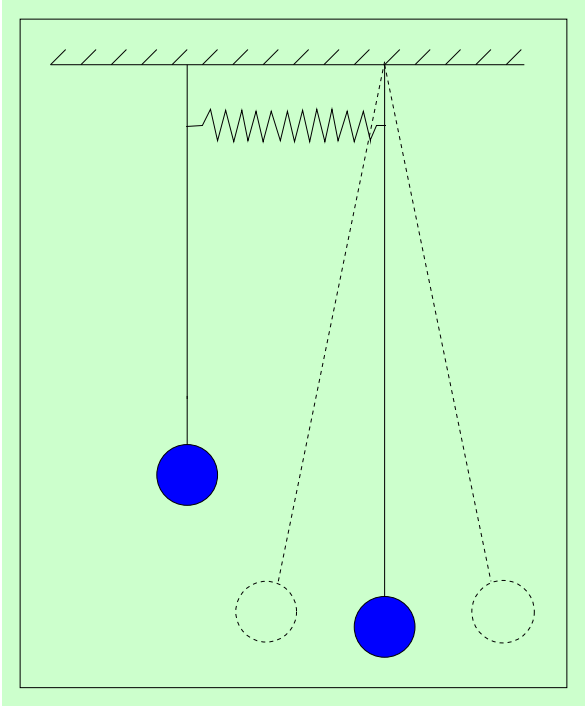

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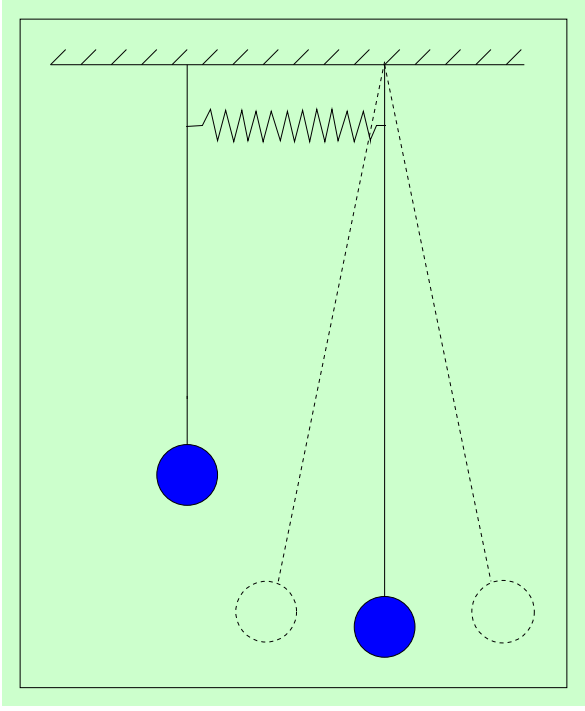

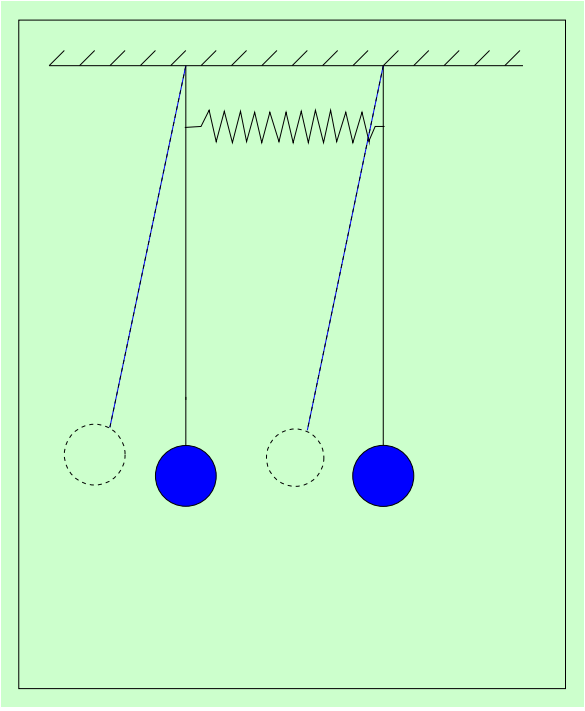

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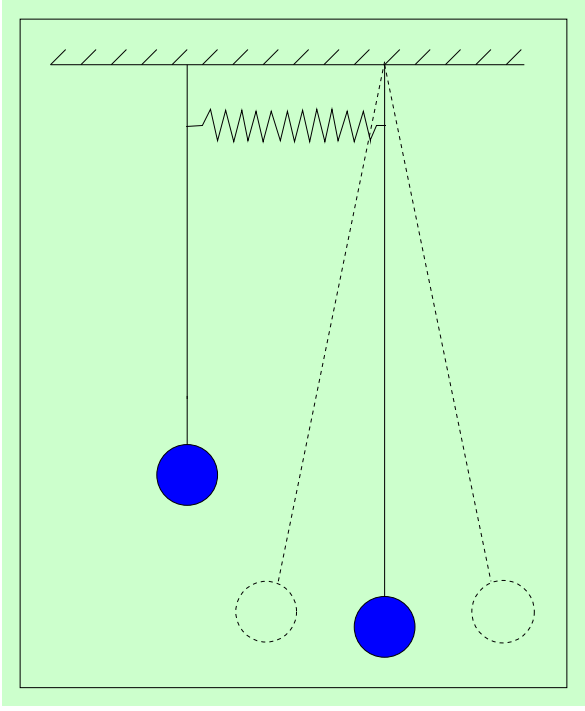
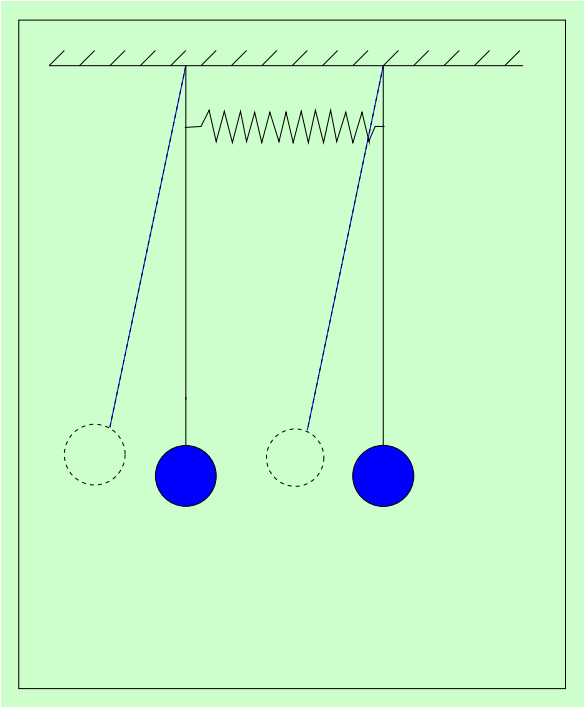
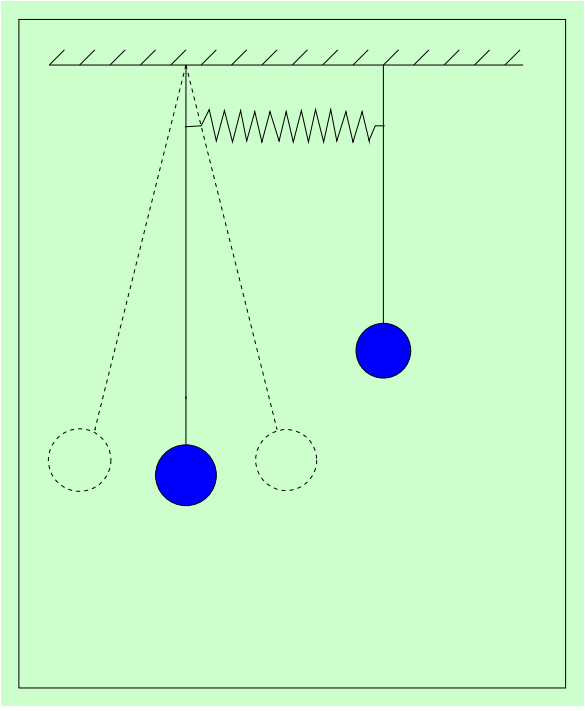






Mechanical model of the MSW effect

Evolution of matter eigenstates

Flavour states in terms of local matter eigenstates:

$$\diamondsuit \qquad |\nu_{\rm fl}\rangle \; = U_m^\dagger(x)\,|\nu_{\rm matt}\rangle$$

Evolution equation: $i\frac{d}{dx}|\nu_{\rm fl}\rangle = H_{\rm fl}^m(x)|\nu_{\rm fl}\rangle \qquad \Rightarrow$

$$\diamondsuit \qquad i\frac{d}{dx}|\nu_{\rm matt}\rangle \; = \; \left[U_m H_{\rm fl}^m U_m^\dagger \; - \; iU_m(U_m^\dagger)'\right]|\nu_{\rm matt}\rangle$$

For the 2f case: $\quad U_m \; = \begin{pmatrix} c_m & s_m \\ -s_m & c_m \end{pmatrix} \qquad \Rightarrow$

$$\diamondsuit \qquad i\frac{d}{dx}\begin{pmatrix} \nu_{1m} \\ \nu_{2m} \end{pmatrix} = \begin{pmatrix} \mathcal{E}_1(x) & -i\theta_m'(x) \\ i\theta_m'(x) & \mathcal{E}_2(x) \end{pmatrix}\begin{pmatrix} \nu_{1m} \\ \nu_{2m} \end{pmatrix}$$

$\mathcal{E}_1(x)$, $\mathcal{E}_2(x)$ – local eigenvals. of $H_{\rm fl}^m$ at a given x.

Adiabatic regime

$$\diamond \quad |\mathcal{E}_2(x) - \mathcal{E}_1(x)| = \sqrt{\left[\frac{\Delta m^2}{2E}\cos 2\theta - \sqrt{2}G_F N_e(x)\right]^2 + \left(\frac{\Delta m^2}{2E}\right)^2 \sin^2 2\theta}$$

If $|\mathcal{E}_2 - \mathcal{E}_1| \gg 2|\theta'_m|$ (adiabatic regime) $\Rightarrow$ matter eigenstates ν_{1m} and ν_{2m} evolve independently. Adiabaticity condition:

$$\frac{|\mathcal{E}_2 - \mathcal{E}_1|_{\rm res}}{2|\theta'_m|} = \frac{\Delta m^2}{2E}\frac{\sin^2 2\theta}{\cos 2\theta} L_\rho \gg 1$$

$L_\rho \equiv |N'_e/N_e|^{-1}$ – scale height of electron number density. Let $|\nu(0)\rangle = |\nu_e\rangle$:

$$|\nu(0)\rangle = c_m(0)|\nu_{1m}\rangle + s_m(0)|\nu_{2m}\rangle$$

In the adiabatic regime:

$$\diamond \quad |\nu(x)\rangle = c_m(0)\, e^{-i\int_0^x \mathcal{E}_1(x')dx'}|\nu_{1m}\rangle + s_m(0)\, e^{-i\int_0^x \mathcal{E}_2(x')dx'}|\nu_{2m}\rangle$$

Adiabatic regime

At the point x the state $|\nu_\mu\rangle$ can be expanded as

$$|\nu_\mu\rangle \;=\; -\,s_m(x)|\nu_{1m}\rangle + c_m(x)|\nu_{2m}\rangle$$

Transition probability: $P_{\rm tr} = |\langle\nu_\mu|\nu(x)\rangle|^2 \quad \Rightarrow$

$$\diamondsuit \qquad \boxed{P_{\rm tr} \;=\; \frac{1}{2} - \frac{1}{2}\cos 2\theta_i \cos 2\theta_f - \frac{1}{2}\sin 2\theta_i \sin 2\theta_f \sin\Phi}$$

$$\theta_i = \theta_m(0), \qquad \theta_f = \theta_m(x), \qquad \Phi = \int_0^x (\mathcal{E}_1 - \mathcal{E}_2)\, dx'$$

◇ Problem: Derive this expression.

If $N_e(0) \gg (N_e)_{\rm res}$ or $\theta_f \ll 1$: the 3rd term can be neglected (also if $\Phi \gg 1$ and averaging is performed) $\Rightarrow$ $P_{\rm tr}$ depends only on θ_i and θ_f.

In the case $N_e(0) \gg (N_e)_{\rm res}$, $N_e(x) \ll (N_e)_{\rm res}$ (i.e. $\theta_i \simeq 90^\circ$, $\theta_f \simeq \theta$)

$\Rightarrow \quad P_{\rm tr} = \cos^2\theta, \quad P_{\rm surv} = \sin^2\theta.$

Violation of adiabaticity

Possible adiabaticty violation can be taken into account.
E.g. in the averaging regime (Parke, 1986):

$$\diamond \quad \boxed{\overline{P}_{\rm tr} \;=\; \frac{1}{2} \;-\; \frac{1}{2}\cos 2\theta_i \cos 2\theta_f \,(1 \;-\; 2P')}$$

P' – probability of $\nu_{1m} \leftrightarrow \nu_{2m}$ transitions between points 0 and x. In the Landau-Zener approximation: $P' \simeq e^{-\frac{\pi}{2}\gamma}$ where γ is the adiab. parameter.

In the extreme non-adaiabatic regime:

$$\diamond \quad i\frac{d}{dx}\begin{pmatrix} \nu_{1m} \\ \nu_{2m} \end{pmatrix} = \begin{pmatrix} 0 & -i\theta'_m(x) \\ i\theta'_m(x) & 0 \end{pmatrix}\begin{pmatrix} \nu_{1m} \\ \nu_{2m} \end{pmatrix}$$

Can be solved exactly by $x \to \tau = \theta_m(x)$, $\frac{d}{d\tau} = \frac{1}{\theta'_m(x)}\frac{d}{dx}$ $\Rightarrow$

$$\diamond \quad P' \;=\; \sin^2(\theta_i - \theta_f)$$

◇ Problem: Derive this expression.

Vacuum oscillation limits

1. The mixing angle and osc. length in matter θ_m, $l^m_{\rm osc}$ go to θ, $l_{\rm osc}$ when

$$V \;=\; \sqrt{2}G_F N_e \;\ll\; \frac{\Delta m^2}{2E}$$

$\Rightarrow \quad P_{\rm osc} \to P^{vac}_{\rm osc}$. In terms of convenient parameters:

$$\sqrt{2}G_F N_e \simeq 7.63\times 10^{-14}\rho({\rm g/cm^3}){\rm Y_e}\ \ {\rm eV}\,, \qquad {\rm Y_e} = \frac{\rm N_e}{\rm N_p + N_n}$$

2. In general (even in the case $V \gg \Delta m^2/2E$) the vacuum oscsill. probability is recovered in the short baseline limit. In matter of constant density:

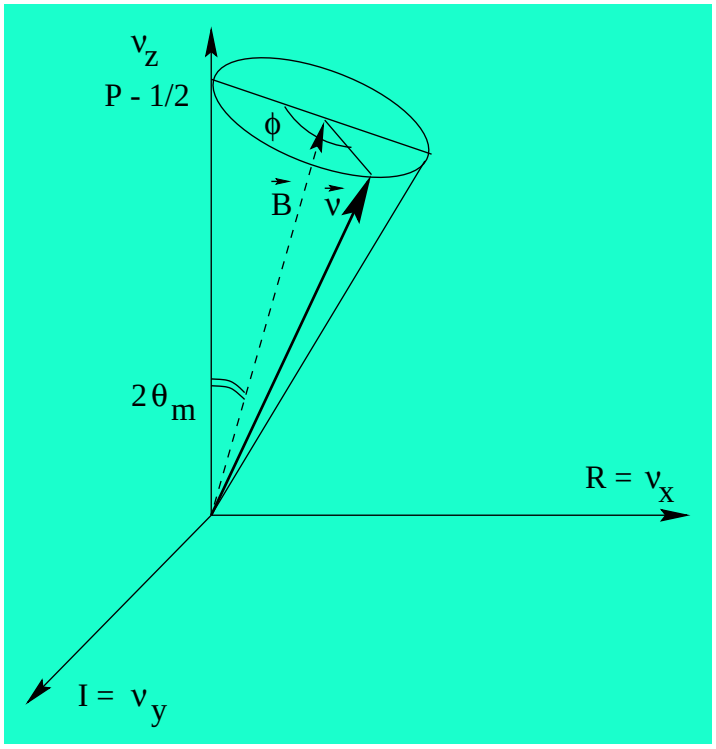
$$P_{\rm tr} = \sin^2 2\theta_m \sin^2(\omega L) = \frac{\sin^2 2\theta\cdot\left(\frac{\Delta m^2}{4E}\right)^2}{\omega^2}\sin^2(\omega L)\,, \qquad \omega \equiv \frac{1}{2}|\mathcal{E}_2 - \mathcal{E}_1|.$$

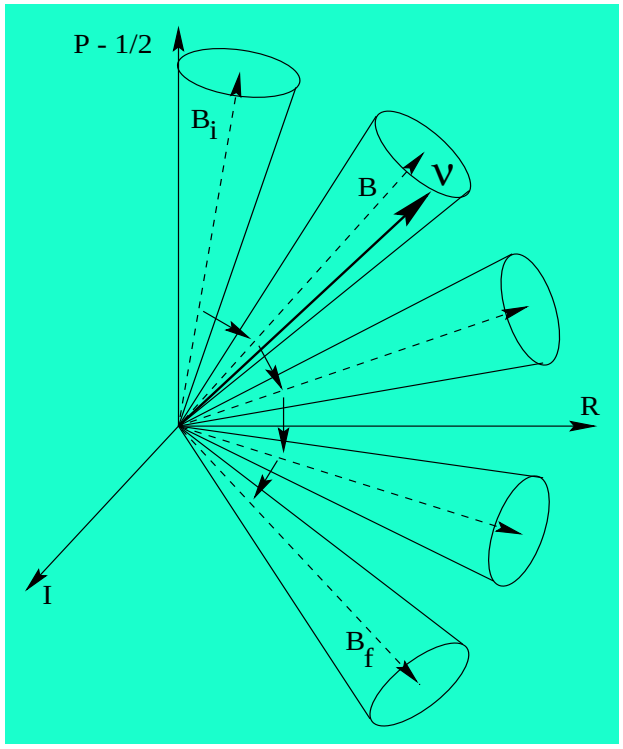
For $\omega L \ll 1$:

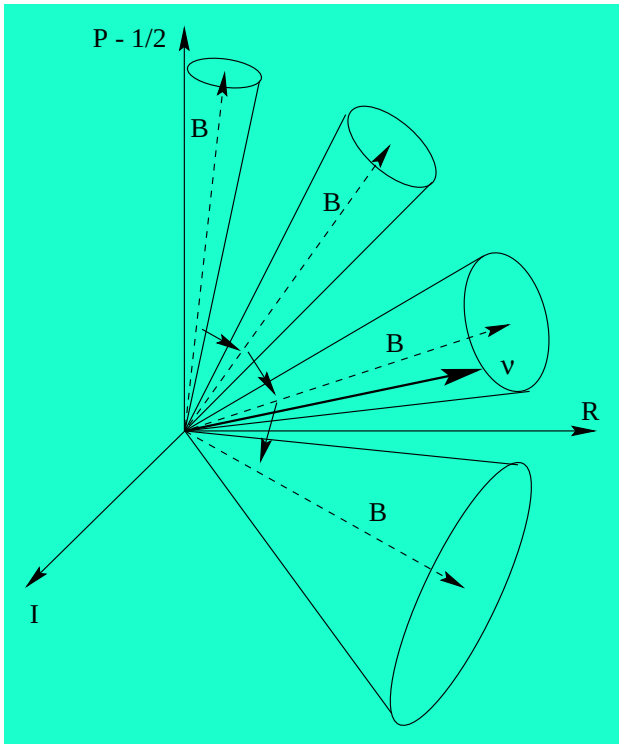
$$P_{\rm tr} \;\simeq\; \sin^2 2\theta\cdot\left(\frac{\Delta m^2}{4E}L\right)^2 \;=\; P^{vac}_{\rm tr} \;\text{ in short } L \text{ limit}\,.$$

Problem (*): Does this hold also for $N_e \neq const.$?

Analogy: Spin precession in a magnetic field







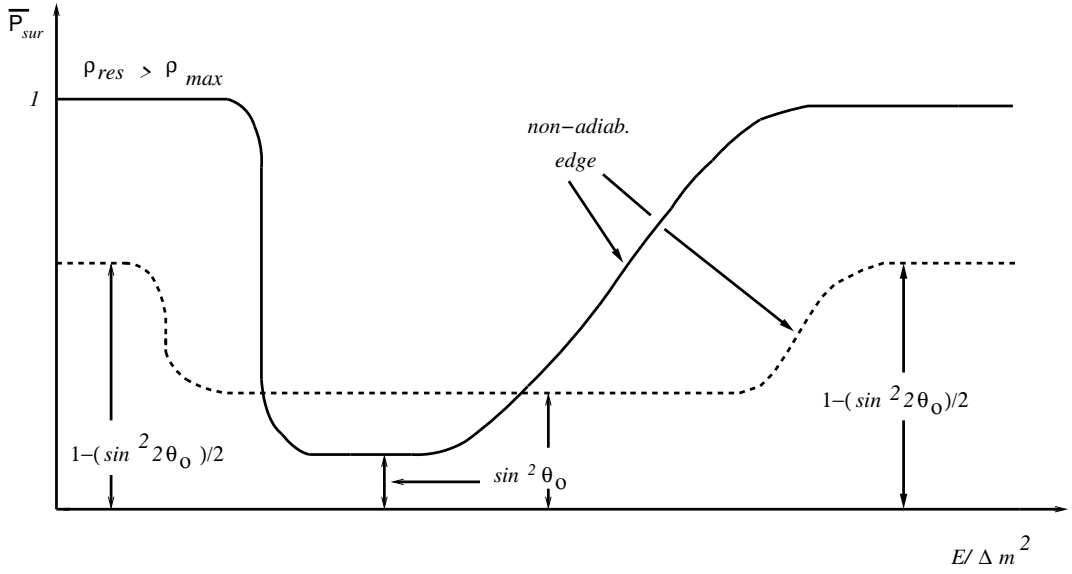
$$\frac{d\vec{S}}{dt} = 2(\vec{B} \times \vec{S})$$

$$\vec{S} = \{\mathrm{Re}(\nu_e^* \nu_\mu)\,,\ \mathrm{Im}(\nu_e^* \nu_\mu)\,,\ \nu_e^* \nu_e - 1/2\}$$

$$\vec{B} = \{(\Delta m^2/4E)\sin 2\theta\,,\ \ 0\,,\ \ V/2 - (\Delta m^2/4E)\cos 2\theta\}$$

MSW effect and solar neutrinos

The survival probability for solar ν_e:



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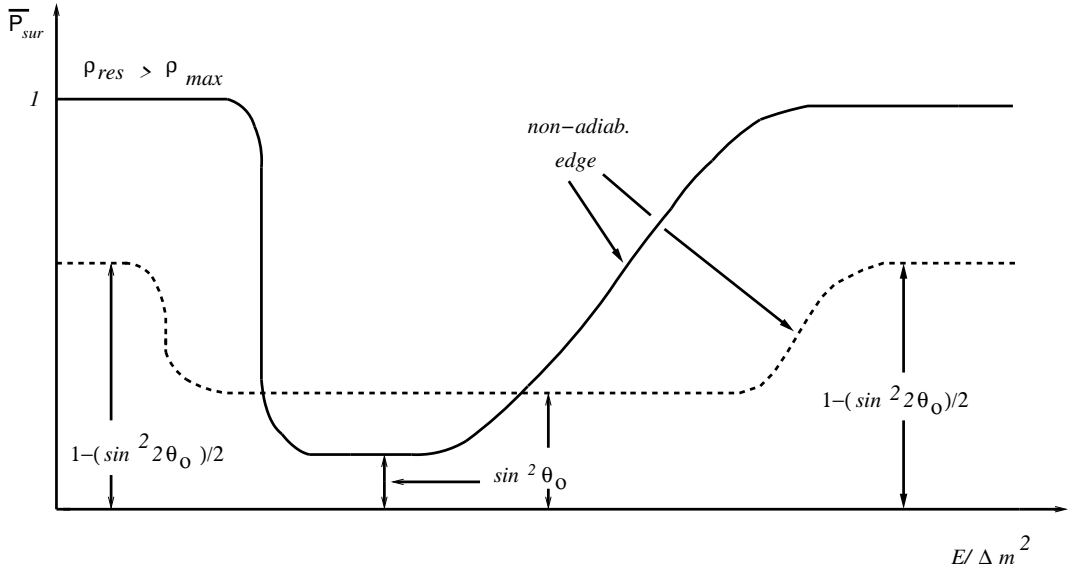


Day-night effect: the probability of finding a solar ν_e after it traverses the Earth

$$P_{SE} = \bar{P}_S + \frac{1 - 2\bar{P}_S}{\cos 2\theta_0} \left(P_{2e} - \sin^2 \theta_0 \right) .$$

Here: $P_{2e} = P(\nu_2 \to \nu_e)$ – probability of oscillations of the second mass eigenstate into electron neutrino inside the Earth.

General properties of $P_{\alpha\beta}$ and CP, T and CPT

General properties of $P_{\alpha\beta}$

3 flavours $\Rightarrow 3 \times 3 = 9$ probabilities

$$P_{\alpha\beta} = P(\nu_\alpha \to \nu_\beta),$$

plus 9 probabilities for antineutrinos $P_{\bar{\alpha}\bar{\beta}}$.
Unitarity conditions (probability conservation):

$$\sum_\beta P_{\alpha\beta} = \sum_\alpha P_{\alpha\beta} = 1 \qquad (\alpha, \beta = e, \mu, \tau)$$

5 indep. conditions $\Rightarrow$ $9 - 5 = 4$ indep. probabilities left.
Additional symmetry: the matrix of matter-induced potentials $\mathrm{diag}(V(t), 0, 0)$ commutes with O_{23} $\Rightarrow$ additional relations between probabilities.

Dependence on θ_{23} and # of indep. $P_{\alpha\beta}$

Define

$$\tilde{P}_{\alpha\beta} = P_{\alpha\beta}(s_{23}^2 \leftrightarrow c_{23}^2,\, \sin 2\theta_{23} \to -\sin 2\theta_{23})$$

(e.g., $\theta_{23} \to \theta_{23} + \pi/2$). Then

$$P_{e\tau} = \tilde{P}_{e\mu} \quad P_{\tau\mu} = \tilde{P}_{\mu\tau} \quad P_{\tau\tau} = \tilde{P}_{\mu\mu}$$

2 out of 3 conditions are independent $\Rightarrow$ $4 - 2 = 2$
indep. probabilities (e.g., $P_{e\mu}$ and $P_{\mu\tau}$) $\Rightarrow$

$\Diamond$ *All 9 neutrino ocillation probabilities can be expressed through just two!*

$$P_{\bar{\alpha}\bar{\beta}} = P_{\alpha\beta}(\delta_{\rm CP} \to -\delta_{\rm CP},\, V \to -V)$$

$\Rightarrow$ *All 18 ν and $\bar{\nu}$ probab. can be expressed through just two*

$\not{CP}$ and $\not{T}$ in ν oscillations in matter

- CP: $\nu_{\alpha,\beta} \leftrightarrow \bar{\nu}_{\alpha,\beta} \quad \Rightarrow \quad U_{\alpha i} \to U^*_{\alpha i} \quad (\{\delta_{\rm CP}\} \to -\{\delta_{\rm CP}\})$
  $V(r) \to -V(r)$

- T: $t \rightleftarrows t_0 \quad \Leftrightarrow \quad \nu_\alpha \leftrightarrow \nu_\beta$
  $\Rightarrow \quad U_{\alpha i} \to U^*_{\alpha i} \quad (\{\delta_{\rm CP}\} \to -\{\delta_{\rm CP}\})$
  $V(r) \to \tilde{V}(r)$

$$\tilde{V}(r) = \sqrt{2} G_F \tilde{N}(r)$$

$\tilde{N}(r)$: corresponds to interchanged positions of ν source and detector.
Symmetric density profiles: $\tilde{N}(r) = N(r)$

◇ *The very presence of matter [with (# of particles) $\neq$ (# of antiparticles)] violates C, CP and CPT!*

$\Rightarrow$ Fake (extrinsic) $\not{CP}$ which may complicate the study of fundamental (intrinsic) $\not{CP}$

$\not{CP}$ in matter

- Exists even in 2f case (in $\geq$ 3f case exists even when all $\{\delta_{\rm CP}\} = 0$) due to matter effects:

$$P(\nu_\alpha \to \nu_\beta) \neq P(\bar{\nu}_\alpha \to \bar{\nu}_\beta)$$

E.g., MSW effect can enhance $\nu_e \leftrightarrow \nu_\mu$ and suppress $\bar{\nu}_e \leftrightarrow \bar{\nu}_\mu$ or vice versa.

- Survival probabilities are not CP-invariant:

$$P(\nu_\alpha \to \nu_\alpha) \neq P(\bar{\nu}_\alpha \to \bar{\nu}_\alpha)$$

To disentangle fundamental $\not{CP}$ from the matter induced one in LBL experiments – need to measure energy dependence of oscillated signal or signal at two baselines

Alternatives:

- Low-E experiments ($E \sim$ 0.1 – 1 GeV) with $L \sim$ 100 – 1000 km
- Indirect measurements: CP-even terms $\sim \cos\delta_{\rm CP}$ or area of leptonic unitarity triangle

$\not{T}$ in matter

CPT not conserved in matter $\Rightarrow$ $\not{CP}$ and $\not{T}$ are not directly related!

- Matter does not necessarily induce $\not{T}$ (only asymmetric matter with $\tilde{N}(r) \neq N(r)$ does)
- There is no $\not{T}$ (either fundamental or matter induced) in 2f case – a consequence of unitarity:

$$P_{ee} + P_{e\mu} = 1$$

$$P_{ee} + P_{\mu e} = 1$$

$$\Downarrow$$

$$P_{e\mu} = P_{\mu e}$$

- In 3f case – only one T-odd probability difference for ν's (and one for $\bar{\nu}$'s) irrespective of matter density profile – a consequence of unitarity in 3f case

$$\Delta P^T_{e\mu} = \Delta P^T_{\mu\tau} = \Delta P^T_{\tau e}$$

Matter-induced $\not{T}$:

- ◇ An interesting, pure 3f matter effect; absent in the case of symmetric density profiles (e.g., $N(r) = const$)
- ◇ Does not vanish in the regime of complete averaging
- ◇ May fake fundamental $\not{T}$ and complicate its study (extraction of $\delta_{\rm CP}$ from the experiment)
- ◇ Vanishes when either $U_{e3} = 0$ or $\Delta m^2_{21} = 0$ (2f limits) $\Rightarrow$ doubly suppressed by both these small parameters

$\Rightarrow$ *Perturbation theory can be used to get analytic expressions*

“CPT in matter”

Is there a relation between $\not{CP}$ and $\not{T}$ in matter?

For symmetric density profiles (i.e. $\tilde{V}(r) = V(r)$)

$$P(\nu_\alpha \to \nu_\beta;\, \delta_{\rm CP},\, V(r)) \;=\; P(\bar{\nu}_\beta \to \bar{\nu}_\alpha;\, \delta_{\rm CP},\, -V(r))$$

(Minakata, Nunokawa & Parke, 2002)

Easy to generalize to the case of an arbitrary density profile:

$$P(\nu_\alpha \to \nu_\beta;\, \delta_{\rm CP},\, V(r)) \;=\; P(\bar{\nu}_\beta \to \bar{\nu}_\alpha;\, \delta_{\rm CP},\, -\tilde{V}(r))$$

Unlike CPT in vacuum, does not directly relate observables

Can be useful for cross-checking theoreticl calculations

Summary – 3f effects in ν oscillations

- Two types of 3f effects – "trivial" (existence of new channels, their inter-dependence through unitarity) and nontrivial (interference of different parameter channels, qualitatively new effects – fundamental CP and T-violation, and matter - induced T violation

- 3f corrections to probabilities of oscillations of solar, atmospheric, reactor and acceler. neutrinos depend on $|U_{e3}| = |\sin\theta_{13}|$; can reach $\sim (5-10)\%$

- Possible interesting 3f effects for SN neutrinos – depend significantly on the value U_{e3} (known now to be not too small)

Summary – contd.

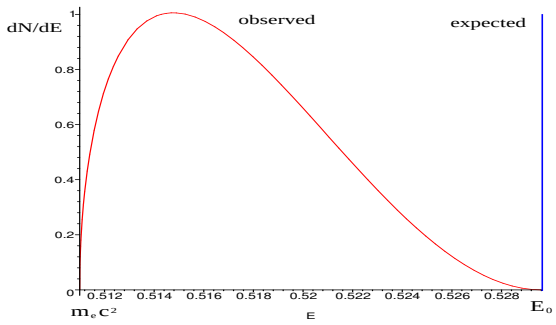
◇ Manifestations of ≥ 3 flavours in neutrino oscillations:

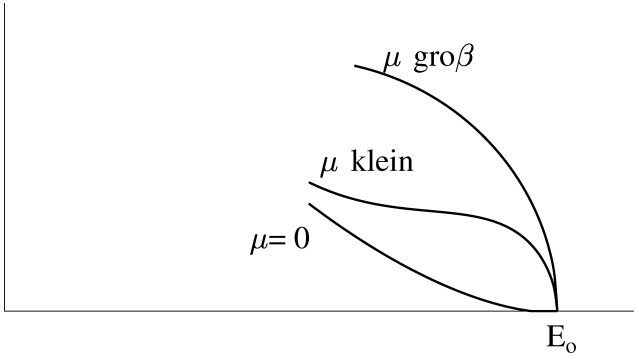
- Fundamental $\not{C}\not{P}$ and $\not{T}$
- Matter-induced $\not{T}$
- Matter effects in $\nu_\mu \leftrightarrow \nu_\tau$ oscillations
- Specific CP and T conserving interference terms in oscillation probabilities

◇ U_{e3} plays a very special role

Direct neutrino mass measurements

Electron spectrum in β decay





E. Fermi, Z. Phys. 1934

Electron spectrum in allowed β decays:

$$N_e(E_e)dE_e \propto F(Z,E_e)\sqrt{E_e^2 - m_e^2}\, E_e(E_0 - E_e)^2\, dE_e\,, \quad (m_\nu = 0);$$

$$N_e(E_e)dE_e \propto F(Z,E_e)\sqrt{E_e^2 - m_e^2}\, E_e(E_0 - E_e)\sqrt{(E_0 - E_e)^2 - m_\nu^2}\, dE_e\,, \quad (m_\nu \neq 0)$$

For n mixed neutrinos:

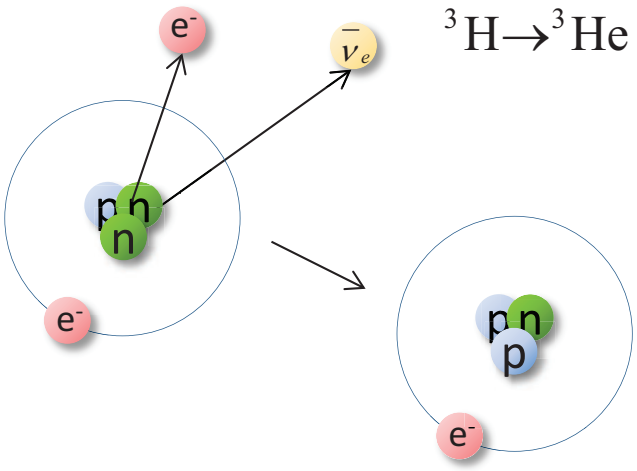
$$m_\nu^2 \to m_\beta^2 \equiv \sum_{i=1}^{n} |U_{ei}|^2 m_i^2$$

Troitsk & Mainz expts. ($^3\mathrm{H} \to {}^3\mathrm{He} + e^- + \bar{\nu}_e$): $m_\beta^2 < (2.2\ \mathrm{eV})^2$ (95% C.L.)

KATRIN (expected sensitivity): $m_\beta < 0.2\ \mathrm{eV}$ (90% C.L.).

Discovery potential: $m_\beta = 0.35\ \mathrm{eV}$ (5σ).

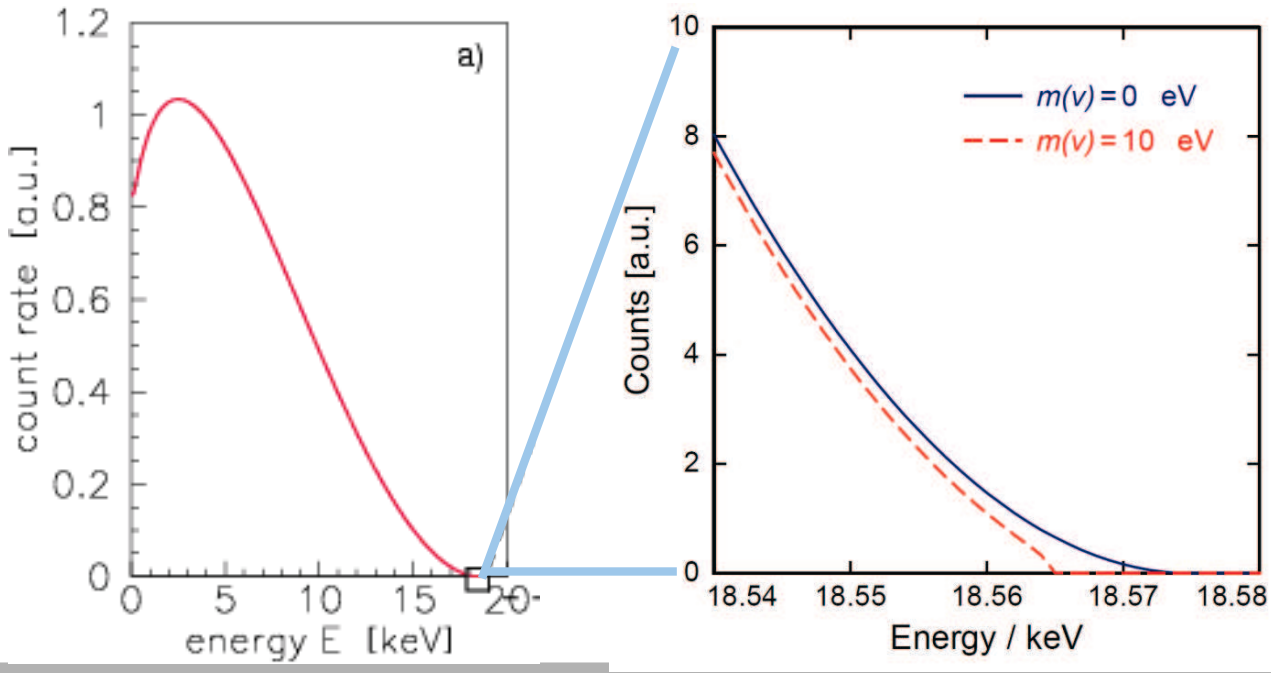
Beta decay of ^{3}H



$$^3\mathrm{H} \rightarrow {}^3\mathrm{He} + e^- + \bar{\nu}_e$$

Precision on the neutrino mass determination relies on

- ✓ Precise modelling of the atomic and molecular final state
- ✓ Background reductions

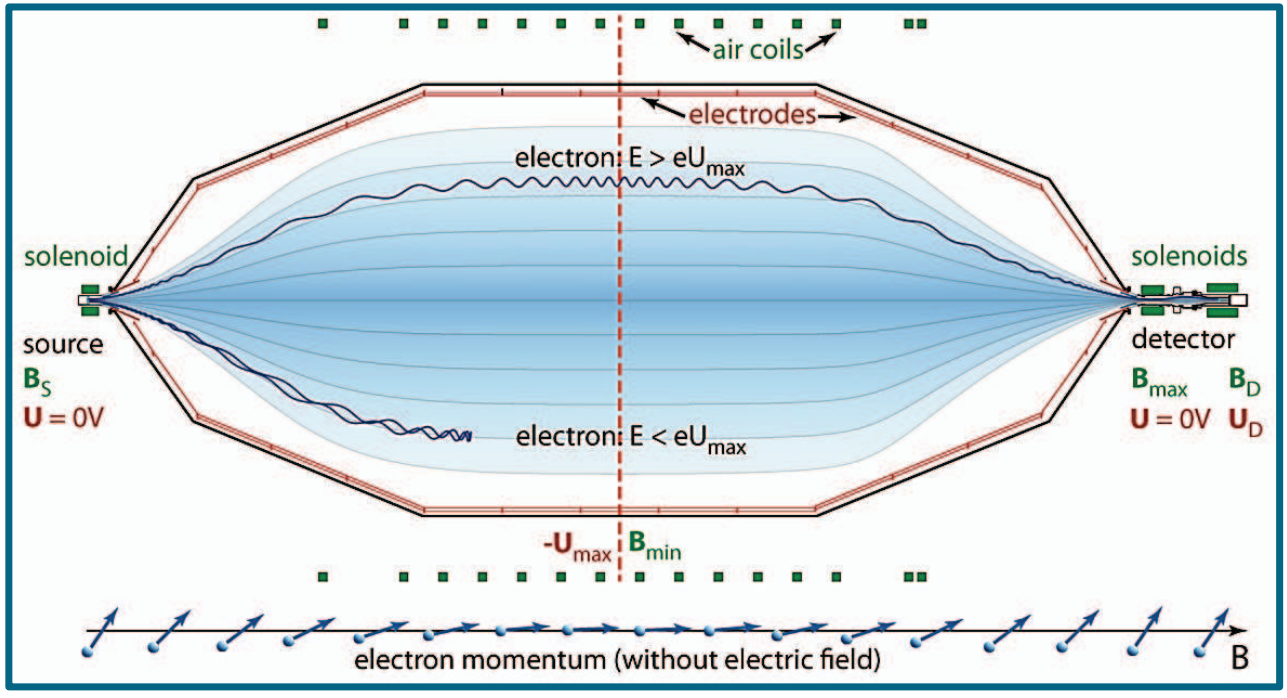


Only a small fraction of events in the last eV below the endpoint: **$2 * 10^{-13}$**

Triutium is present as **bi-atomic molecules**

High resolution β-spectroscopy: MAC-E-Filter

Magnetic **A**diabatic **C**ollimation and **E**lectrostatic Filter:



Magnetic guiding and collimation of e$^-$

- Transform $E_\perp$ to $E_{||}$

$$\mu = \frac{E_\perp}{B} = \text{const.}$$

Electrostatic field for energy analysis

- Sharp transmission depending on:
  - Emission angle
  - Radius at B_{min}

Integrated energy resolution:

$$\Delta E = qU_{\max}\frac{B_{\min}}{B_{\max}}$$

e.g. A. Picard et al., NIM-B63(1992) 345-358

KATRIN experiment in Karlsruhe

main spectrometer: transport

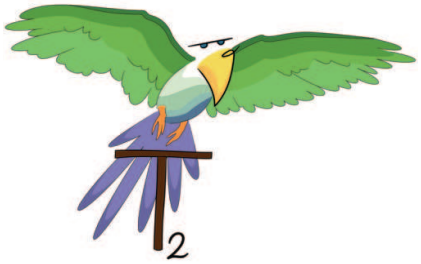
End of the 8800 km voyage

16 **The KATRIN main spectrometer: status of the electrode system**
J.Wolf, KIT SFB-TR27-Meeting 13.06.2008, Karlsruhe

KIT – die Kooperation von Forschungszentrum Karlsruhe GmbH und Universität Karlsruhe (TH)

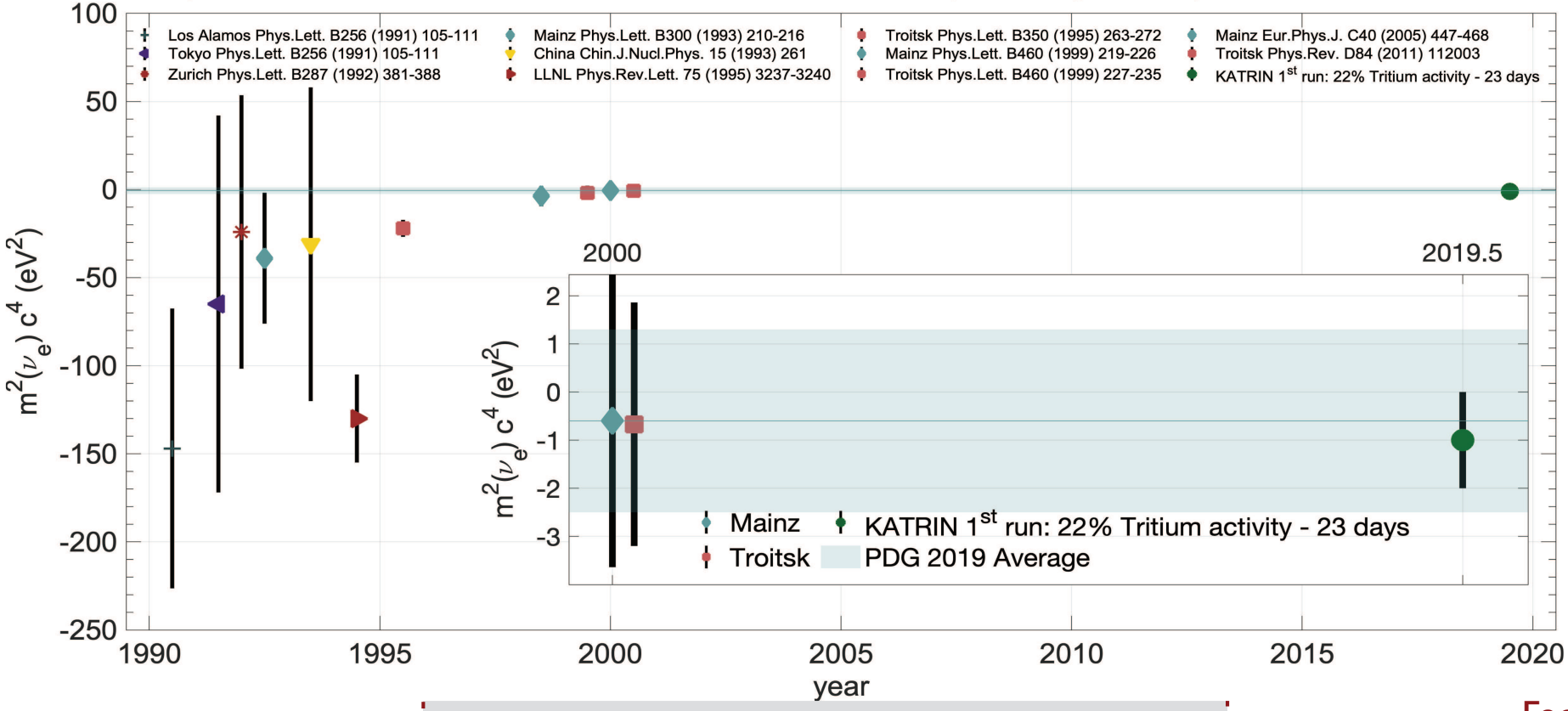
Forschungszentrum Karlsruhe in der Helmholtz-Gemeinschaft

Universität Karlsruhe (TH)



KATRIN's 1st Measurement!

Squared neutrino mass values obtained from tritium β -decay in the period 1990-2019



$$m(\nu_e) < 1.1 \text{ eV } (90\% \text{ C.L.})$$

Factor of 2 improvement in 30 days!

Different technologies

Magnetic calorimeters

e^- capture (^{163}Ho – ECHo, HOLMES, NuMECS...)

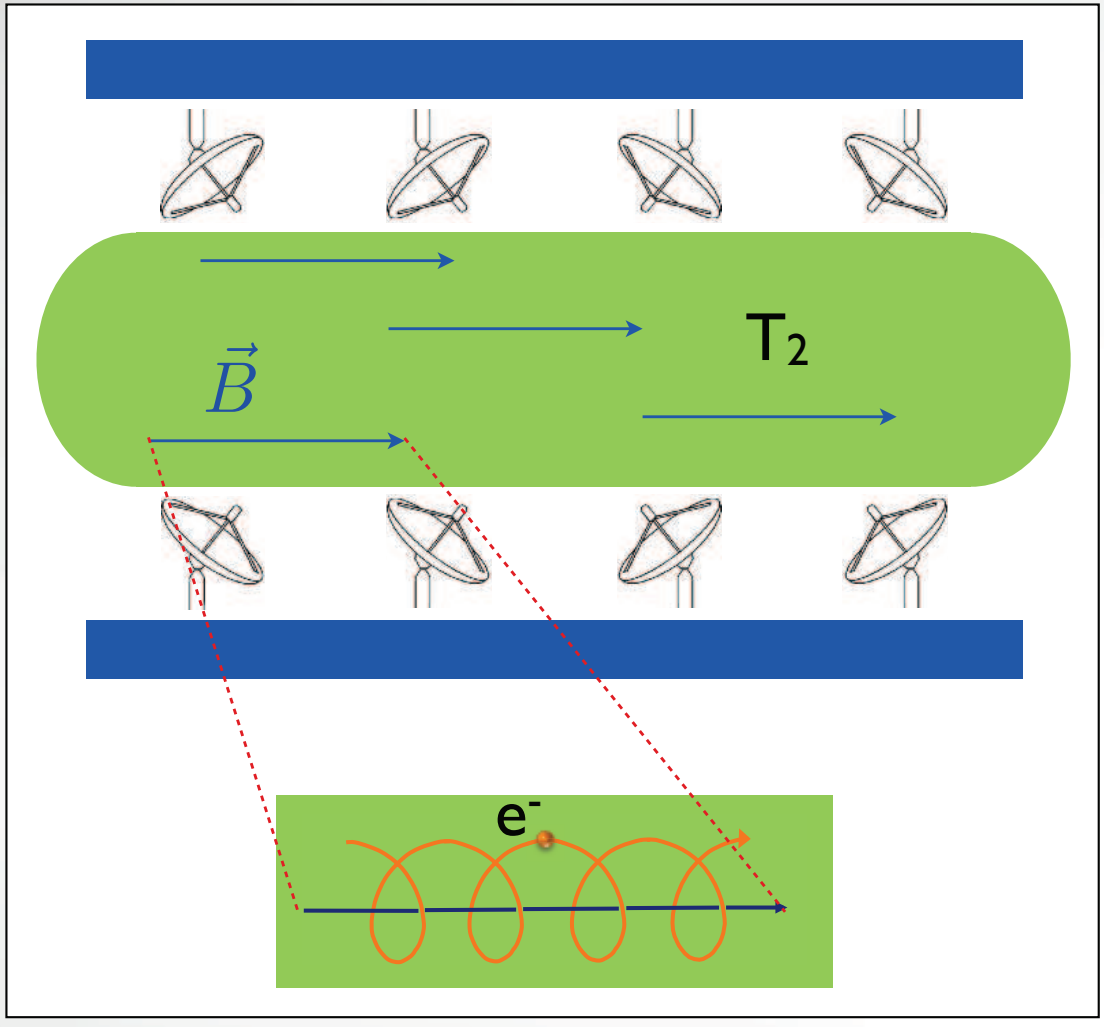
<——————————>

Electron synchrotron radiation (Project 8)

Novel Technique: CRES

Cyclotron Radiation Emission Spectroscopy

- Enclosed volume
- Fill with tritium gas
- Add a magnetic field

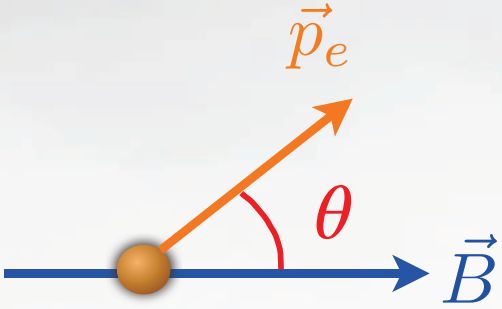


- Decay electrons spiral around field lines
- Add antennas to detect the cyclotron radiation

B. Monreal and J. Formaggio, Phys. Rev. D80 051301 (2009)

Pitch Angle

The angle between
the electron momentum
and the magnetic field

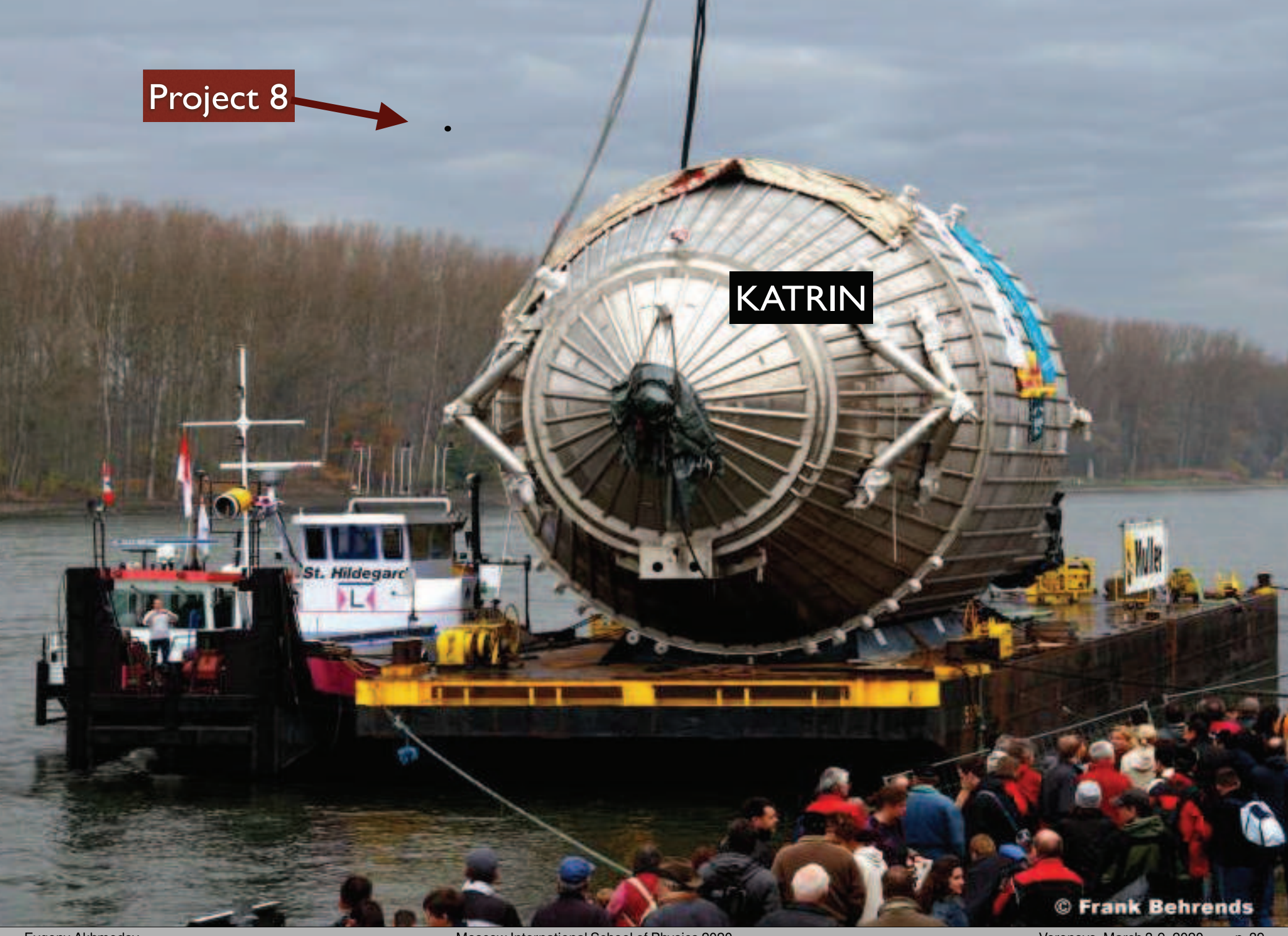


- Correction term for the cyclotron frequency

$$\omega_\gamma = \frac{\omega_0}{\gamma} = \frac{eB}{K + m_e}\left(1 + \frac{\cot^2\theta}{2}\right)$$

- Power emitted

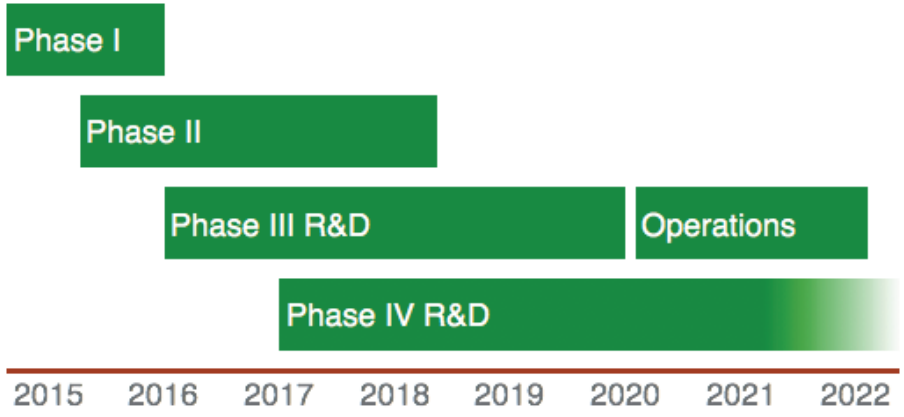
$$P_{\text{tot}} = \frac{1}{4\pi\epsilon_0}\frac{2q^2\omega_c^2}{3c}\frac{\beta^2\sin^2\theta}{1-\beta^2}$$


Project 8
KATRIN
St. Hildegard
© Frank Behrends

Project 8 Experiment

A phased tritium beta endpoint experiment to measure the electron neutrino mass

- **Phase I (Complete)**
  - First demonstration of CRES technique with ^{83m}Kr
- **Phase II (2015-2018)**
  - First tritium measurement with CRES
  - Endpoint determination to ~30 eV
  - *see also Mathieu Guigue, Thurs. parallel*
- **Phase III (2016-2022)**
  - CRES demonstration in 200 cm^3 free space volume
  - Neutrino mass sensitivity of ~2 eV
- **Phase IV (2017+)**
  - Atomic tritium endpoint measurement with m_ν ~ 40 meV projected sensitivity



Cosmological constraints

Cosmology: constraints on $\sum m_\nu$. Strongly depend on what is taken into account.

- Typically range from $\sum m_\nu < 0.32$ eV (Planck, . . .)
  down to $\sum m_\nu < 0.12$ eV (Planck + Lyman α) (95% C.L.).
- In a foreseeable future may start probing hierarchical neutrino masses.
- eV - range sterile neutrinos ruled out (if thermalized).
- keV - scale sterile neutrino (warm dark matter) allowed

2β decay

Decay modes for Double Beta Decay

Double Beta Decay is a very rare, second-order weak nuclear transition which is possible for a few tens of even-even nuclides

Two decay modes are usually discussed:

① $(A,Z) \rightarrow (A,Z+2) + 2e^- + 2\bar{\nu}_e$ ← 2ν Double Beta Decay allowed by the Standard Model already observed – $\tau \geq 10^{19}$ y

② $(A,Z) \rightarrow (A,Z+2) + 2e^-$ ← neutrinoless Double Beta Decay (0ν-DBD) never observed (except a discussed claim) $\tau > 10^{25}$ y

Process ② would imply new physics beyond the Standard Model

violation of lepton number conservation

Observation of **0ν-DBD** → $m_\nu \neq 0$, $\bar{\nu} \equiv \nu$

2β decay

Is possible for $A(Z,N)$ when the decay into the "neighbouring" nucleus $A(Z\pm 1, N\mp 1)$ is energetically forbidden, but decay into the next nucleus $A(Z\pm 2, N\mp 2)$ is allowed. ^{82}Se, ^{76}Ge, ^{100}Mo, ^{130}Te, ^{96}Zr, ^{48}Ca, ^{136}Xe, ...

Extremely rare decays $(\Gamma \propto G_F^4)$, $T_{1/2}(2\beta 2\nu) > 10^{19}$ yr.

Usually $2\beta^-$ decays (only few canidates for $2\beta^+$ decays known, expected $T_{1/2}$ very large due to small Q values).

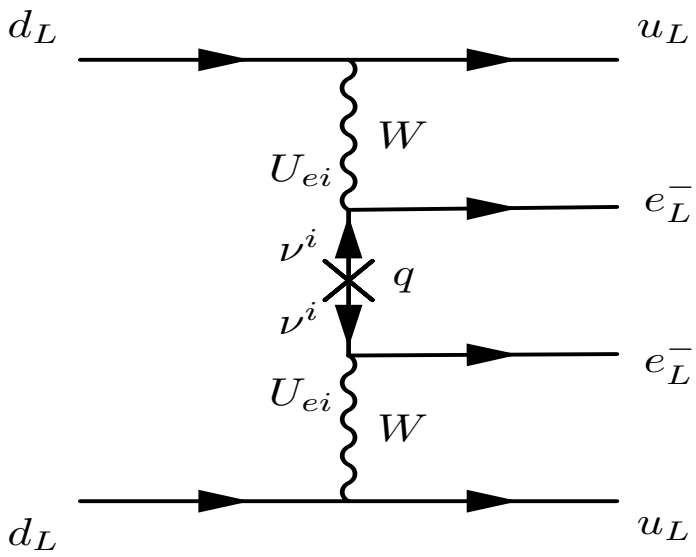
Neutrinoless 2β decay – $\Delta L = 2$ process; would be an unambiguous evidence for Majorana nature of neutrino!

$2\beta 0\nu$ decay not yet experimentally established (only lower bounds on $T_{1/2}(2\beta 0\nu)$ exist). Only one (controversial) claim by part of Heidelberg-Moscow collavoration (Kalpdor-Kleingrothaus et al.). – contradicts data of GERDA expt.

Main uncertainty in the interpretation of the results related to inaccuracy in the theoretciacal calculations of the nuclear matrix elements.

Mechanisms of $2\beta0\nu$ decay

The standard mechanism with a light Majorana neutrino:



$\diamondsuit \quad \mathcal{A}_{2\beta0\nu} \propto \sum_i m_i U_{ei}^2 \equiv m_{\beta\beta}$

In the basis where m_l is diagonalized $m_{\beta\beta}$ is the ee entry of m_ν: $m_{\beta\beta} = m_{ee}$

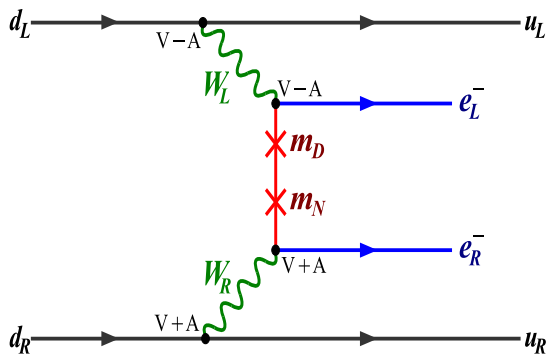
Depends on Majorana-type $\not{CP}$ phases! In the 3f case:

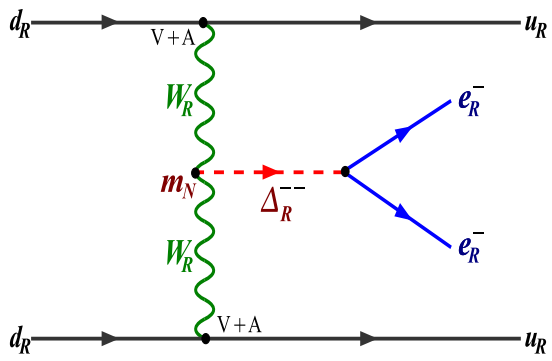
$$\diamondsuit \quad \boxed{m_{\beta\beta} = c_{13}^2 c_{12}^2 m_1 + c_{13}^2 s_{12}^2 e^{2i\sigma_1} m_2 + s_{13}^2 e^{2i(\sigma_2 - \delta_{CP})} m_3.}$$

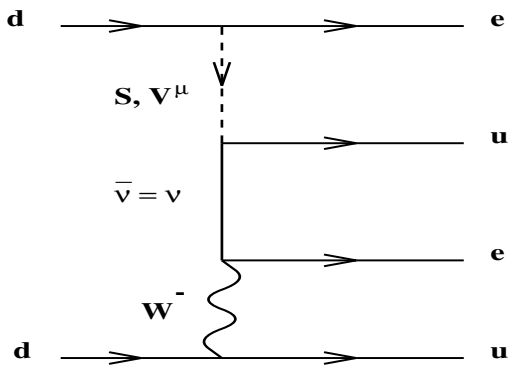
In the case of NH, cancellation possible!

Other mechanisms in extensions of the SM

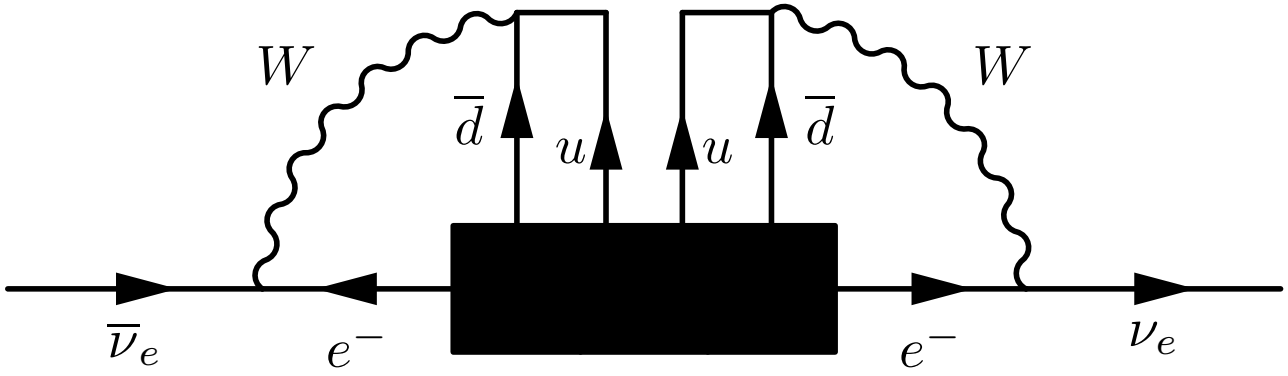
Contributions of W_R, N_R, triplet Higgses, SUSY particles, leptoquarks, ...





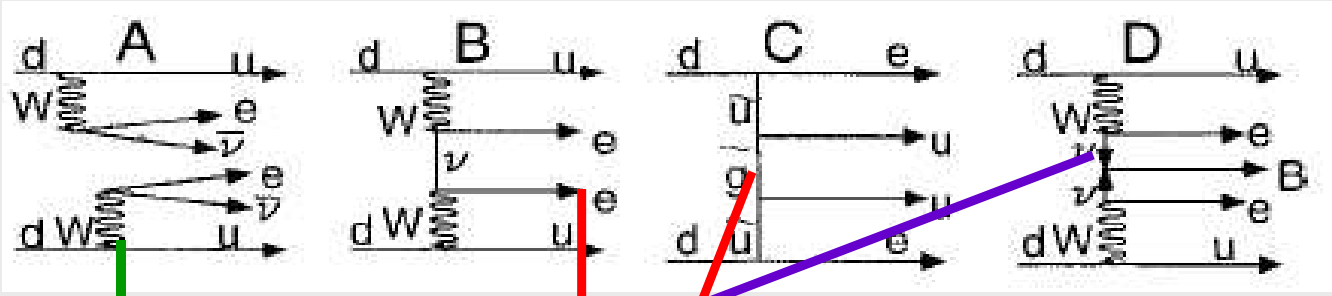


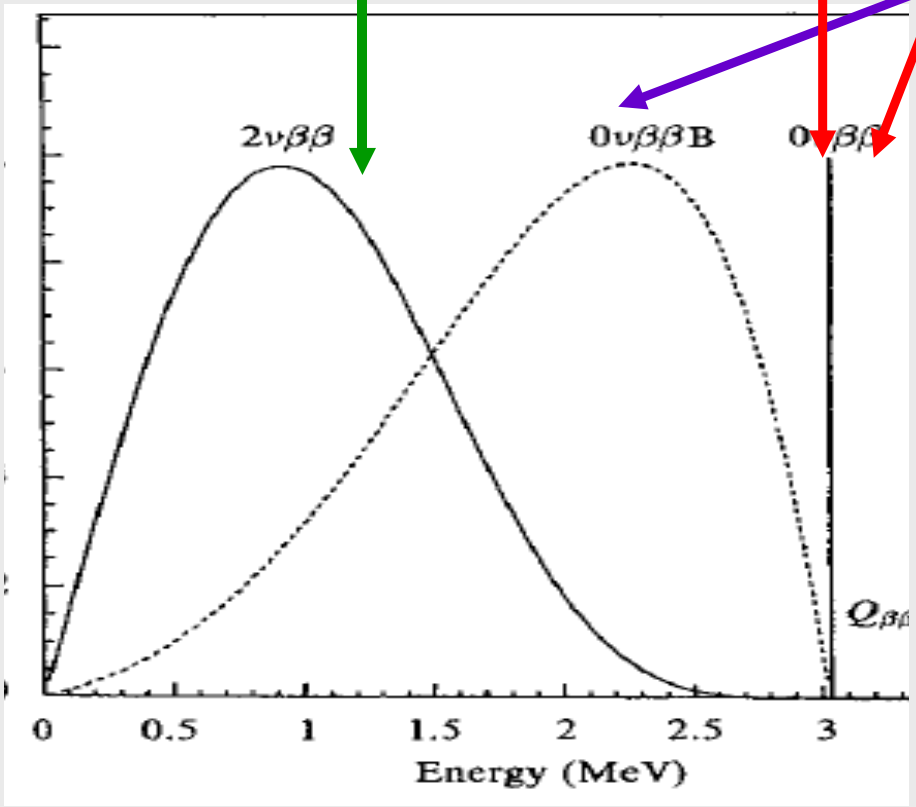
Independently of the $2\beta0\nu$ decay mechanism, neutrino gets Majorana mass term $\Rightarrow$ ν's are Majorana particles! The black box argument:



(Schechter & Valle, 1982)

$0\nu\beta\beta$ by RHC, Heavy ν, SUSY, and others





$A^{0\nu}$ = LHC + RHC

<m> +SUSY $\langle\lambda\rangle \sim k(M_L/M_R)^2$

LHC / RHC

Θ_{21} and E_{12} correlations

LHC m_ν / SUSY

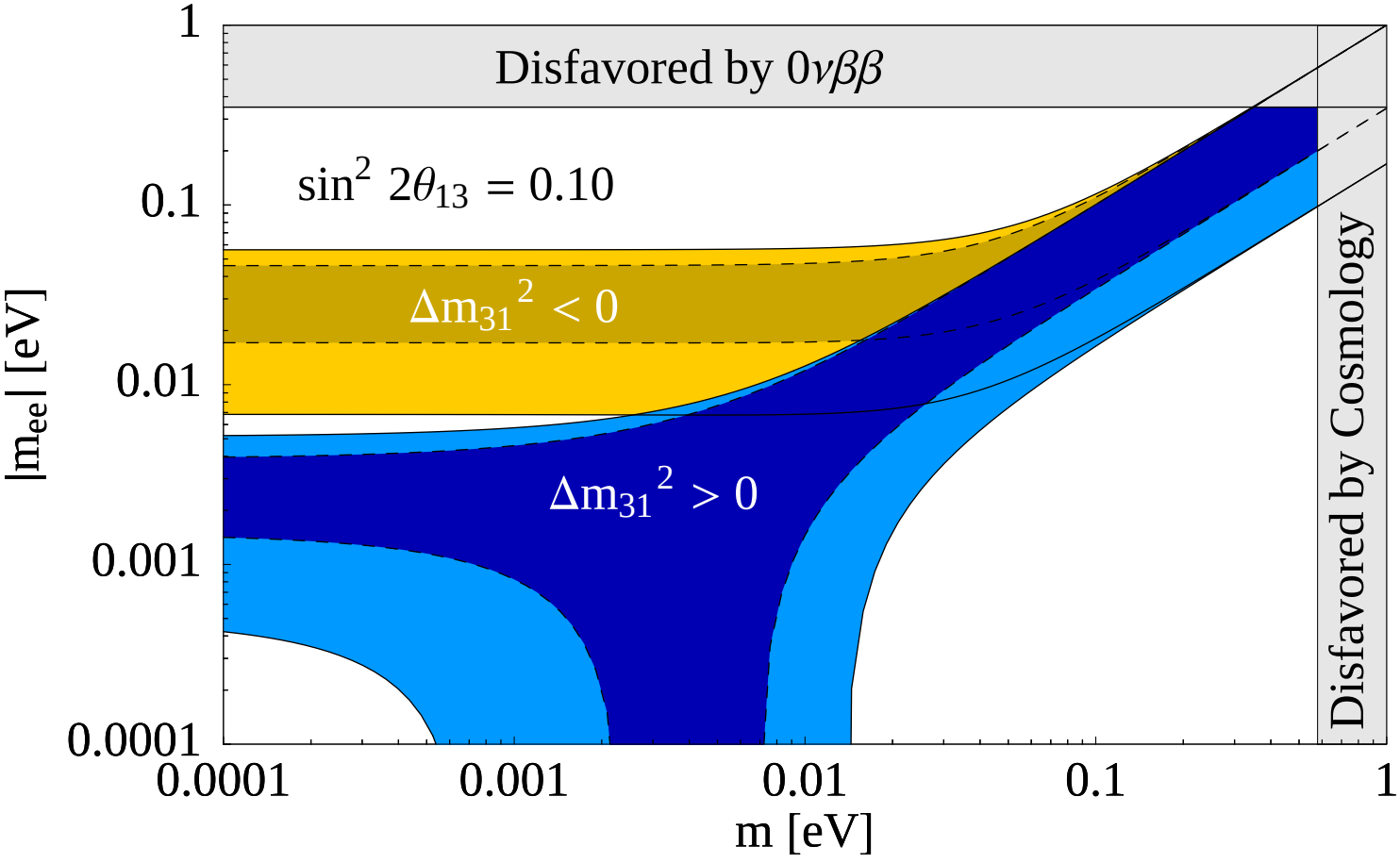
m_ν $M^{0\nu} + kM^S$

different isotopes and states with different M

$A^{2\nu}=GM^{2\nu}$ $A^M=\langle g_M\rangle M$

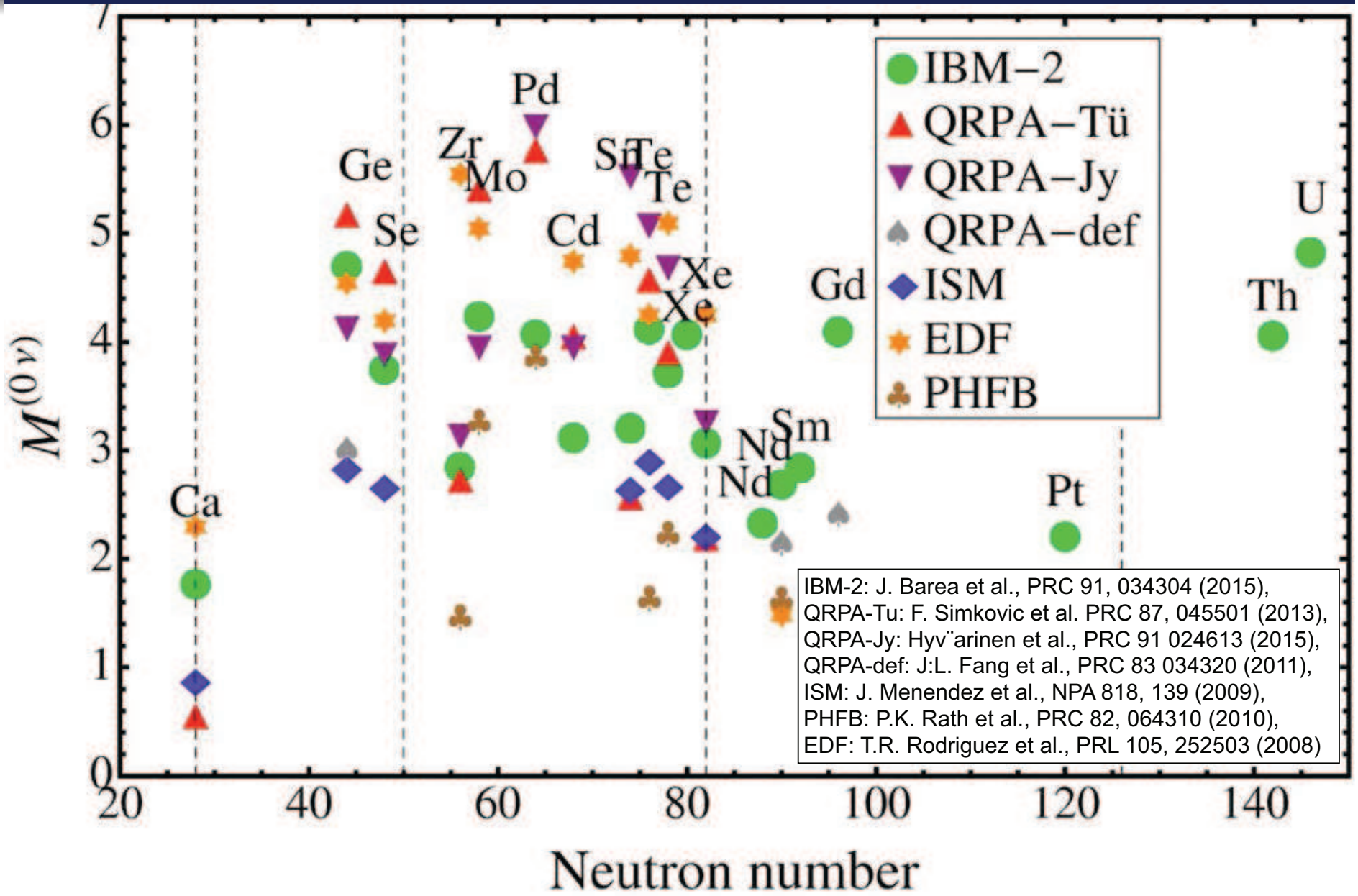
Energy spectra 4,3,2 body

$m_{\beta\beta}$ as a function of $m_{lightest}$



Blue – normal mass ordering, yellow – inverted mass ordering

NME status



IBM-2: J. Barea et al., PRC 91, 034304 (2015),
QRPA-Tu: F. Simkovic et al. PRC 87, 045501 (2013),
QRPA-Jy: Hyv¨arinen et al., PRC 91 024613 (2015),
QRPA-def: J:L. Fang et al., PRC 83 034320 (2011),
ISM: J. Menendez et al., NPA 818, 139 (2009),
PHFB: P.K. Rath et al., PRC 82, 064310 (2010),
EDF: T.R. Rodriguez et al., PRL 105, 252503 (2008)

Experiments

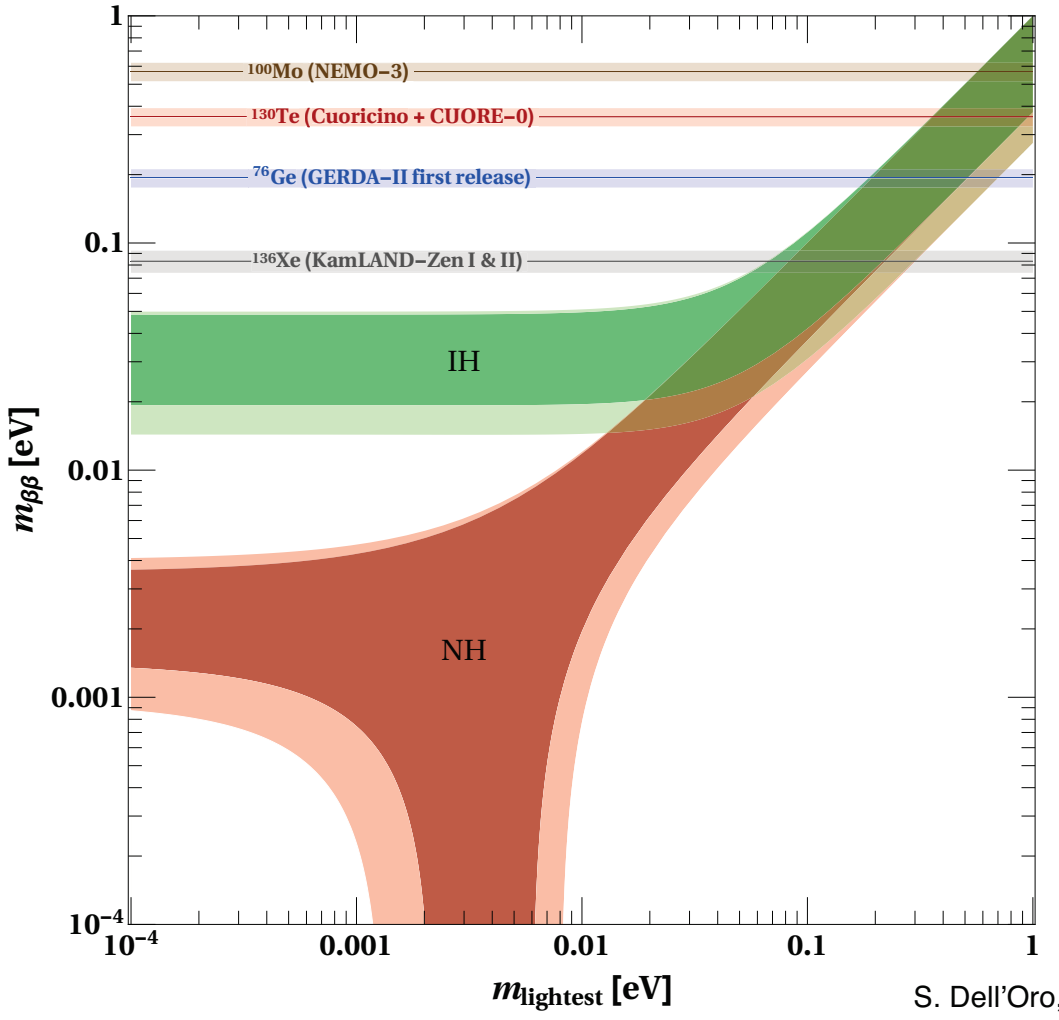
Collaboration	Isotope	Technique	mass ($0\nu\beta\beta$ isotope)	Status
CANDLES	^{48}Ca	305 kg CaF_2 crystals - liq. scint	0.3 kg	Operating
CARVEL	^{48}Ca	$^{48}CaWO_4$ crystal scint.	16 kg	R&D
GERDA I	^{76}Ge	Ge diodes in LAr	15 kg	Complete
GERDA II	^{76}Ge	Point contact Ge in active LAr	44 kg	Operating
MAJORANA DEMONSTRATOR	^{76}Ge	Point contact Ge in Lead	30 kg	Operating
LEGEND 200	^{76}Ge	Point contact Ge in active LAr	200 kg	Construction
LEGEND 1000	^{76}Ge	Point contact Ge in active LAr	1 tonne	R&D
NEMO3	$^{100}Mo/^{82}Se$	Foils with tracking	6.9 kg/0.9 kg	Complete
SuperNEMO Demonstrator	^{82}Se	Foils with tracking	7 kg	Construction
SELENA	^{82}Se	Se CCDs	<1 kg	R&D
NvDEx	^{82}Se	SeF6 high pressure gas TPC	50 kg	R&D
AMoRE	^{100}Mo	CaMoO4 bolometers (+ scint.)	5 kg	Construction
CUPID	^{100}Mo	Scintillating Bolometers	250 kg	R&D
COBRA	$^{116}Cd/^{130}Te$	CdZnTe detectors	10 kg	Operating
CUORE-0	^{130}Te	TeO_2 Bolometer	11 kg	Complete
CUORE	^{130}Te	TeO_2 Bolometer	206 kg	Operating
SNO+	^{130}Te	0.3% ^{nat}Te in liquid scint.	800 kg	Construction
SNO+ Phase II	^{130}Te	3% ^{nat}Te in liquid scint.	8 tonnes	R&D
KamLAND-Zen 400	^{136}Xe	2.7% in liquid scint.	370 kg	Complete
KamLAND-Zen 800	^{136}Xe	2.7% in liquid scint.	750 kg	Operating
KamLAND2-ZEN	^{136}Xe	2.7% in liquid scint.	~tonne	R&D
EXO-200	^{136}Xe	Xe liquid TPC	160 kg	Complete
nEXO	^{136}Xe	Xe liquid TPC	5 tonnes	R&D
NEXT-WHITE	^{136}Xe	High pressure GXe TPC	~5 kg	Operating
NEXT-100	^{136}Xe	High pressure GXe TPC	100 kg	Construction
PandaX	^{136}Xe	High pressure GXe TPC	~tonne	R&D
DARWIN	^{136}Xe	Xe liquid TPC	3.5 tonnes	R&D
AXEL	^{136}Xe	High pressure GXe TPC	~tonne	R&D
DCBA	^{150}Nd	Nd foils & tracking chambers	30 kg	R&D

R&D | Construction | Operating | Complete

Fro

Present experiments ($m_{\beta\beta}$)

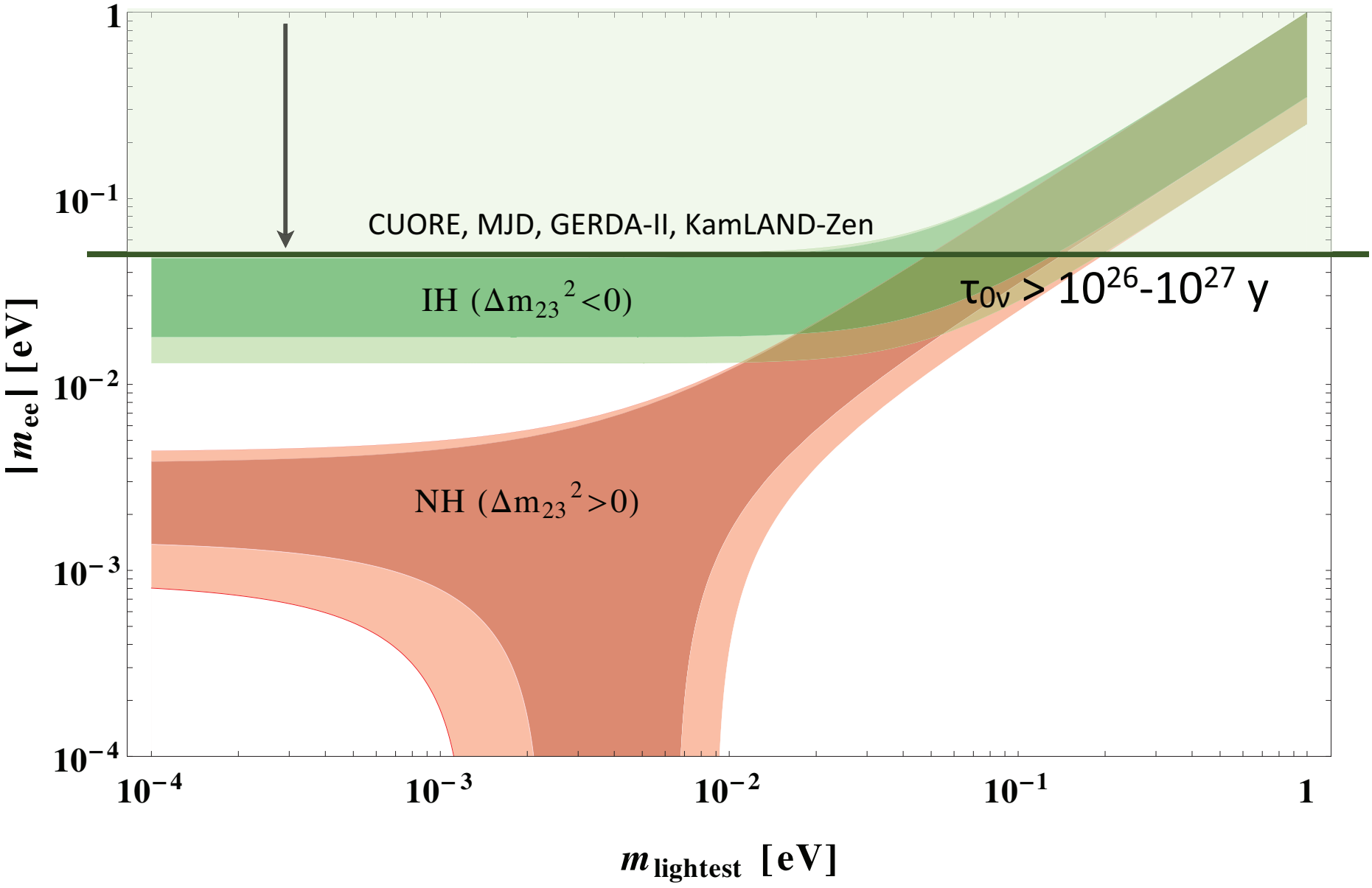
Presently best available published limits for each isotope



S. Dell'Oro, Presentation at NuFact 2016

NOW 2016 - Neutrino Oscillation Workshop Otranto (Lecce, Italy), September 4-11, 2016

Status: near future



Backup slides

Do we need 2β-decay experiments?

Neutrinos are Majorana particles – proven logically :-)

The proof: $\Rightarrow$

(Boris Kayser, 2019)

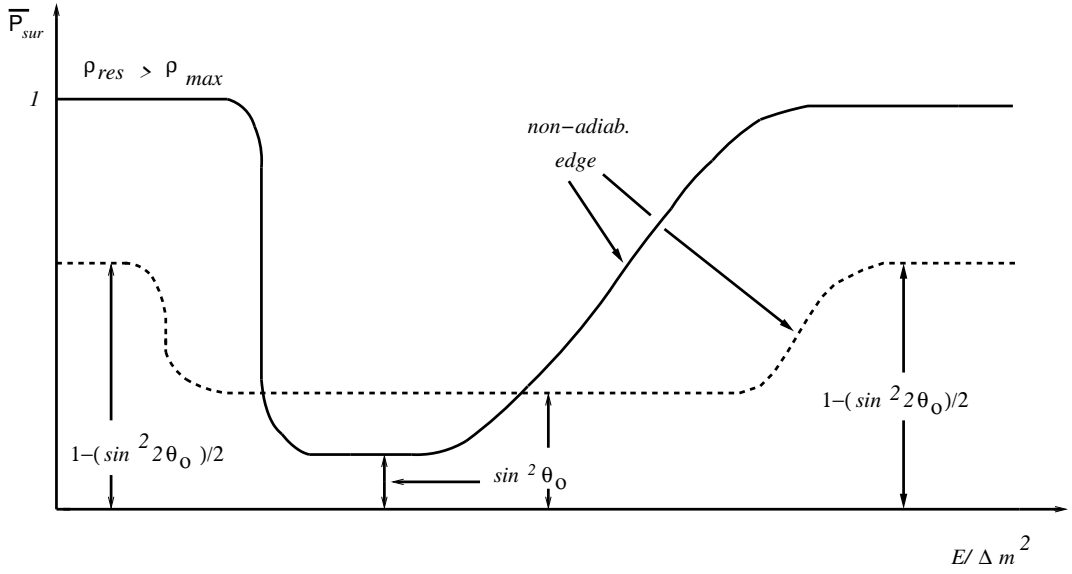
1. There are three phrases on this slide

2. Exactly two of them are wrong

3. Neutrinos are Majorana particles

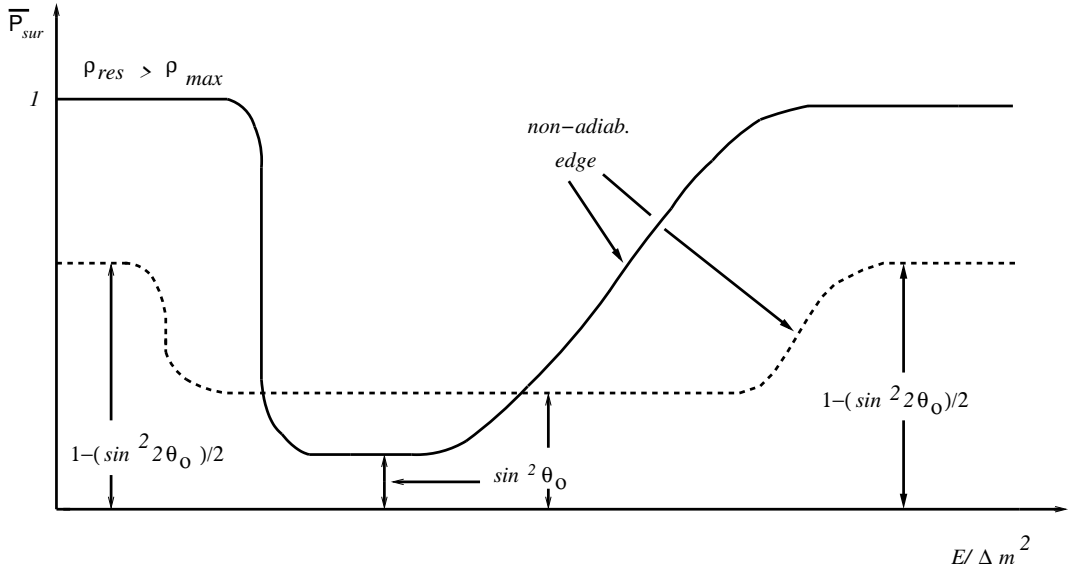
MSW effect and solar neutrinos

The survival probability for solar ν_e:



MSW effect and solar neutrinos

The survival probability for solar ν_e:



Day-night effect: the probability of finding a solar ν_e after it traverses the Earth

$$P_{SE} = \bar{P}_S + \frac{1 - 2\bar{P}_S}{\cos 2\theta_0} \left(P_{2e} - \sin^2 \theta_0 \right) .$$

Here: $P_{2e} = P(\nu_2 \to \nu_e)$ – probability of oscillations of the second mass eigenstate into electron neutrino inside the Earth.

How is it obtained?

Neutrino state at the surface of the Sun:

$$|\nu_\odot\rangle \;=\; a_1|\nu_1\rangle \;+\; a_2\, e^{i\phi_S}\,|\nu_2\rangle \qquad (a_{1,2} \;-\; \text{real})$$

Averaged ν_e survival probability in the Sun:

$$\overline{P}_S = \overline{|\langle\nu_e|\nu_\odot\rangle|^2} = a_1^2\cos^2\theta + a_2^2\sin^2\theta \quad \Rightarrow$$

$$a_2^2 \;=\; 1 - a_1^2 \;=\; \frac{\cos^2\theta - \overline{P}_S}{\cos 2\theta}$$

Solar neutrinos arrive at the Earth as an incoherent sum of ν_1 and ν_2 $\Rightarrow$

$$P_{SE} = a_1^2 P_{1e} + a_2^2 P_{2e} = a_1^2(1 - P_{2e}) + a_2^2 P_{2e} = \bar{P}_S + \frac{1 - 2\bar{P}_S}{\cos 2\theta}\,(P_{2e} - \sin^2\theta)\,.$$

In vacuum $P_{2e} = \sin^2\theta \quad \Rightarrow \quad P_{SE} = \overline{P}_S$.

How is it obtained?

For matter of constant density:

$$\diamondsuit \quad P_{2e} - \sin^2\theta \;=\; \frac{V\delta}{4\omega^2}\sin^2 2\theta\,\sin^2\,(\omega L)$$

Here:

$$\delta \;\equiv\; \frac{\Delta m^2_{21}}{2E}\,, \qquad \theta \;=\; \theta_{12}\,. \qquad \omega = \sqrt{(\cos 2\theta\cdot\delta - V)^2 + \delta^2\sin^2 2\theta^2}$$

Pre-sine^2 factor in $P_{2e} - \sin^2\theta$ reaches its max. at $V = \delta$ (not at $V = \delta\cdot\cos 2\theta$ which would correspond to the MSW resonance!)

$$(P_{2e} - \sin^2\theta)_{max.\ ampl.} \;=\; \cos^2\theta\sin^2(\sin\theta\cdot\delta\cdot L)$$

In the (realistic) case $V \ll \delta$:

$$\diamondsuit \quad P_{2e} - \sin^2\theta \;=\; \frac{V}{\delta}\sin^2 2\theta\sin^2\left(\frac{1}{2}\delta\cdot L\right)$$

3f oscillations in matter

3f neutrino oscillations in matter

Evolution equation:

$$i\frac{d}{dt}\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix}=\left[U\begin{pmatrix}E_1 & 0 & 0\\ 0 & E_2 & 0\\ 0 & 0 & E_3\end{pmatrix}U^\dagger+\begin{pmatrix}V(t) & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}\right]\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix}$$

$$E_i=\sqrt{p^2+m_i^2}\simeq p+\frac{m_i^2}{2p}\,; \qquad t\simeq r$$

$$V(t)=[V(\nu_e)]_{CC}=\sqrt{2}G_F N_e(t)$$

$[V(\nu_e)]_{NC}=[V(\nu_\mu)]_{NC}=[V(\nu_\tau)]_{NC}$ – do not contribute

(Modulo tiny radiative corrections)

Evolution in the rotated basis

Evolution matrix $S(t,\, t_0)$: $\nu(t) \;=\; S(t,\, t_0)\, \nu(t_0)$. Satisfies

$$\diamondsuit \quad i\frac{d}{dt} S(t,\, t_0) \;=\; H_{\rm fl}\, S(t,\, t_0) \qquad \text{with} \quad S(t_0,\, t_0) \;=\; \mathbb{1}\,.$$

$$H_{\rm fl} \;=\; (O_{23}\, \Gamma_\delta\, O_{13}\, \Gamma_\delta^\dagger\, O_{12}\,)\, {\rm diag}(0,\, \delta,\, \Delta)\; (O_{12}^T\, \Gamma_\delta O_{13}^T\, \Gamma_\delta^\dagger\, O_{23}^T) \;+\; {\rm diag}(V(t),\, 0,\, 0)$$

$$=\; (O_{23}\, \Gamma_\delta\, O_{13}\;\, O_{12}\,)\, {\rm diag}(0,\, \delta,\, \Delta)\, (O_{12}^T\, O_{13}^T\, \Gamma_\delta^\dagger\, O_{23}^T) \;+\; {\rm diag}(V(t),\, 0,\, 0)$$

where

$$\delta \;\equiv\; \frac{\Delta m_{21}^2}{2E}\,, \qquad \Delta \;\equiv\; \frac{\Delta m_{31}^2}{2E}$$

Oscillation probabilities:

$$P_{\alpha\beta} \;=\; |S_{\beta\alpha}|^2$$

Define

$$O'_{23} = O_{23}\, \Gamma_\delta$$

The matrix ${\rm diag}(V(t), 0, 0)$ commutes with O'_{23} $\Rightarrow$ go to the rotated basis

Evolution in the rotated basis – contd.

$$\nu \;=\; O'_{23}\,\nu'\,, \qquad \text{or} \qquad S(t,\,t_0) \;=\; O'_{23}\,S'(t,\,t_0)\,{O'_{23}}^{\dagger}\,,$$

In the rotated basis $H' \;=\; O'_{23}\,H_{\rm fl}\,{O'_{23}}^{\dagger}$. Explicitly:

$$H'(t) = \begin{pmatrix} s_{12}^2 c_{13}^2 \delta + s_{13}^2 \Delta + V(t) & s_{12} c_{12} c_{13} \delta & s_{13} c_{13} \left(\Delta - s_{12}^2 \delta\right) \\ s_{12} c_{12} c_{13} \delta & c_{12}^2 \delta & -s_{12} c_{12} s_{13} \delta \\ s_{13} c_{13} \left(\Delta - s_{12}^2 \delta\right) & -s_{12} c_{12} s_{13} \delta & c_{13}^2 \Delta + s_{12}^2 s_{13}^2 \delta \end{pmatrix}$$

Dependence on θ_{23} and $\delta_{\rm CP}$ can be obtained in the general case by rotating back to the original flavour basis. Also: easy to apply PT approximations

- If $\frac{\Delta m_{21}^2}{2E} L \ll 1$ – neglect $\delta \;=\; \frac{\Delta m_{21}^2}{2E}$
- As θ_{13} is relatively small – neglect s_{13}

or use expansion in these small parameters

General dependence on $\delta_{\rm CP}$

Another use of essentially the same symmetry: rotate by

$$O'_{23} = O_{23} \times \mathrm{diag}(1, 1, e^{i\delta_{\rm CP}})$$

From commutativity of $\mathrm{diag}(V(t), 0, 0)$ with O'_{23} $\Rightarrow$
General dependence of probabilities on $\delta_{\rm CP}$:

$$P_{e\mu} = A_{e\mu} \cos\delta_{\rm CP} + B_{e\mu} \sin\delta_{\rm CP} + C_{e\mu}$$

$$P_{\mu\tau} = A_{\mu\tau} \cos\delta_{\rm CP} + B_{\mu\tau} \sin\delta_{\rm CP} + C_{\mu\tau}$$

$$+ D_{\mu\tau} \cos 2\delta_{\rm CP} + E_{\mu\tau} \sin 2\delta_{\rm CP}$$

General structure of T-odd probability diff.

$$\Delta P^T_{e\mu} \;=\; \underbrace{\sin\delta_{\rm CP}\cdot Y}_{\text{fundam. } \not{T}} \;+\; \underbrace{\cos\delta_{\rm CP}\cdot X}_{\text{matter}-\text{ind. } \not{T}}$$

In adiabatic approximation: $X = J_{\rm eff}\cdot$ (oscillating terms) ,

$$\diamondsuit \quad J_{\rm eff} \;=\; s_{12}\,c_{12}\,s_{13}\,c_{13}^2\,s_{23}\,c_{23}\,\frac{\sin(2\theta_1-2\theta_2)}{\sin 2\theta_{12}}$$

Compare with the vacuum Jarlskog invariant:

$$J \;=\; s_{12}\,c_{12}\,s_{13}\,c_{13}^2\,s_{23}\,c_{23}\,\sin\delta_{\rm CP}$$

$$\Rightarrow \qquad \sin\delta_{\rm CP} \;\Leftrightarrow\; \frac{\sin(2\theta_1-2\theta_2)}{\sin 2\theta_{12}}$$

Matter-induced $\mathcal{T}$:

- ◊ Negligible effects in terrestrial experiments
- ◊ Cannot be observed in supernova ν oscillations due to experimental indistinguishability of low-energy ν_μ and ν_τ
- ◊ Can affect the signal from $\sim$GeV neutrinos produced in annihilations of WIMPs inside the Sun

Backup

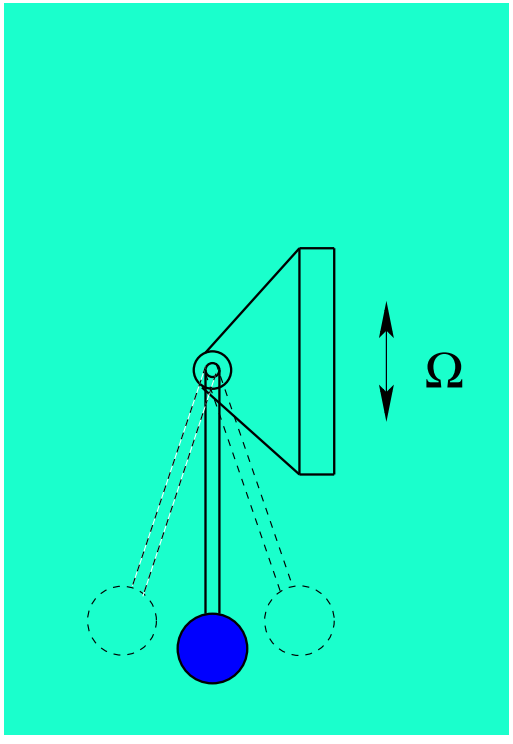
Another possible matter effect

Parametric resonance in neutrino oscillations

Parametric resonance in oscillating systems with varying parameters: occurs when the rate of the parameter change is correlated in a certain way with the values of the parameters themselves

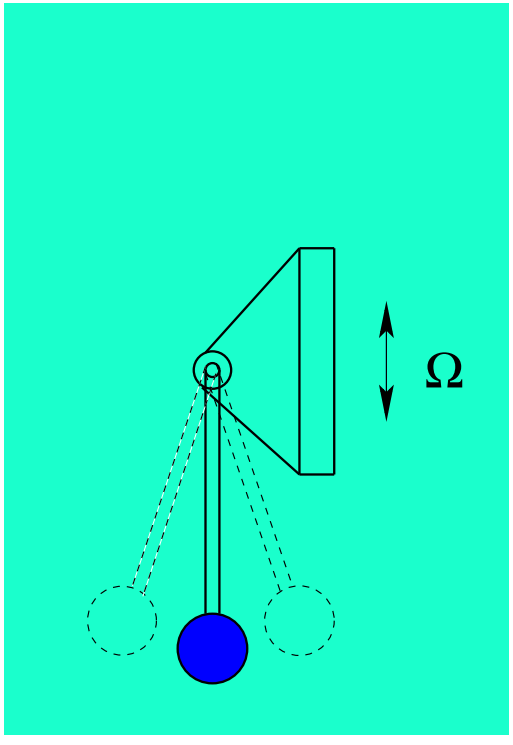
Parametric resonance in neutrino oscillations

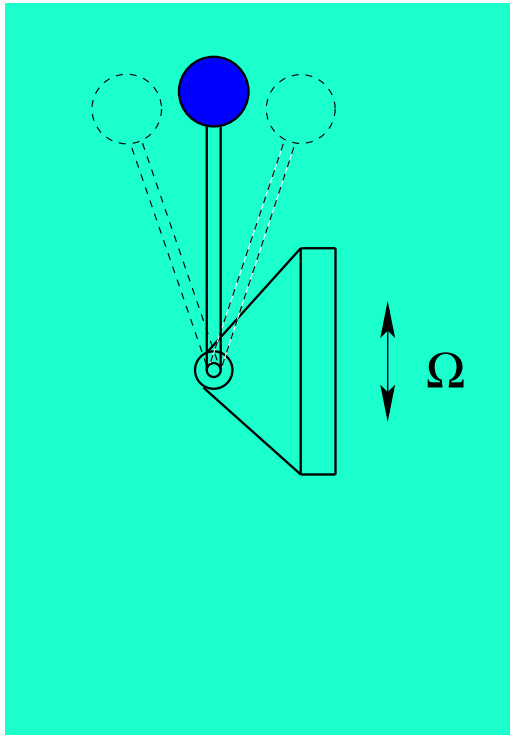
Parametric resonance in oscillating systems with varying parameters: occurs when the rate of the parameter change is correlated in a certain way with the values of the parameters themselves



Parametric resonance in neutrino oscillations

Parametric resonance in oscillating systems with varying parameters: occurs when the rate of the parameter change is correlated in a certain way with the values of the parameters themselves





Parametric resonance in neutrino oscillations

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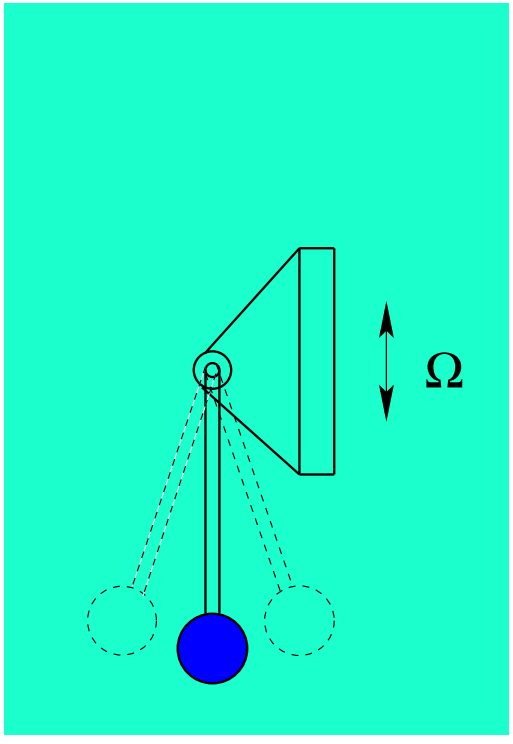
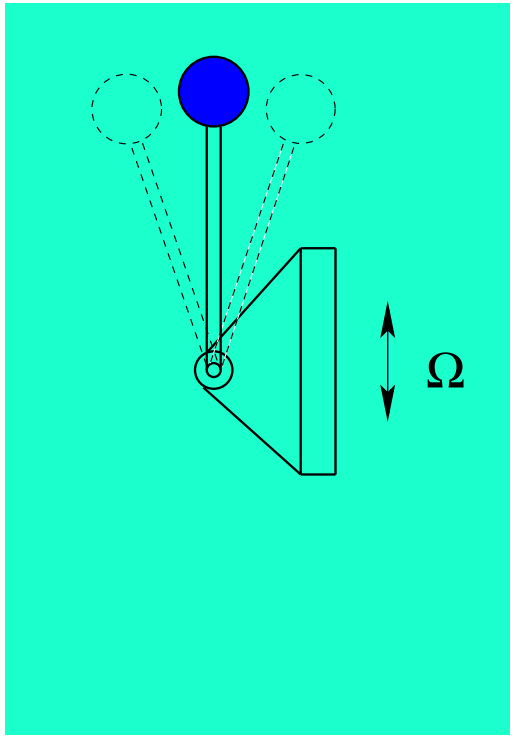




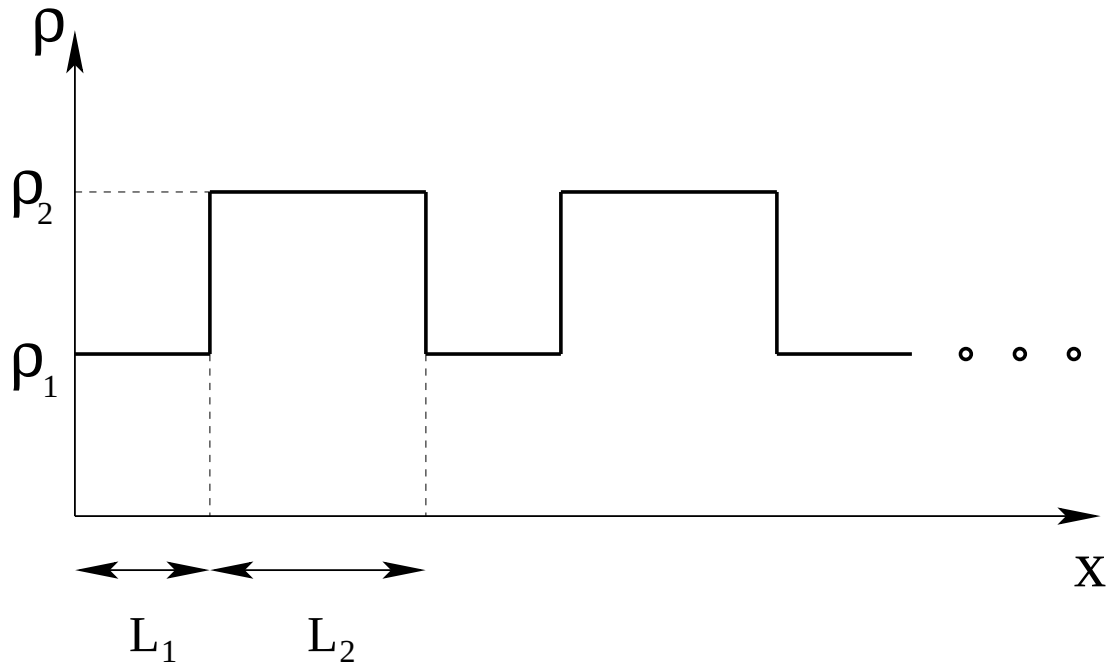
For small-ampl. osc.:

$$\Omega_{\rm res} = \frac{2\omega}{n}$$

$n = 1, 2, 3...$

Different from MSW eff. – no level crossing !

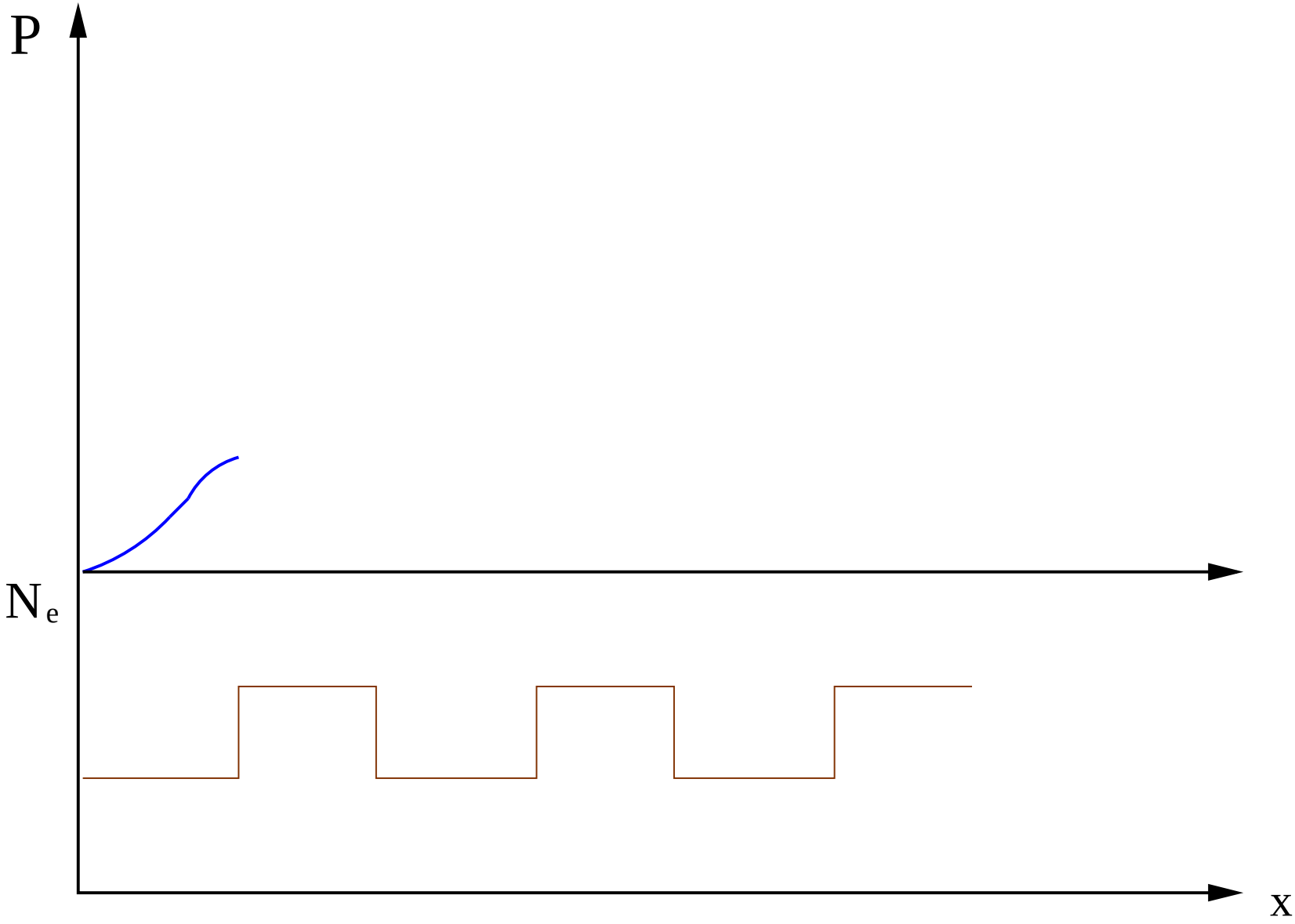
An example admitting an exact analytic solution – “castle wall” density profile (E.A., 1987, 1998):

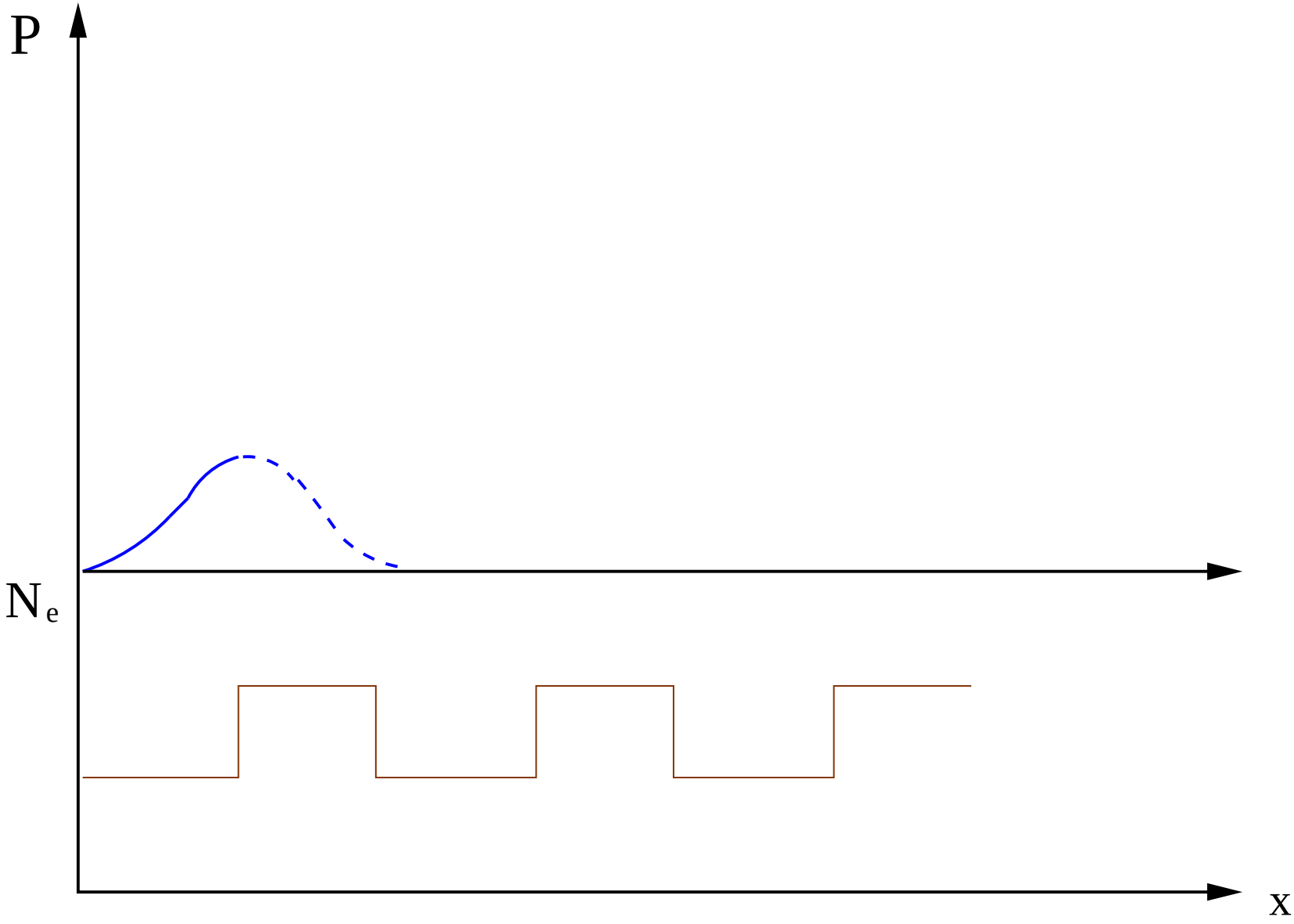


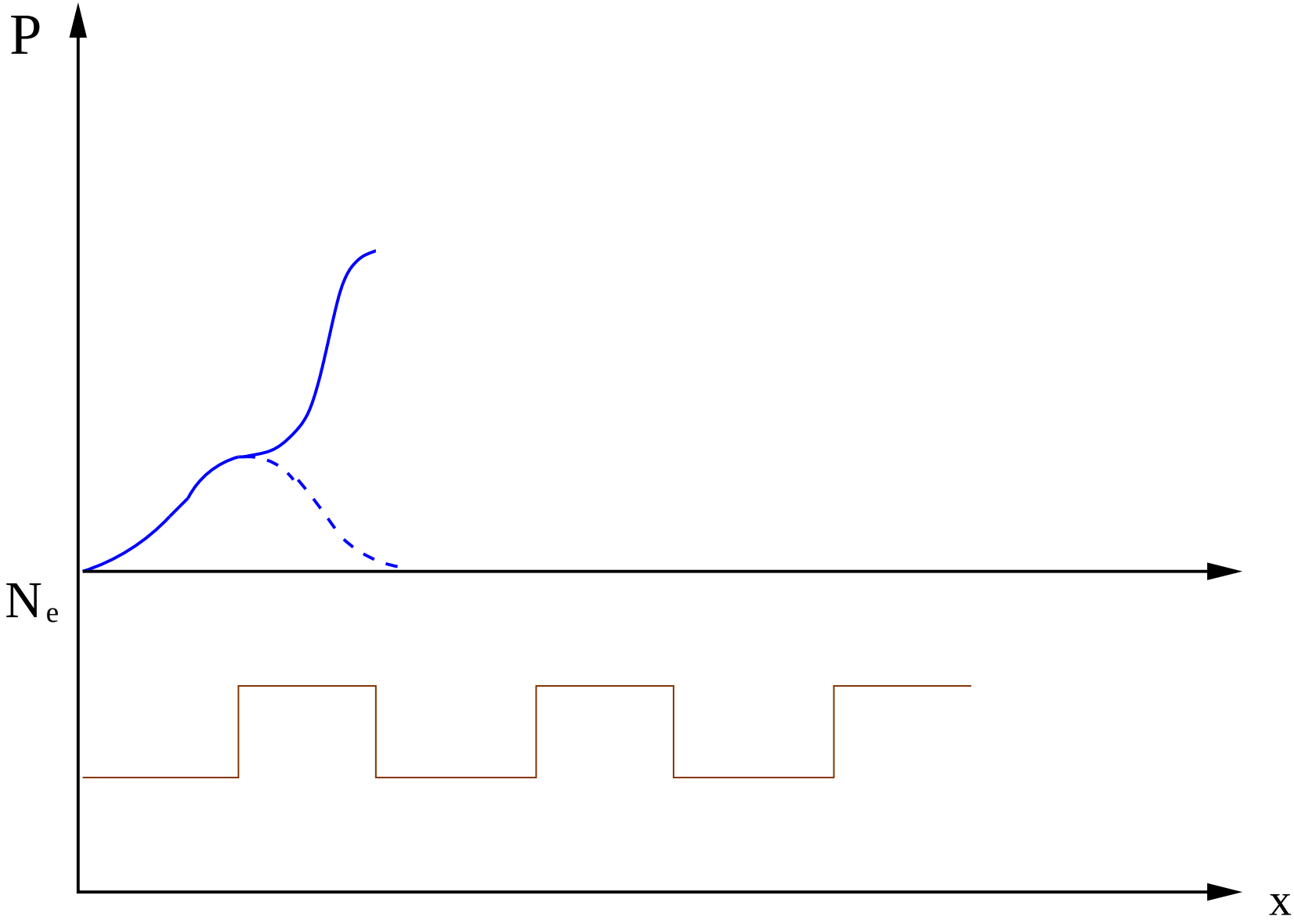
Resonance condition:

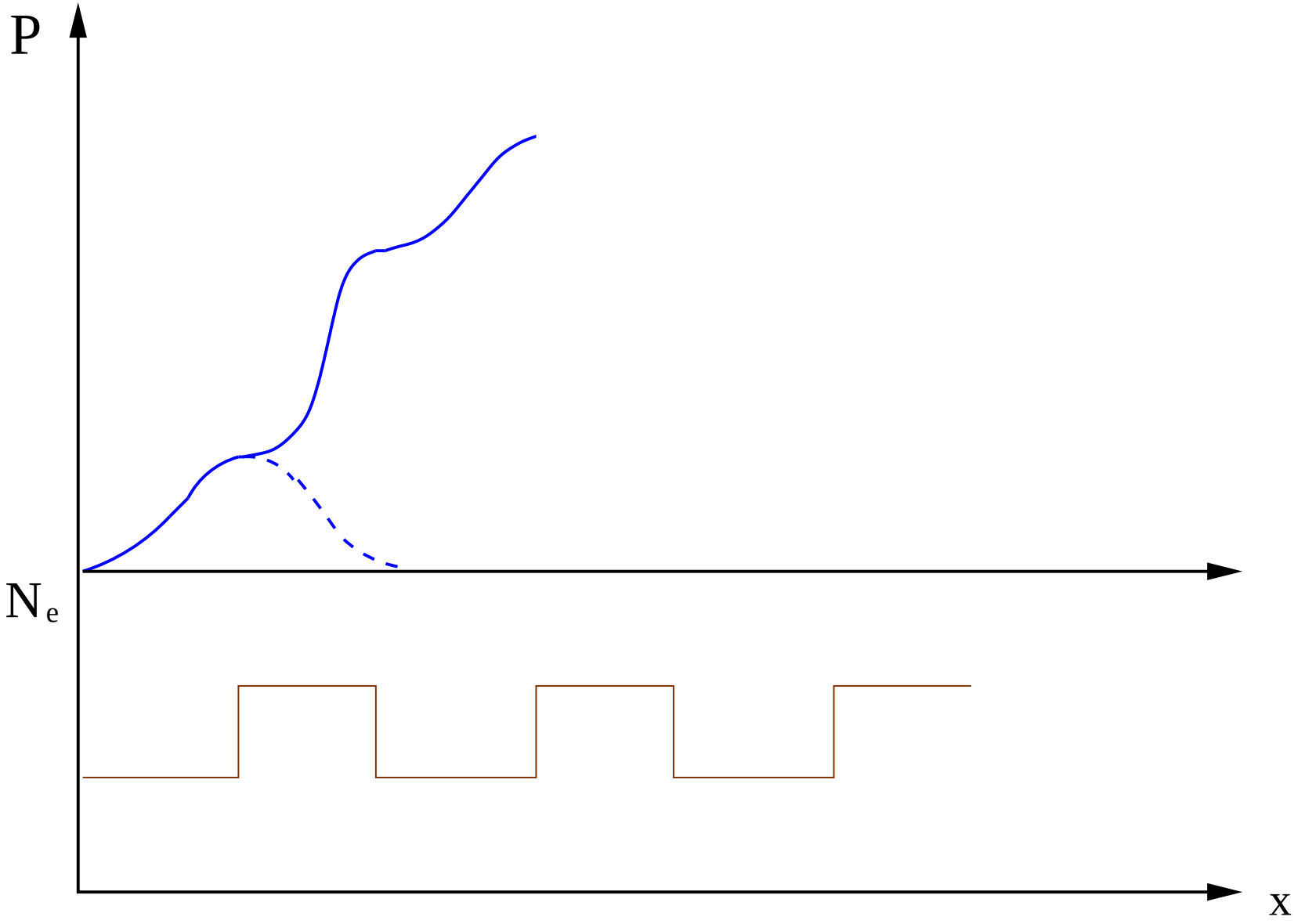
$$X_3 \equiv -(\sin\phi_1 \cos\phi_2 \cos 2\theta_{1m} + \cos\phi_1 \sin\phi_2 \cos 2\theta_{2m}) = 0$$

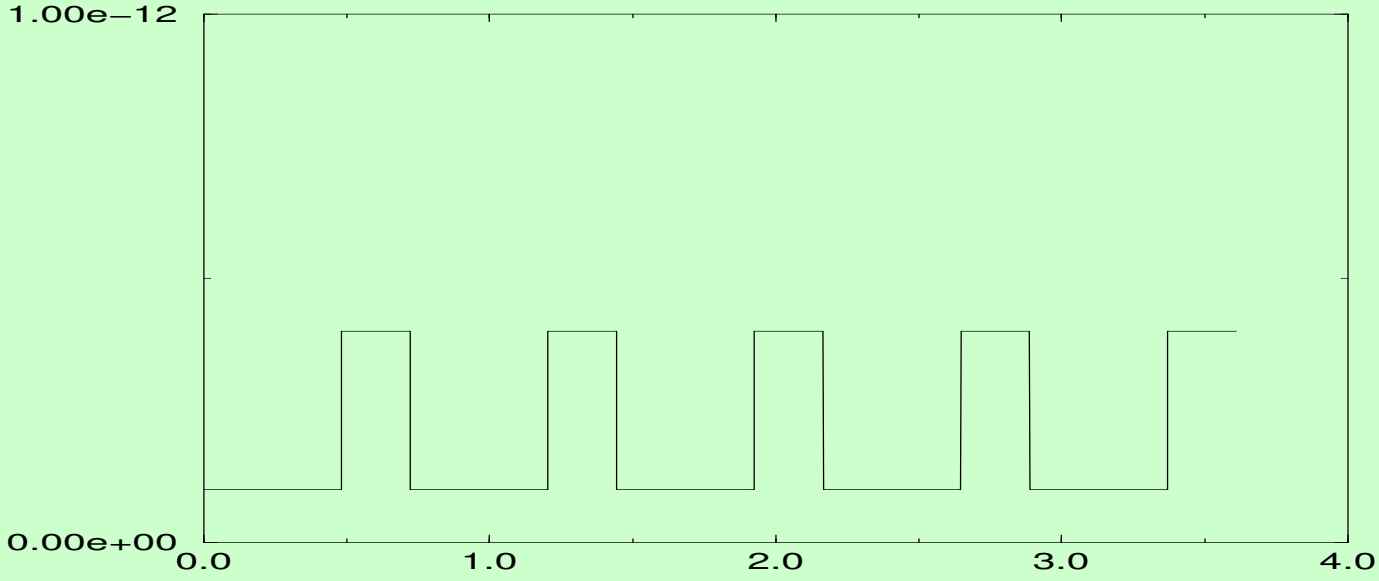
$\phi_{1,2}$ – oscillation phases acquired in layers 1, 2

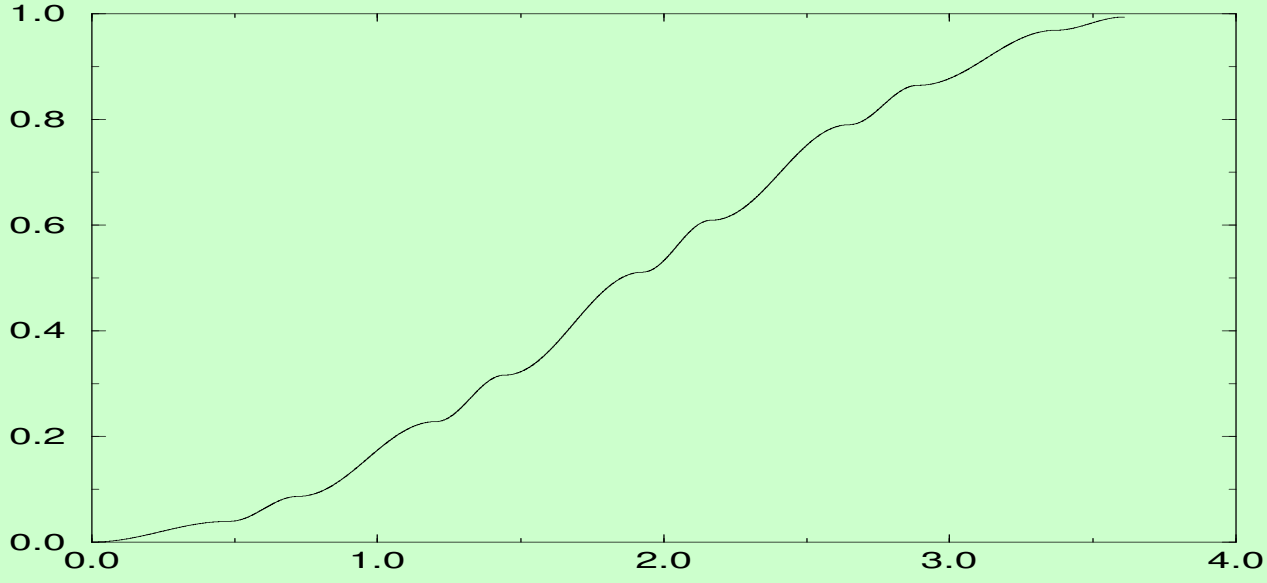

P
N_e
x


P
N_e
x

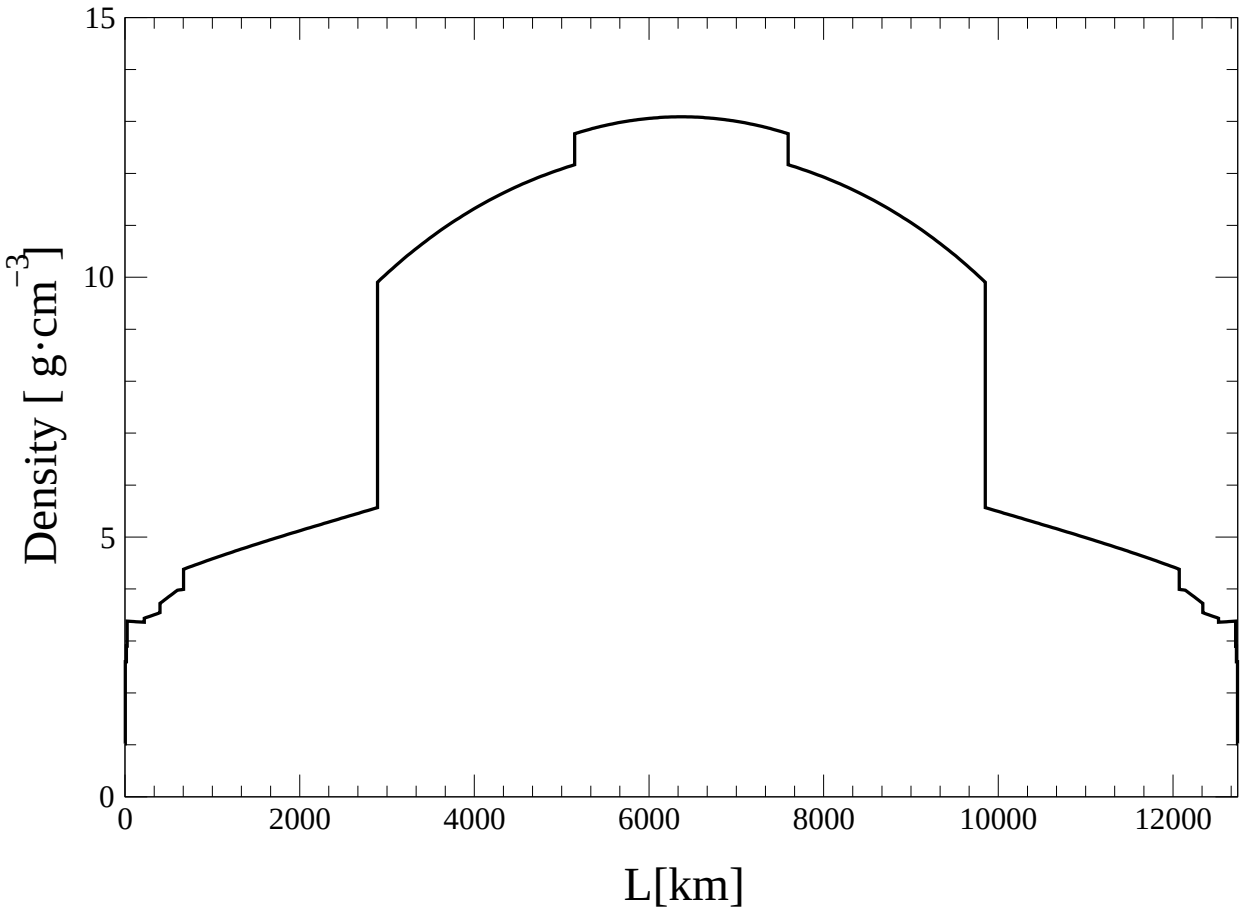

P
N_e
x


P
N_e
x

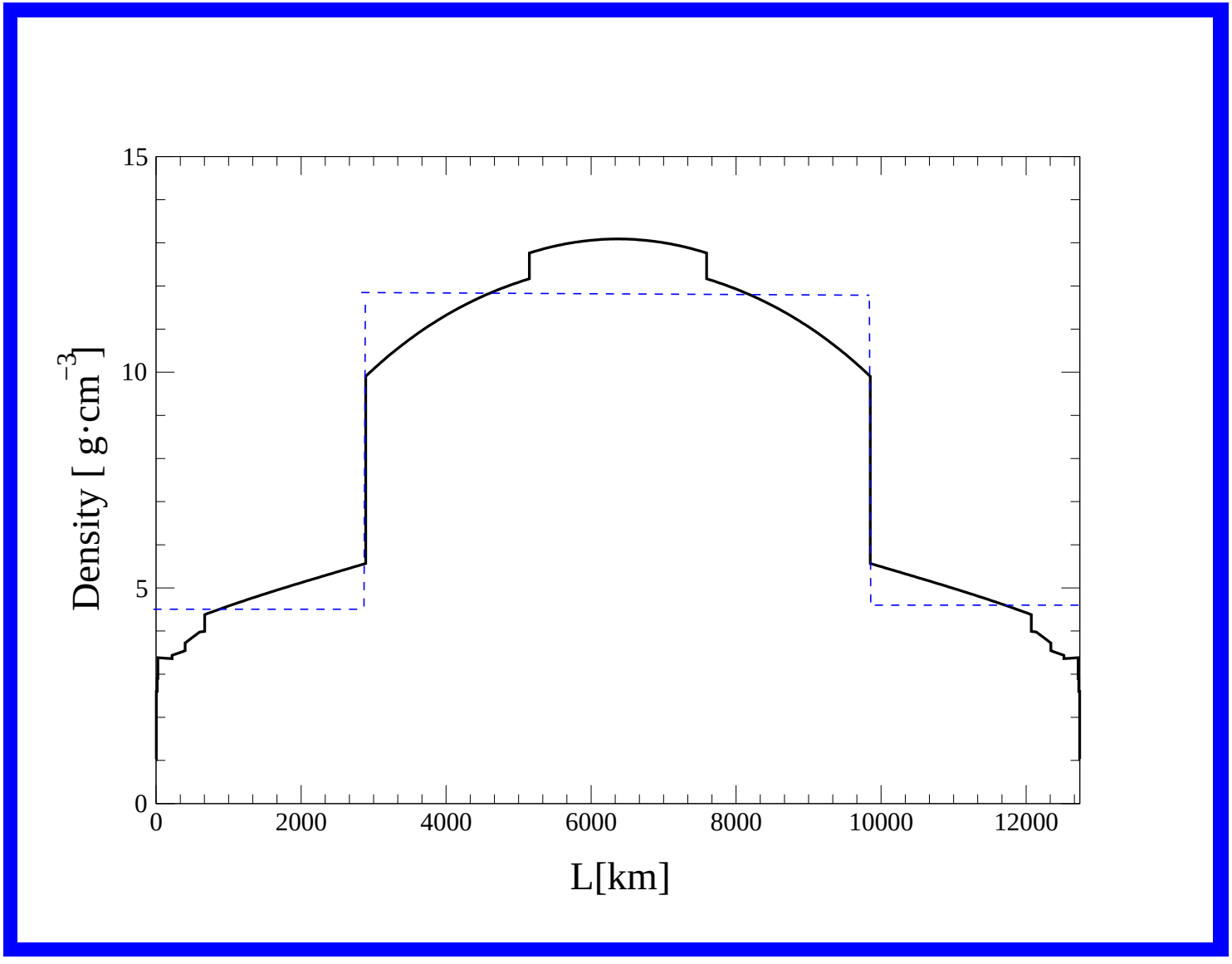

1.00e−12
0.00e+00
0.0
1.0
2.0
3.0
4.0


1.0
0.8
0.6
0.4
0.2
0.0
0.0
1.0
2.0
3.0
4.0

Earth's density profile (PREM model) :

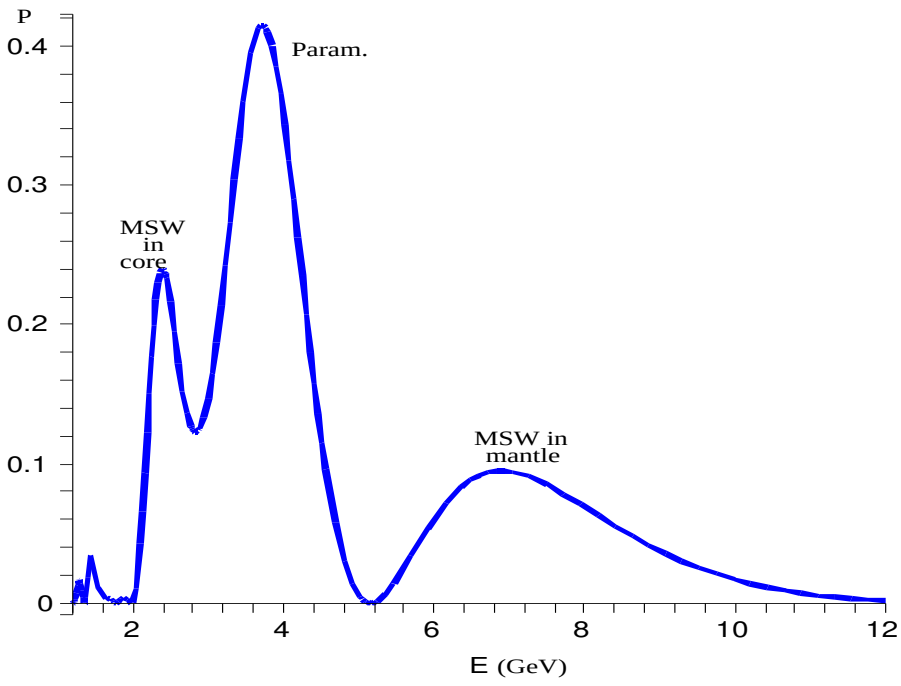


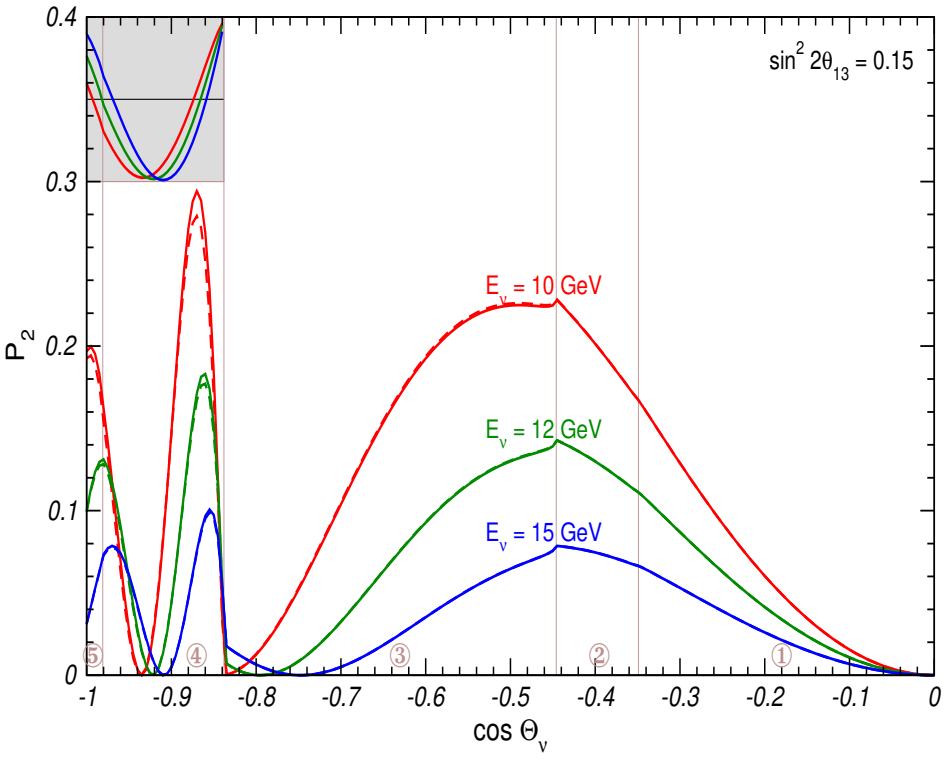
Earth's density profile (PREM model) :



Param. res. condition: $(l_{\rm osc})_{\rm matt} \simeq l_{\rm density\ mod.}$

Fulfilled for $\nu_e \leftrightarrow \nu_{\mu.\tau}$ oscillations of core-crossing ν's in the Earth for a wide range of energies and zenith angles !





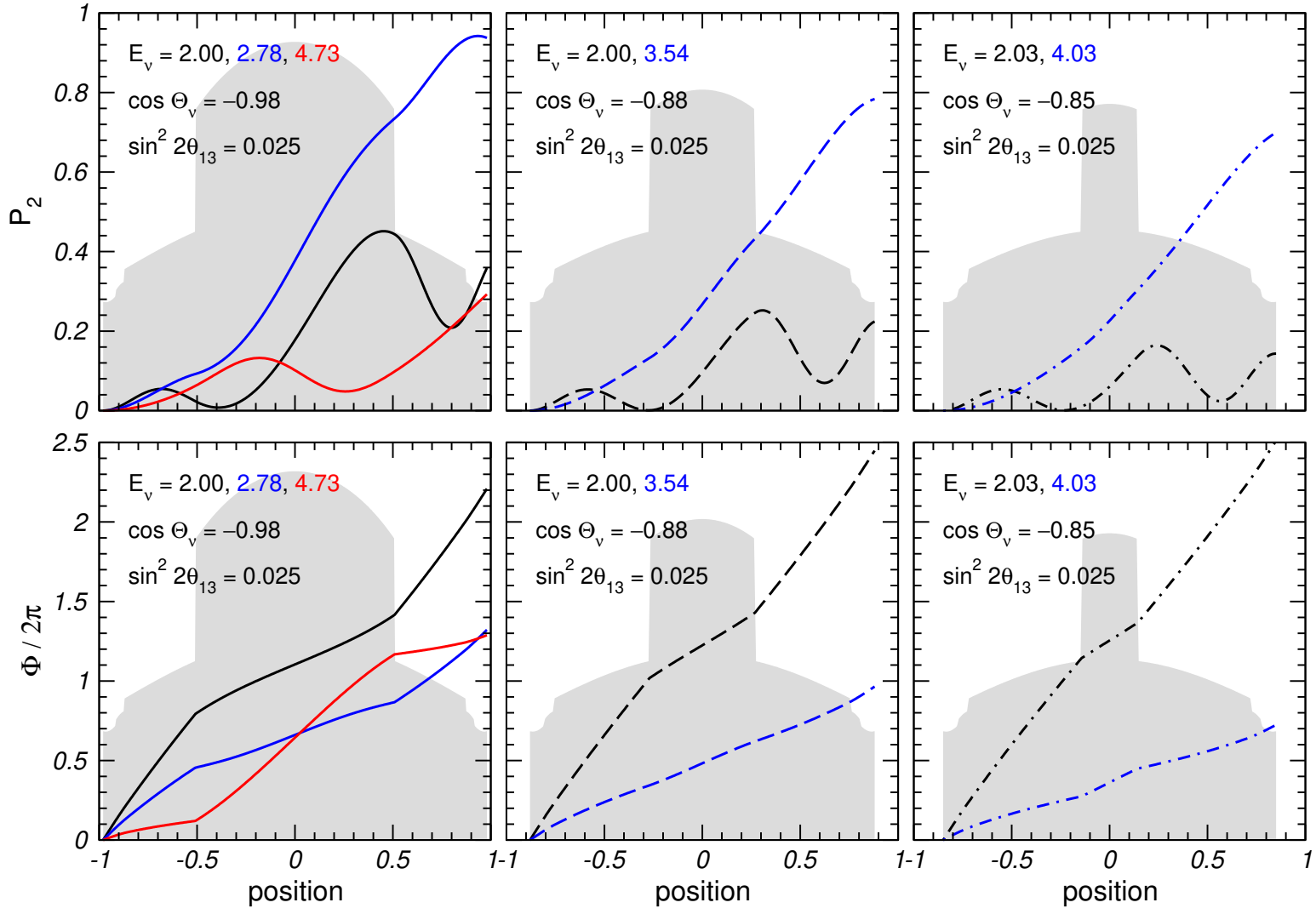
Intermed. energies

$\cos\Theta = -0.89$ $\sin^2 2\theta_{13} = 0.01$

(Liu, Smirnov, 1998; Petcov, 1998; E.A. 1998)

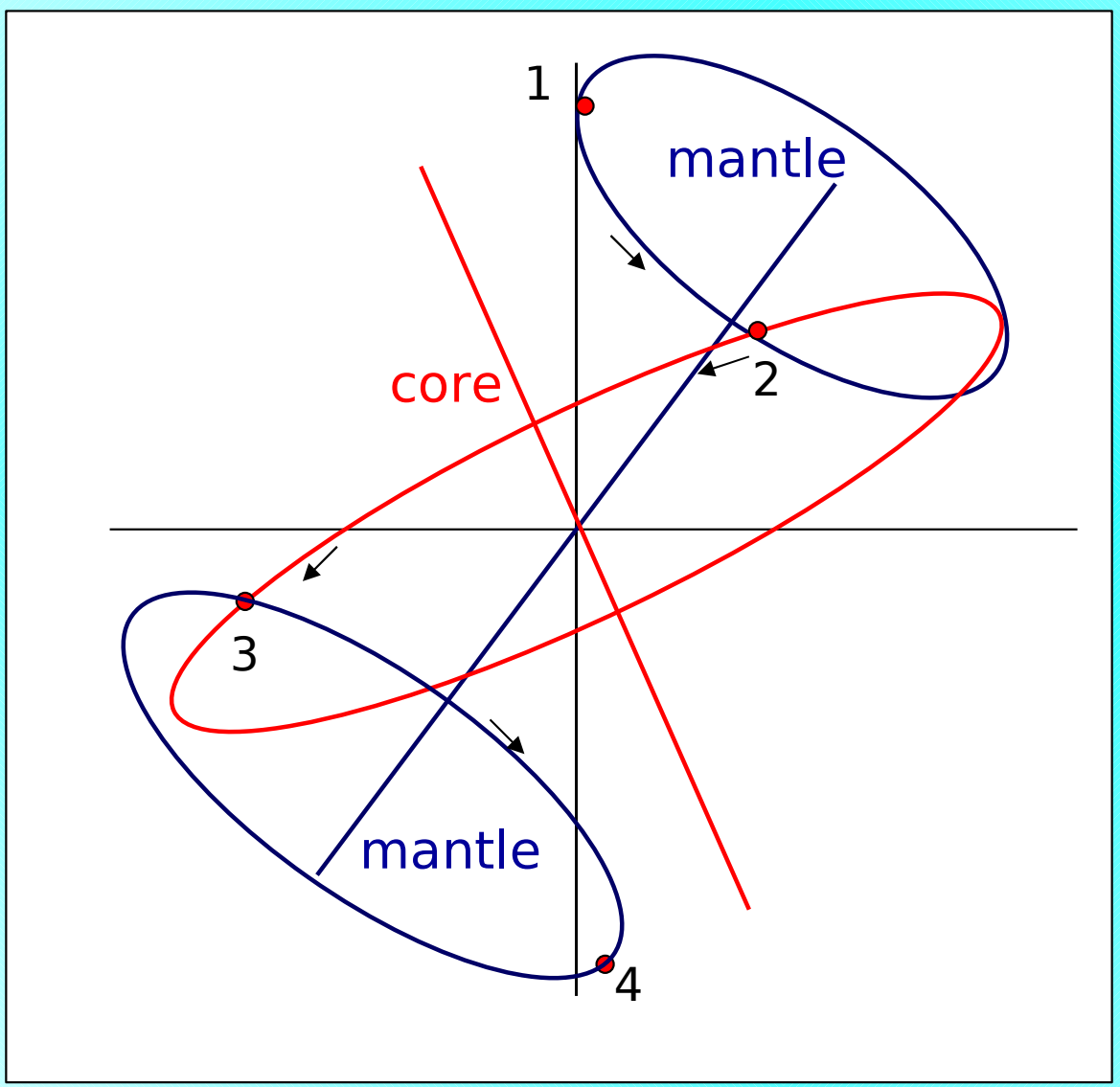
High energies, $\cos\Theta$ - dependence

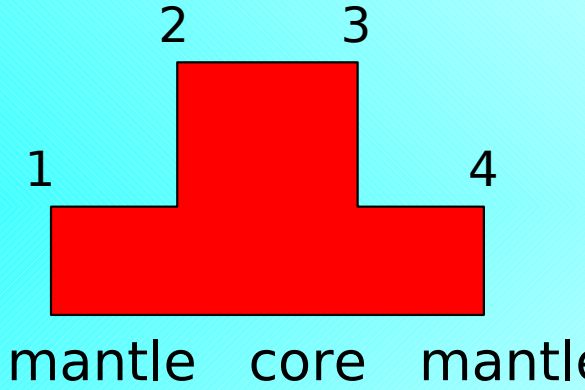
(E.A., Maltoni & Smirnov, 2005)



◇ Parametric resonance of ν oscillations in the Earth: can be observed in future atmospheric or accelerator experiments if θ_{13} is not much below its current upper limit

Parametric enhancement in the Earth





Neutrino oscillations in the Earth

A coherent description in terms of different realizations of just 2 conditions – amplitude and phase conditions

Matter with $N_e = const$:

$$\diamondsuit \quad P_{\rm tr} = \sin^2 2\theta_m \sin^2 \phi_m$$

- amplitude condition = MSW resonance condition $(\theta_m = 45^\circ)$
- phase condition: $\phi_m = \pi/2 + \pi n$

Neutrino oscillations in the Earth

"Castle wall" density profile:

$$\diamondsuit \quad P_{\rm tr}^{(n)} = \frac{X_1^2 + X_2^2}{X_1^2 + X_2^2 + X_3^2} \sin^2 n\Phi$$

Evolution matrix: $\nu(t) = S(t, t_0)\,\nu(0)$. For 2 layers:

$$S^{(2)}(t, t_0) = \begin{pmatrix} Y - iX_3 & -i(X_1 - iX_2) \\ -i(X_1 + iX_2) & Y + iX_3 \end{pmatrix}, \qquad Y^2 + \mathbf{X}^2 = 1$$

- amplitude condition = parametric resonance condition ($X_3 = 0$)
- phase condition: $\Phi \equiv \arccos Y = \pi/2 + \pi n$

Neutrino oscillograms of the Earth

Contours of equal osc. probabilities in $(\Theta_\nu,\, E_\nu)$ plane

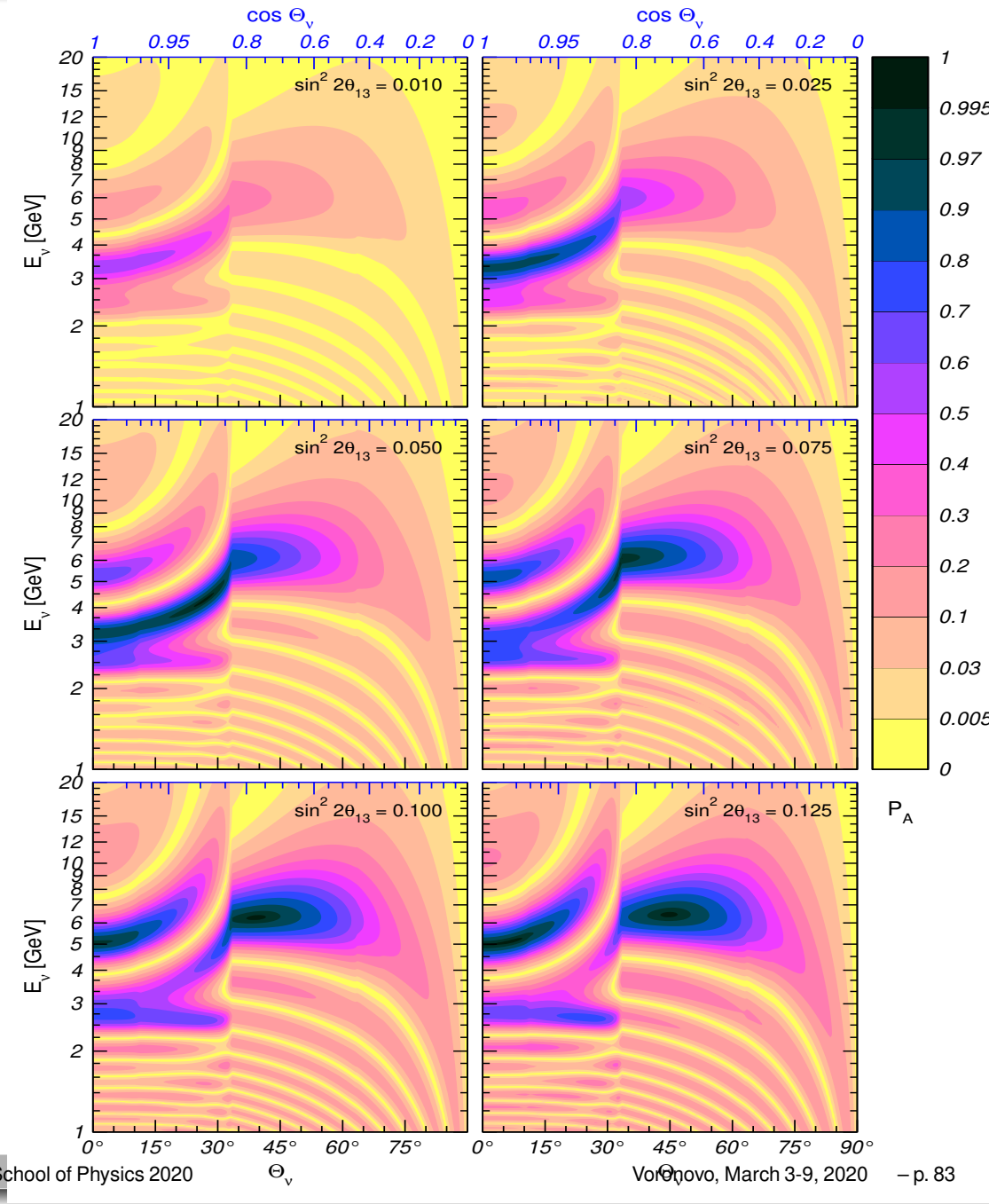
Θ_{13} - dependense of P_A $\Rightarrow$

P_A – effective 2f transition probability $(\Delta m^2_{\rm sol} \to 0)$

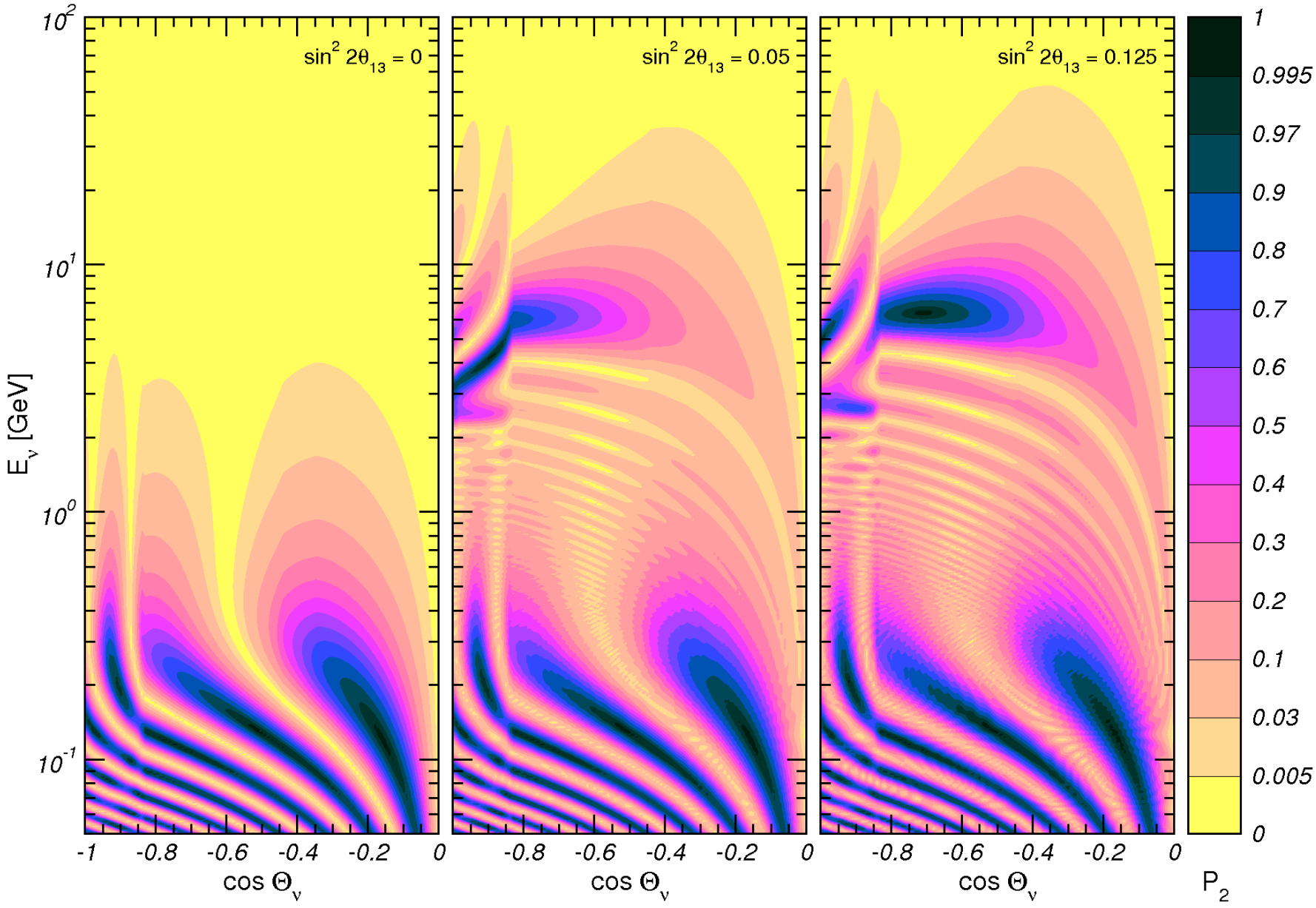
$$P_{e\mu} = s^2_{23} P_A$$

$$P_{e\tau} = c^2_{23} P_A$$

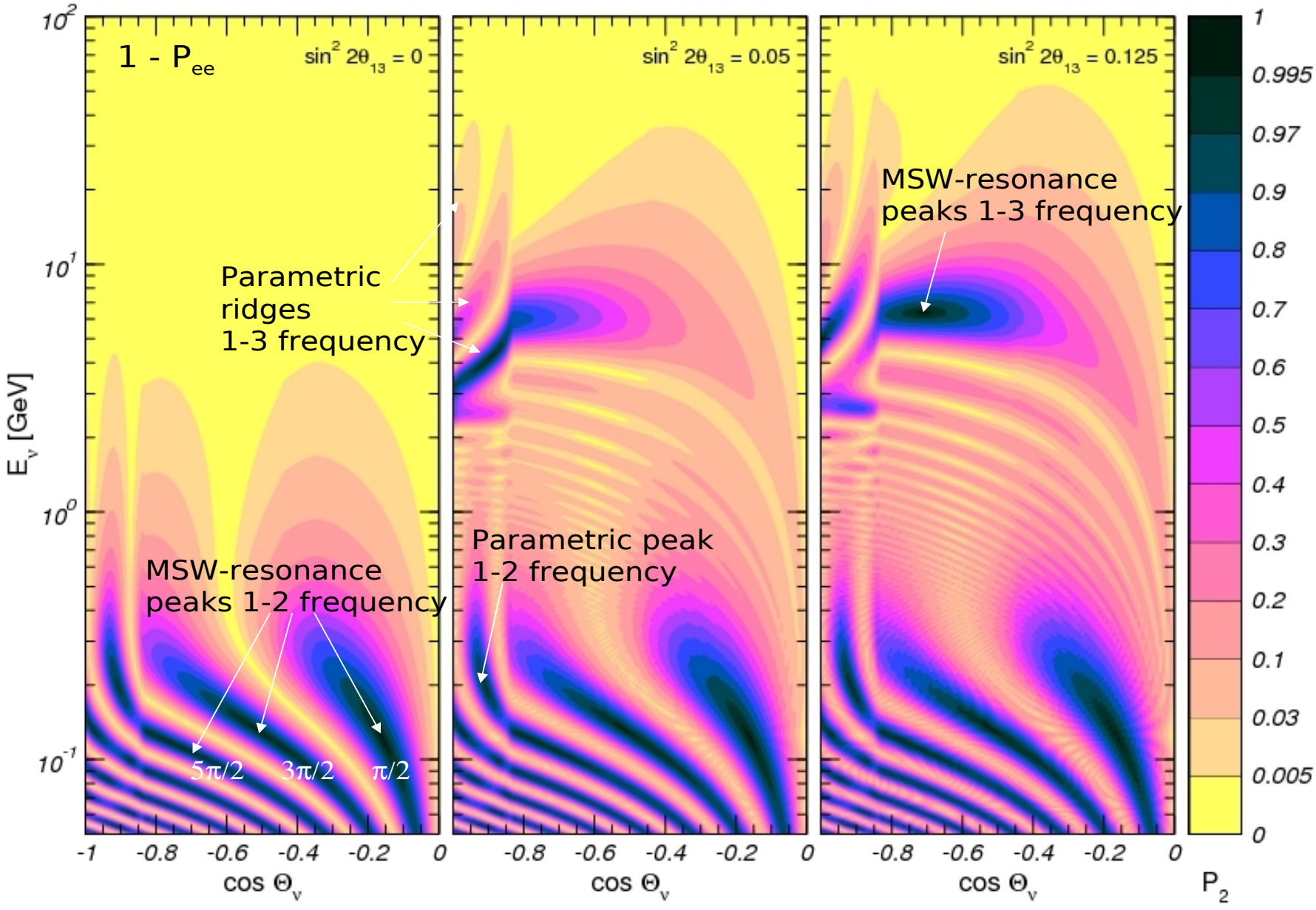
(E.A., Maltoni & Smirnov, 2006)



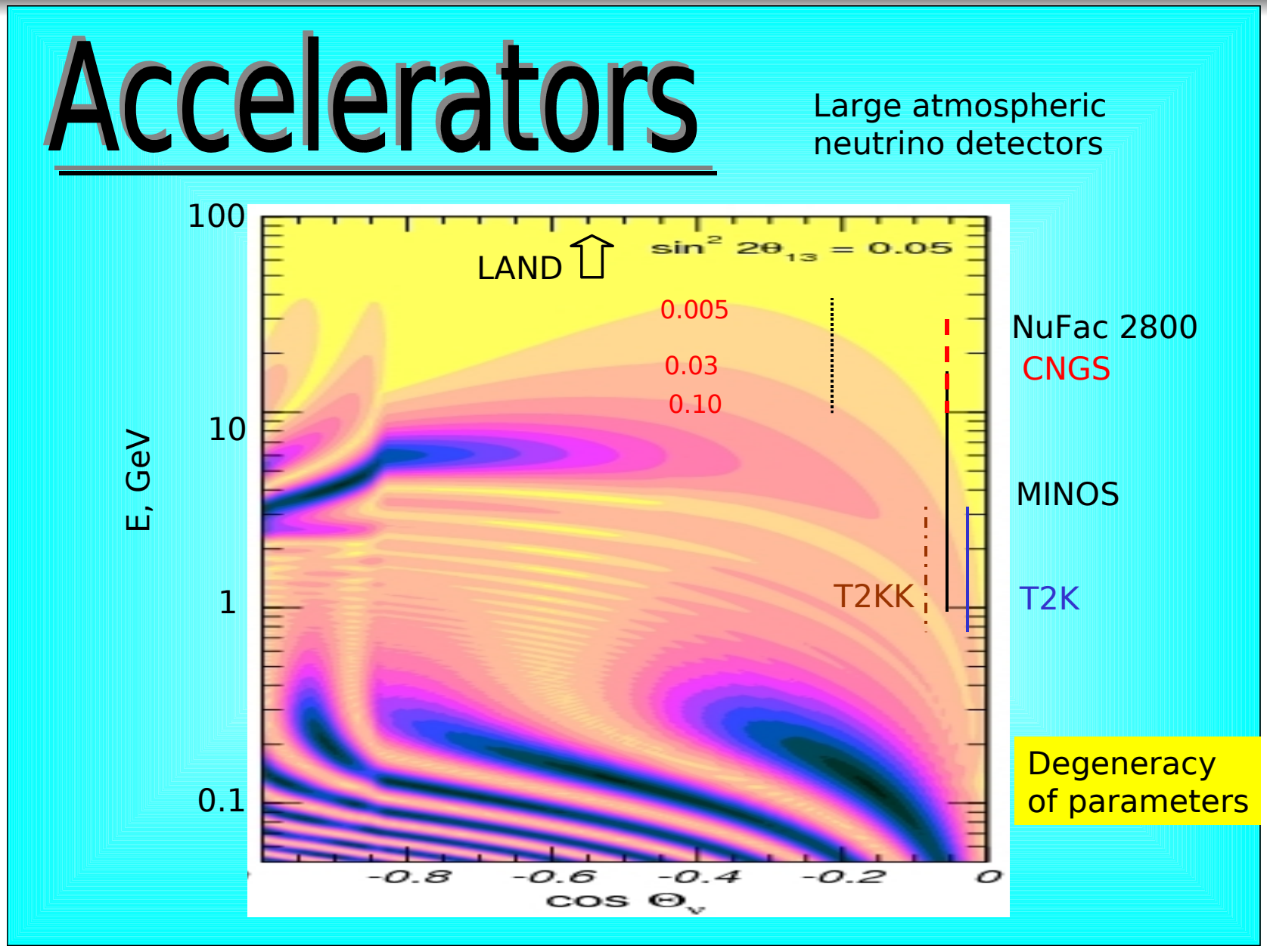
Including the effects of $\Delta m^2_{\rm sol}$: $(1 - P_{ee})$



Including the effects of $\Delta m^2_{\rm sol}$: $(1 - P_{ee})$



Producing the oscillograms



A. Smirnov, UCLA seminar

Is T reversal in matter equivalent to $\nu_a \leftrightarrow \nu_b$?

No explicit closed form solution in general.

Still, easy to answer !

T reversal: $t \rightleftarrows t_0 \quad \Leftrightarrow \quad S(t,\, t_0) \;\Rightarrow\; S(t_0,\, t)$

One has:

$$S(t_0,\, t) \;=\; S(t,\, t_0)^{-1} \;=\; S(t,\, t_0)^{\dagger} \;=\; [S(t,\, t_0)^T]^*$$

Therefore

$$|[S(t_0,\, t)]_{\alpha\beta}|^2 \;=\; |[S(t,\, t_0)]_{\beta\alpha}|^2$$

$\Rightarrow$ In matter with arbitrary density profile, as well as in vacuum, time reversal is equivalent to interchanging the initial and final neutrino flavours

To extract fundamental $\not{T}$ need to measure:

$$\Delta P_{\alpha\beta} \equiv P_{\rm dir}(\nu_\alpha \to \nu_\beta) - P_{\rm rev}(\nu_\beta \to \nu_\alpha) \propto \sin\delta_{\rm CP}$$

Even survival probabilities $P_{\alpha\alpha}$ $(\alpha = \mu, \tau)$ can be used!

$$P_{\rm dir}(\nu_\alpha \to \nu_\alpha) - P_{\rm rev}(\nu_\alpha \to \nu_\alpha) \sim \sin\delta_{\rm CP} \quad (\alpha \neq e)$$

In 3f case P_{ee} does not depend on $\delta_{\rm CP}$ – not true if $\nu_{\rm sterile}$ is present!

Matter-induced $\not{T}$ in LBL experiments due to imperfect sphericity of the Earth density distribution cannot spoil the determination of $\delta_{\rm CP}$ if the error in $\delta_{\rm CP}$ is $> 1\%$ at 99% C.L.

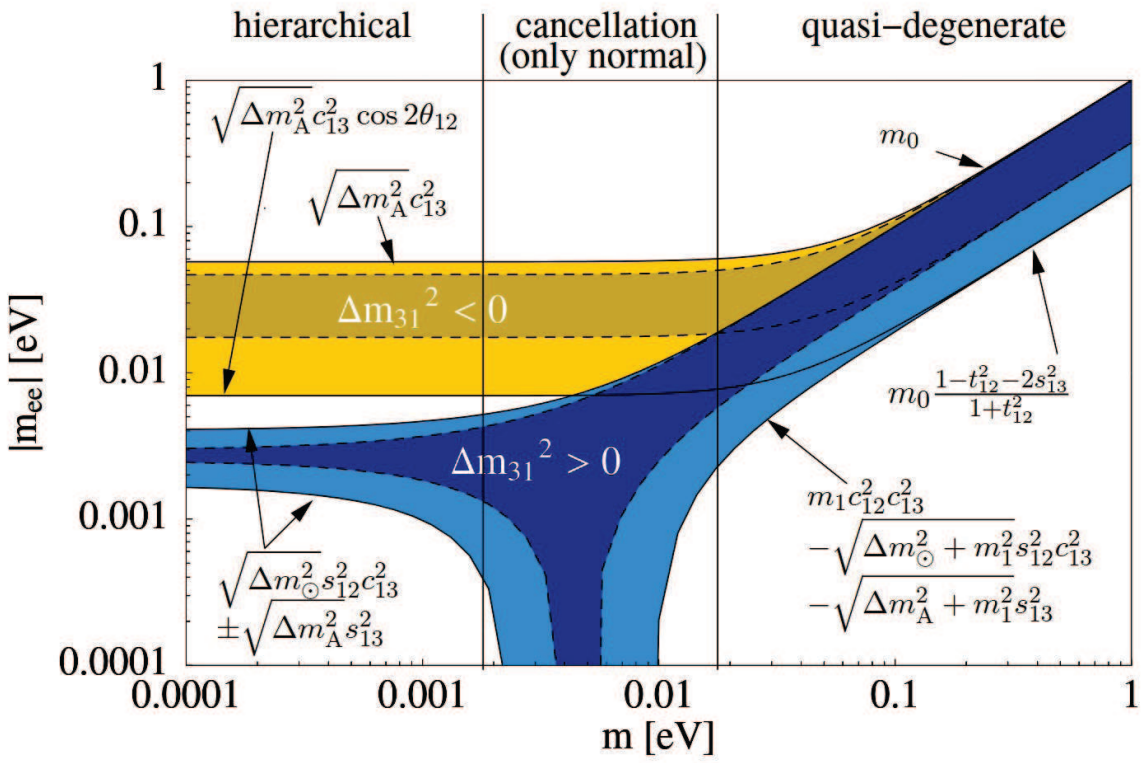
$\Rightarrow$ *No need to interchange positions of ν source and detector!*

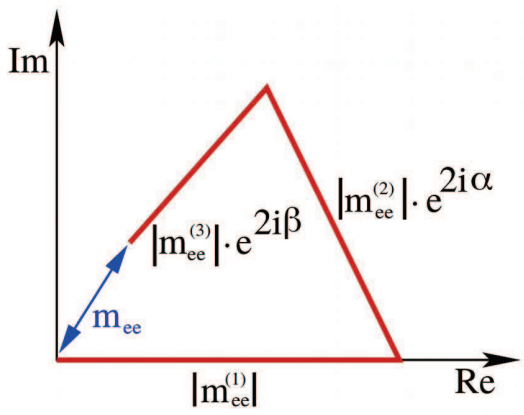
Experimental study of $\not{T}$ difficult because of problems with detection of $e^\pm$

Neutrino Oscillations & 0νββ

$$\langle m_{ee} \rangle = \left| c_{12}^2 c_{13}^2 \, m_1 + s_{12}^2 c_{13}^2 \, m_2 \, e^{2i\phi_{12}} + s_{13}^2 \, m_3 \, e^{2i\phi_{13}} \right|$$

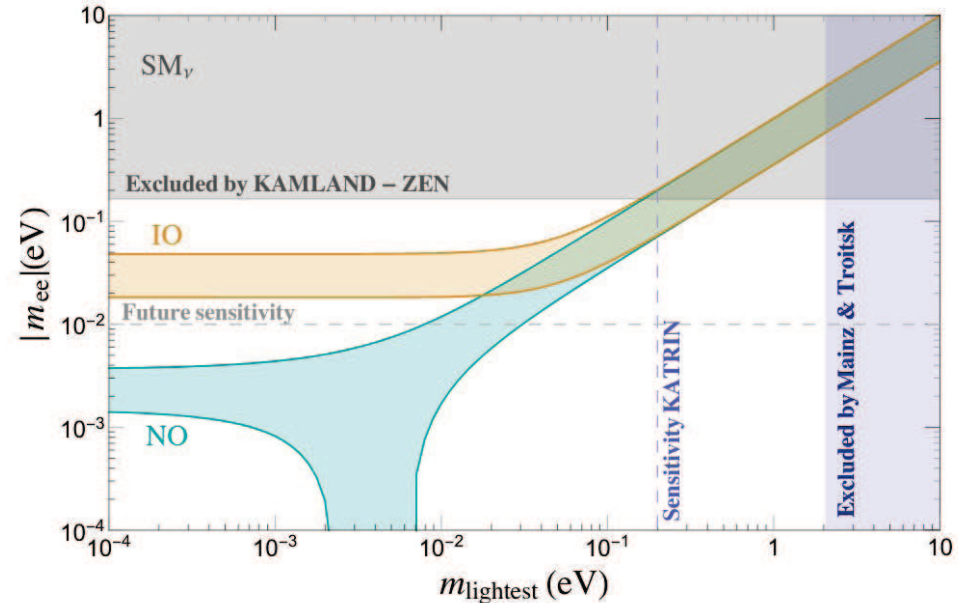
- Uncertainty from **unknown Majorana phase**
- **Quasi-degenerate** region above 0.2 eV
- Accidental **cancellation** for NO

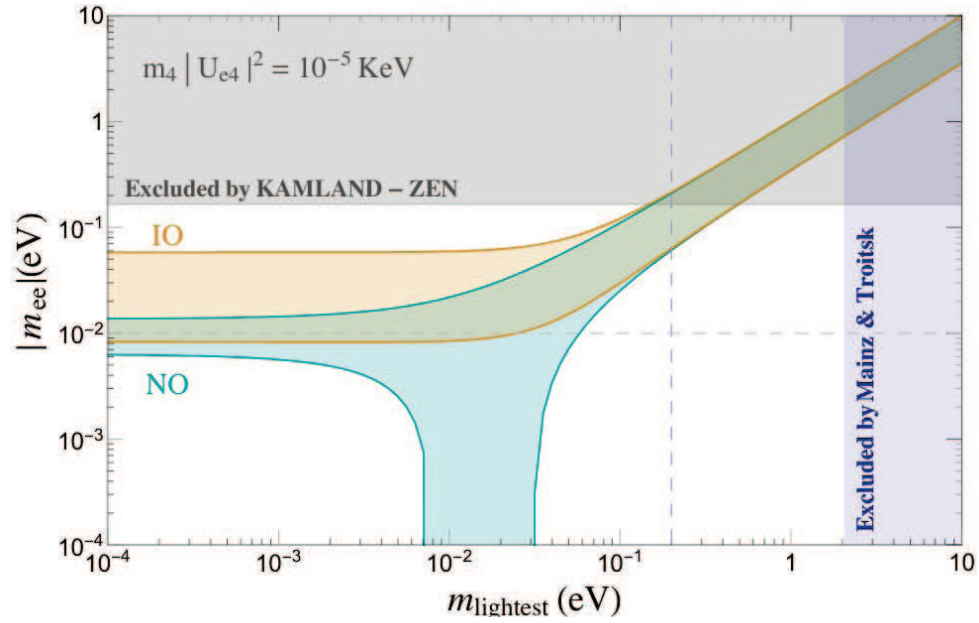


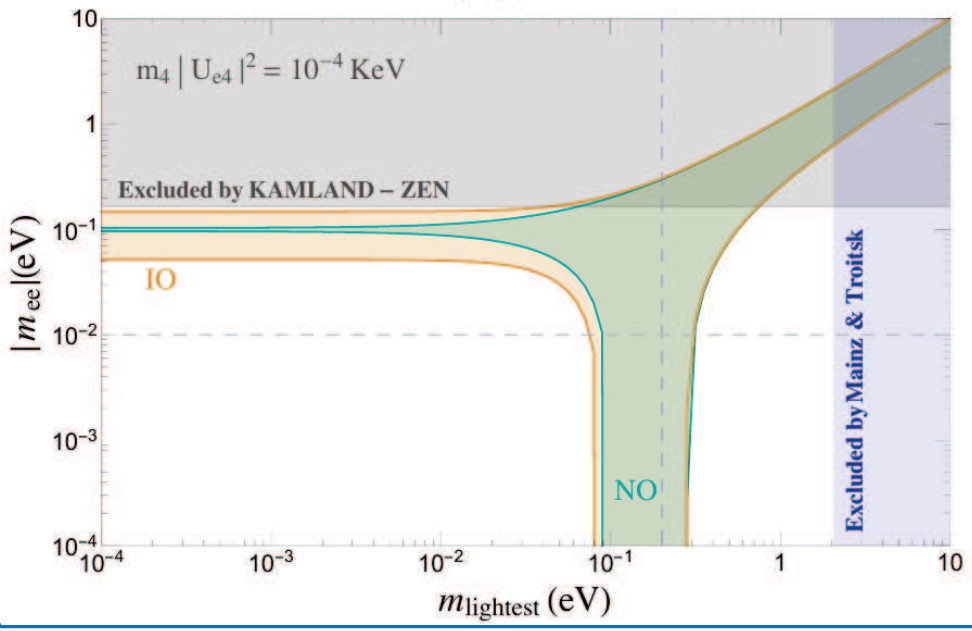


Lindner, Merle, Rodejohann (2006)

Light Sterile Neutrinos – Interplay 0νββ & KATRIN







- possible kink @ KATRIN would imply that IO and NO might **not be distinguishable** anymore with 0νββ
- **Observation** of 0νββ would not necessarily imply IO
- **Non-observation** would not rule out IO due to cancellations for large enough $m_4 U^2_{e4}$

Abada, Hernandez-Cabezudo, Marcano (2019)